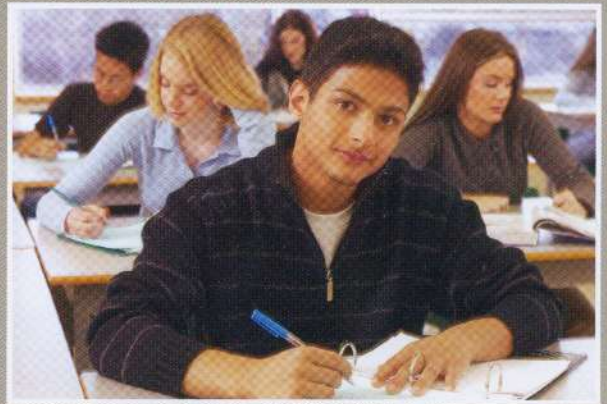


# Physics 12

## UNIVERSITY PREPARATION (SPH4U)

# THE KEY

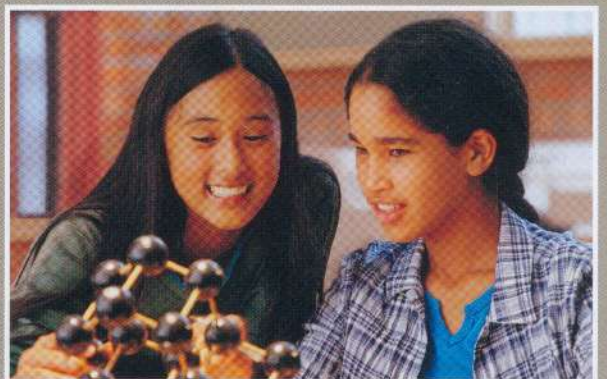
## STUDY GUIDE



- ▶ 100% aligned with the Ontario curriculum
- ▶ Includes unit reviews, practice questions, and tests
- ▶ Contains answers, explanations, and detailed solutions
- ▶ Complements classroom instruction
- ▶ Reviewed by respected Ontario educators

**Castle Rock**  
Research Corp  
[www.castlerockresearch.com](http://www.castlerockresearch.com)

Ontario  
Edition



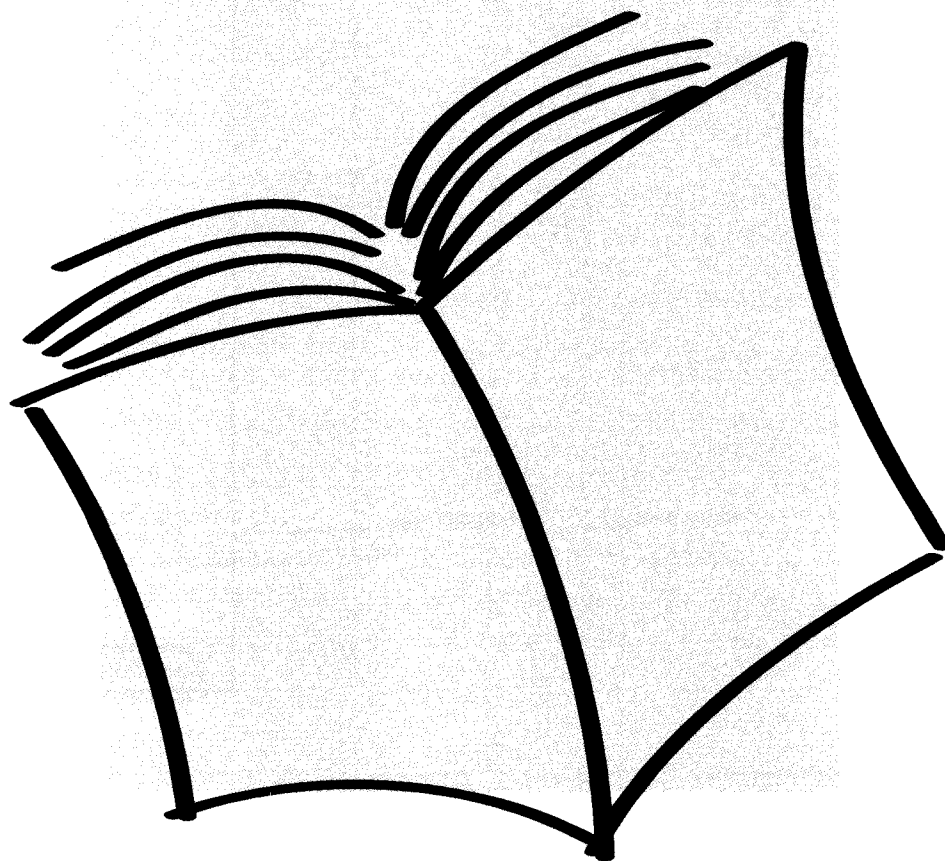
# THE KEY

## STUDENT STUDY GUIDE

**THE KEY** student study guide is designed to help students achieve success in school. The content in each study guide is 100% curriculum aligned and serves as an excellent source of material for review and practice. To create this book, teachers, curriculum specialists, and assessment experts have worked closely to develop the instructional pieces that explain each of the key concepts for the course. The practice questions and sample tests have detailed solutions that show problem-solving methods, highlight concepts that are likely to be tested, and point out potential sources of errors. **THE KEY** is a complete guide to be used by students throughout the school year for reviewing and understanding course content, and to prepare for assessments.

**Physics 12**

University Preparation (SPH4U)



Published 2008  
Copyright © 1999–2008 Castle Rock Research Corporation

All rights reserved. No part of this book covered by the copyright hereon may be reproduced or used in any form or by any means graphic, electronic, or mechanical, including photocopying, recording, taping, or information storage and retrieval systems without the express permission of the publisher.

Some material has been reproduced with the permission from Alberta Education, and any reproduction from this booklet is strictly prohibited, unless authorization is obtained from Alberta Education and the publishers of this document.

Rao, Gautam, 1961 –  
**THE KEY** – Physics 12 University Preparation (2009 Edition)  
Ontario

1. Physics – Juvenile Literature. I. Title

Castle Rock Research Corporation  
2340 Manulife Place  
10180 – 101 Street  
Edmonton, AB T5J 3S4

1 2 3 FP 10 09 08

Printed in Canada

**Publisher**

Gautam Rao

**Contributors**

Lloyd Barker  
Dave Gargus  
Powell Jones  
Kati Kovacs  
Rob Schultz  
Steve Sprysak  
Venkat Sundaram

**Castle Rock**  
Research Corp

*Dedicated to the memory of Dr. V. S. Rao*

## **THE KEY—Physics 12 University Preparation**

**THE KEY** consists of the following sections:

**KEY Tips for Being Successful at School** gives examples of study and review strategies. It includes information about learning styles, study schedules, and note taking for test preparation.

**Class Focus** includes a unit on each area of the curriculum. Units are divided into sections, each focusing on one of the specific expectations, or main ideas, that students must learn about in that unit. Examples, definitions, and visuals help to explain each main idea. Practice questions on the main ideas are also included. At the end of each unit is a test on the important ideas covered. The practice questions and unit tests help students identify areas they know and those they need to study more. They can also be used as preparation for tests and quizzes. Most questions are of average difficulty, though some are easy and some are hard—the harder questions are called *Challenger Questions*. Each unit is prefaced by a **Table of Correlations**, which correlates questions in the unit (and in the practice tests at the end of the book) to the specific curriculum expectations. Answers and solutions are found at the end of each unit.

**KEY Strategies for Success on Tests** helps students get ready for tests. It shows students different types of questions they might see, word clues to look for when reading them, and hints for answering them.

**Practice Tests** includes one to three tests based on the entire course. They are very similar to the format and level of difficulty that students may encounter on final tests. In some regions, these tests may be reprinted versions of official tests, or reflect the same difficulty levels and formats as official versions. This gives students the chance to practice using real-world examples. Answers and complete solutions are provided at the end of the section.

*For the complete curriculum document (including specific expectations along with examples and sample problems),*  
<http://www.edu.gov.on.ca/eng/curriculum/secondary/science1112curr.pdf>.

**THE KEY Study Guides** are available for many courses. Check [www.castlerockresearch.com](http://www.castlerockresearch.com) for a complete listing of books available for your area.

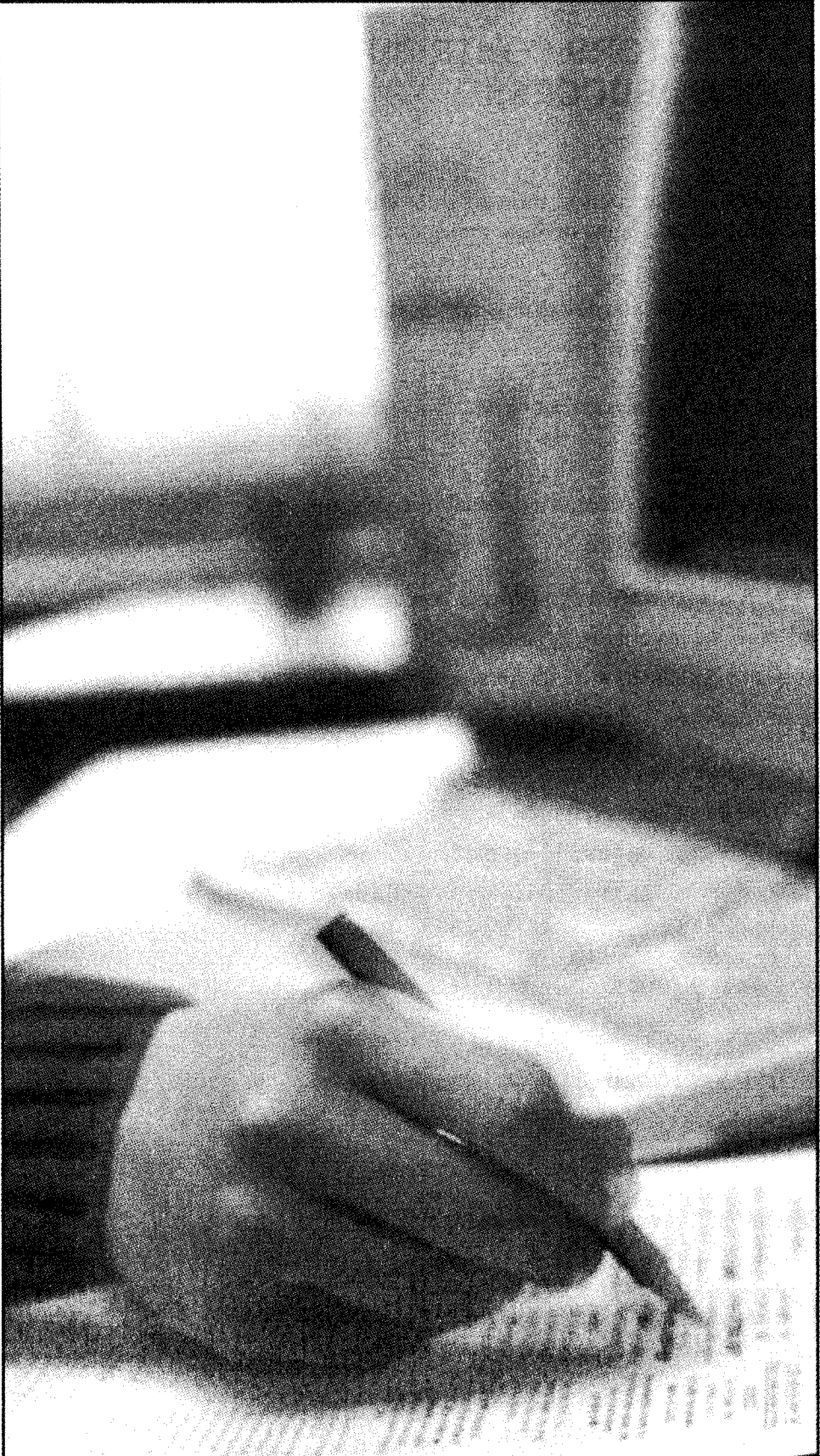
For information about any of our resources or services, please call Castle Rock Research at 905.625.3332 or visit our website at <http://www.castlerockresearch.com>.

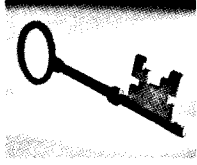
*At Castle Rock Research, we strive to produce an error-free resource. If you should find an error, please contact us so that future editions can be corrected.*

# CONTENTS

<b>KEY TIPS FOR SUCCESS AT SCHOOL .....</b>	<b>1</b>
 <b>CLASS FOCUS</b>	
Forces and Motion .....	9
Solutions–Forces and Motion .....	32
Unit Test .....	39
Solutions–Unit Test .....	45
Energy and Motion .....	51
Solutions–Energy and Motion .....	75
Unit Test .....	79
Solutions–Unit Test .....	85
Electric, Gravitational, and Magnetic Fields .....	89
Solutions–Electric, Gravitational, and Magnetic Fields .....	105
Unit Test .....	109
Solutions–Unit Test .....	114
The Wave Nature of Light .....	117
Solutions–The Wave Nature of Light .....	137
Unit Test .....	141
Solutions–Unit Test .....	145
Matter-Energy Interface .....	147
Solutions–Matter-Energy Interface .....	170
Unit Test .....	175
Solutions–Unit Test .....	182
 <b>KEY STRATEGIES FOR SUCCESS ON TESTS .....</b>	 <b>187</b>
 <b>PRACTICE TEST .....</b>	 <b>193</b>
Solutions–Practice Test .....	209
 <b>APPENDICES .....</b>	 <b>217</b>

# Success at School

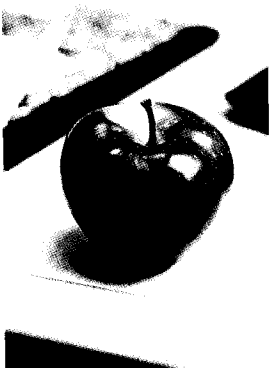




## KEY FACTORS CONTRIBUTING TO SCHOOL SUCCESS

In addition to learning the contents of your courses, there are some other things that you can do to help you do your best at school. Some of these strategies are listed below.

- **ATTEND SCHOOL REGULARLY** so you do not miss any classes, notes, or important activities that will help you learn.
- **KEEP A POSITIVE ATTITUDE.** Always reflect on what you can already do and what you already know.
- **BE PREPARED TO LEARN.** Have the necessary materials (pencils, pens, notebooks, and other required materials) with you in class.
- **COMPLETE ALL OF YOUR ASSIGNMENTS.** Do your best to finish all of your assignments. Even if you know the material well, practice will reinforce your knowledge. If an assignment or question is difficult for you, work through it as far as you can so your teacher can see exactly where you are having difficulty.
- **SET SMALL GOALS** for yourself when you are learning new material. For example, when learning formulas, do not try to learn everything in one night. Work on only one formula each study session. When you understand one particular formula and have memorized it, move on to another one. Continue this process until you have learned and memorized all of the required formulas.
- **REVIEW YOUR CLASSROOM WORK** regularly at home to be sure you understand the material you learned in class.
- **ASK YOUR TEACHER FOR HELP** when you do not understand something or when you are having difficulty completing your assignments.
- **GET PLENTY OF REST AND EXERCISE.** Concentrating in class is hard work. It is important to be well-rested and have time to relax and socialize with your friends. This helps you to keep a positive attitude about your school work.
- **EAT HEALTHY MEALS.** A balanced diet keeps you healthy and gives you the energy you need for studying at school and at home.



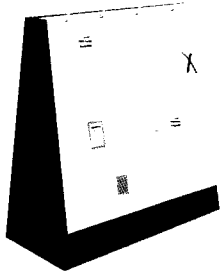
## HOW TO FIND YOUR LEARNING STYLE

Every student has a certain manner in which it seems easier for him or her to learn. The manner in which you learn best is called your learning style. By knowing your learning style, you can increase your success at school. Most students use a combination of learning styles.

Do you know what type of learner you are? Read the following descriptions. Which of these common learning styles do you use most often?

- **Do you need to say things out loud?** You may learn best by saying, hearing, and seeing words. You are probably really good at memorizing dates, places, names, and facts. To learn the steps in a process, a formula, or the actions that lead up to a significant event, you may need **to write them and then read them out loud.**
- **Do you need to read or see things?** You may learn best by looking at and working with pictures. You are probably really good at puzzles, imagining things, and reading maps and charts. You may need to use strategies like **mind mapping and webbing** to organize your information and study notes.
- **Do you need to draw or write things down?** You may learn best by touching, moving, and figuring things out using manipulatives. You are probably really good at physical activities and learning through movement. You may need to **draw your finger over a diagram** to remember it, **tap out the steps** needed to solve a problem, or **feel yourself writing** or typing a formula.

## SCHEDULING STUDY TIME



You should review your class notes regularly to be sure you have a clear understanding of all the new material you learned. Reviewing your lessons on a regular basis helps you to learn and remember ideas and concepts. It also reduces the quantity of material you need to study prior to a test. Creating a study schedule will help you to make the best use of your time.

Regardless of the type of study schedule you use, you may want to consider the following strategies for making the most of your study time and effort:

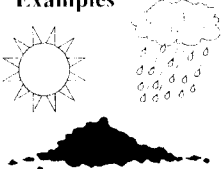

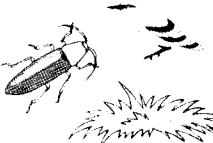
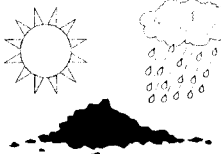
- Organize your work so you begin with the most challenging material first.
- Divide the subject content into small, manageable chunks.
- Alternate regularly between your different subjects and types of study activities in order to maintain your interest and motivation.
- Make a daily list with the headings *must do*, *should do*, and *could do*.
- Begin each study session by quickly reviewing what you studied the day before.
- Maintain your usual routine of eating, sleeping, and exercising to help you concentrate better for extended periods of time.

## CREATING STUDY NOTES



### MIND-MAPPING OR WEBBING

- Use the key words, ideas, or concepts from your class notes to create a *mind map* or *mind web*, which is a diagram or visual representation of the given information. A mind map or web is sometimes referred to as a *knowledge map*.
- Write the key word, concept, theory, or formula in the centre of your page.
- Write down related facts, ideas, events, and information and then link them to the central concept.
- The following examples of a Frayer Model illustrate how this technique can be used to study scientific vocabulary.

<p><b>Definition</b> - Non-living components of an ecosystem</p>	<p><b>Notes</b> The abiotic factors of an area determine biotic factors that can live there</p>	<p><b>Definition</b> Living components of an ecosystem</p>	<p><b>Notes</b> - The types of living things that can live in an area depends on the ability of the abiotic factors to meet their needs</p>
<p><b>Abiotic</b></p>		<p><b>Biotic</b></p>	
<p><b>Examples</b></p> 	<p><b>Non-examples</b></p> 	<p><b>Examples</b></p> 	<p><b>Non-examples</b></p> 

**INDEX CARDS**

To use index cards while studying, follow these steps:

- Write a key word or question on one side of an index card.
- On the other side, write the definition of the word, answer to the question, or any other important information you want to remember.

**What is the difference  
between heat and  
thermal energy?**

**What is the difference between  
heat and thermal energy?**

Thermal energy is the total energy of the particles in a solid, liquid, or gas. Heat is the amount of the thermal energy that is transferred between two objects.

**SYMBOLS AND STICKY NOTES—IDENTIFYING IMPORTANT INFORMATION**

- Use symbols to mark your class notes. For example, an exclamation mark (!) might be used to point out something that must be learned well because it is a very important idea. A question mark (?) may highlight something you are not certain about, and a diamond (◇) or asterisk (\*) could mark interesting information you want to remember.
- Use sticky notes to mark a page in a book that contains an important diagram, formula, or explanation.

## KEY STRATEGIES FOR REVIEWING



Reviewing textbook material, class notes, and handouts should be an ongoing activity. Spending time reviewing becomes more critical when you are preparing for tests. You may find some of the following review strategies useful when studying during your scheduled study time.

- Before reviewing a unit, note the headings, charts, graphs, and chapter questions.
- Highlight mathematical key concepts, vocabulary, definitions, and formulas.
- Carefully read over each step in a procedure.
- Draw a picture or diagram to help make the concept clearer.

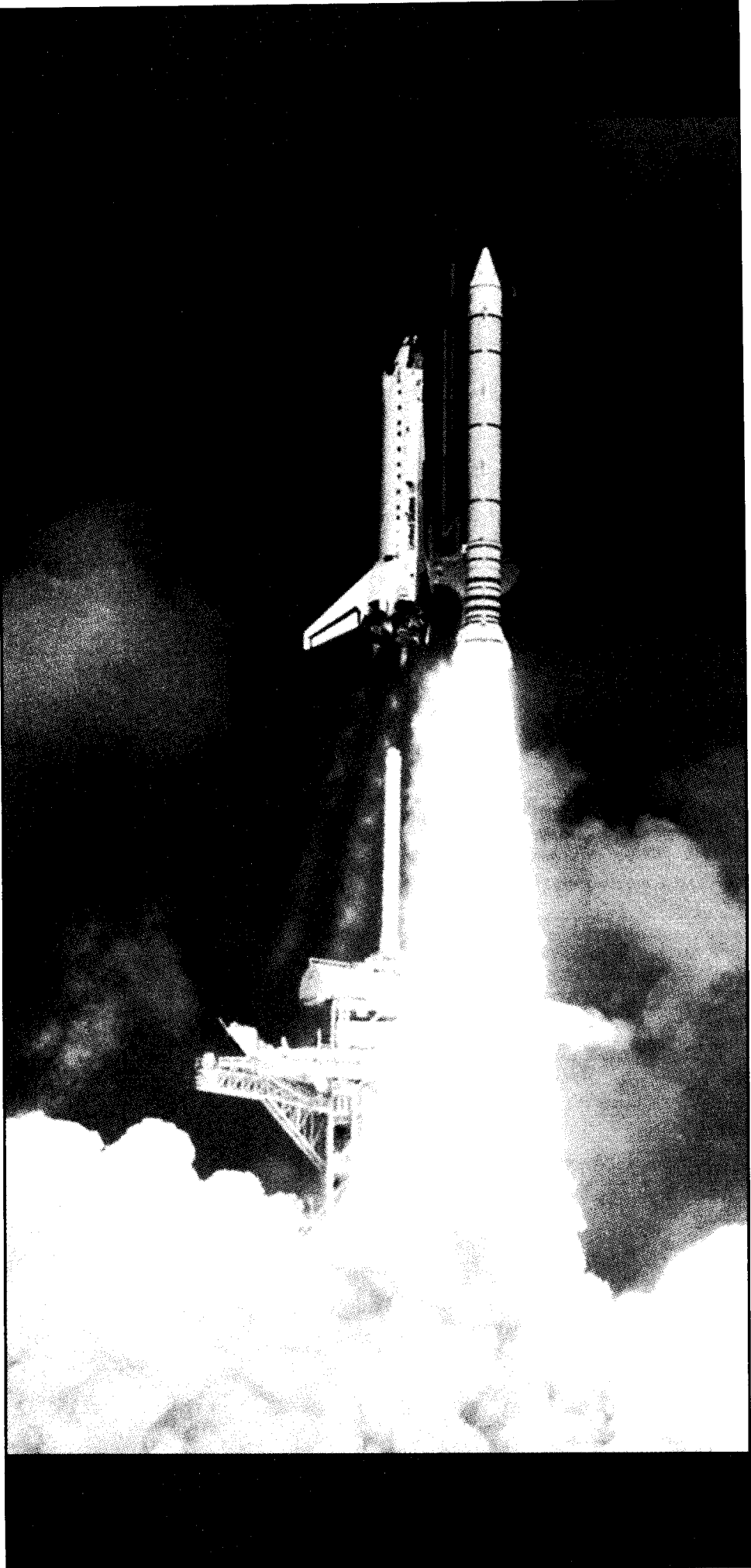
## KEY STRATEGIES FOR SUCCESS—A CHECKLIST

*Review, review, review:* that is a huge part of doing well at school and preparing for tests. Below is a checklist for you to keep track of how many suggested strategies for success you use. Read each question and then put a check mark (✓) in the correct column. Look at the questions for which you have checked the *No* column. Think about how you might try using some of these strategies to help you do your best at school.

<b>KEY Strategies for Success</b>	<b>Yes</b>	<b>No</b>
Do you attend school regularly?		
Do you know your personal learning style—how you learn best?		
Do you spend 15 to 30 minutes each day reviewing your notes?		
Do you study in a quiet place at home?		
Do you clearly mark the most important ideas in your study notes?		
Do you use sticky notes to mark texts and research books?		
Do you practice answering multiple-choice and written-response questions?		
Do you ask your teacher for help when you need it?		
Do you maintain a healthy diet and sleep routine?		
Do you participate in regular physical activity?		



# Forces and Motion





# Forces and Motion

## Table of Correlations

Specific Expectation	Practice Questions	Unit Test Questions
<b>12.1.1</b> Understanding Basic Concepts		
<b>12.1.1.1</b> <i>define and describe the concepts and units related to dynamics</i>	1, 2	1
<b>12.1.1.3</b> <i>analyse and predict, in quantitative terms, and explain the motion of a projectile with respect to the horizontal and vertical components of its motion</i>	9, 10, 11, 12, 13	5, 6
<b>12.1.1.4</b> <i>analyse and predict, in quantitative terms, and explain uniform circular motion in the horizontal and vertical planes with reference to the forces involved</i>	14, 15, 16, 17, 18	7, 8, 9
<b>12.1.1.5</b> <i>distinguish between inertial and accelerating (non-inertial) frames of reference, and predict velocity and acceleration in a variety of situations</i>	19	10
<b>12.1.1.6</b> <i>describe Newton's law of universal gravitation, apply it quantitatively, and use it to explain planetary and satellite motion</i>	20, 21, 22, 23	11, 12, 13
<b>12.1.1.2</b> <i>analyse and predict, in quantitative terms, and explain the linear motion of objects in horizontal, vertical, and inclined planes</i>	3, 4, 5, 6, 7, 8	2, 3, 4
<b>12.1.3</b> Relating Science to Technology, Society and the Environment		
<b>12.1.3.1</b> <i>describe, or construct prototypes, of technologies based on the concepts and principles related to projectile and circular motion</i>	32	20a, 20b
<b>12.1.3.2</b> <i>analyse the principles of dynamics and describe, with reference to these principles, how the motion of human beings, objects, and vehicles can be modified</i>	33	21
<b>12.1.2</b> Developing Skills of Inquiry and Communication		
<b>12.1.2.1</b> <i>analyse experimental data, using vectors, graphs, trigonometry, and the resolution of vectors into perpendicular components, to determine the net force acting on an object and its resulting motion</i>	24, 25, 26	14, 15
<b>12.1.2.3</b> <i>predict the motion of an object, and then design and conduct an experiment to test the prediction</i>	29	17
<b>12.1.2.4</b> <i>investigate, through experimentation, the relationships among centripetal acceleration, radius of orbit, and the period and frequency of an object in uniform circular motion; analyse the relationships in quantitative terms; and display the relationships using a graph</i>	30, 31	18, 19
<b>12.1.2.2</b> <i>carry out experiments or simulations involving objects moving in two dimensions, and analyse and display the data in an appropriate form</i>	27, 28a, 28b	16



**12.1.1.1** define and describe the concepts and units related to dynamics

**VECTOR FORCES**

A force is any push or a pull on an object. A force is a vector, which means that it has both magnitude and direction. Forces affect motion cumulatively, so a resulting motion is caused by a net force.

A net force is the vector sum of all forces acting on an object. If two forces acting on an object are equal and opposite to one another, it is the same as though no force acted on the object at all. If the net force is not equal to zero, then the movement of the object being pushed or pulled will change.

The following list contains examples of different forces:

- friction
- tension (a force exerted through a string or cable)
- normal force (a force exerted by a surface onto an object that pushes on the surface)
- gravitational force

The force of gravity on an object is usually referred to as the object's weight. This force is equal to the mass of the object multiplied by the acceleration due to gravity.

$$\vec{F}_g = m\vec{g}$$

**Example**

What is the mass of an object that weighs 150 N on Earth?

*Solution*

$$\begin{aligned} \vec{F} &= m\vec{g} \\ -150 \text{ N} &= m(-9.81 \text{ m/s}^2) \\ m &= 15.3 \text{ kg} \end{aligned}$$

**Example**

What is the weight of a crate with a mass of 250 kg resting on the floor?

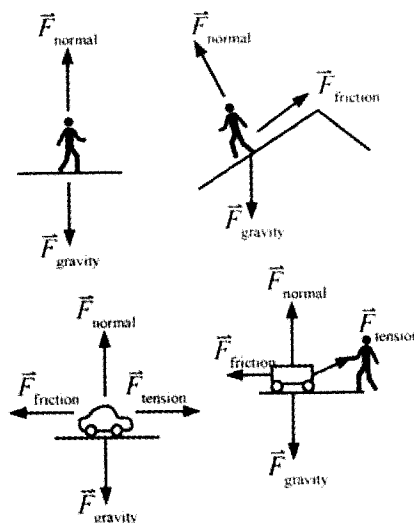
*Solution*

$$\begin{aligned} \vec{F}_g &= m\vec{g} = (250 \text{ kg})(-9.81 \text{ m/s}^2) \\ &= -2453 \text{ N} = -2.45 \text{ kN} \end{aligned}$$

One of the most important tasks when studying motion is to identify all the forces acting on an object.

**Example**

Identify the forces acting on the objects in each of these diagrams.



**Practice**

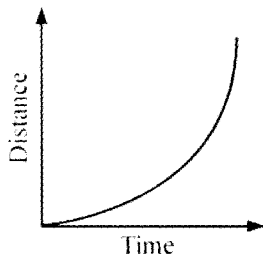
1. The correct SI unit for force is the
 

A. kilogram	B. newton
C. joule	D. dyne



**Open Response**

Use the following information to answer the next question.



An object's motion along a straight path is described qualitatively by the given graph. A student suggests the

motion being described could be any of the following three scenarios:

- a. An object accelerates uniformly in a straight line.
- b. An object travels in a circular horizontal path with a constant speed.
- c. An object drops from a bridge into a river.

2. Which of the student's suggestions are plausible and is the object whose motion is shown moving in an inertial frame of reference? Explain your answer.

---

---

---

---

---

---

---

---

**12.1.1.2** *analyse and predict, in quantitative terms, and explain the linear motion of objects in horizontal, vertical, and inclined planes*

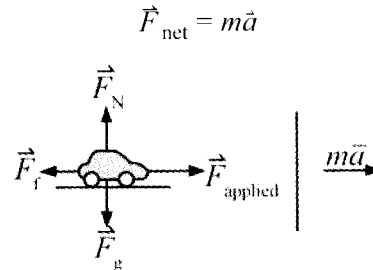
**LINEAR MOTION PROBLEMS**

In order to analyze problems involving forces, draw a free-body diagram. This will help you identify each of the forces involved and the direction of the net force that results. As well, recall that any object on a horizontal surface will have a normal force equal to the object's weight acting upward on it.

**Example**

A person pushes a 50.0-kg model car with a force of 250 N. The car accelerates at 2.00 m/s<sup>2</sup>. Find the force of friction on the car.

First, draw a free-body diagram of the situation. This diagram will show all of the forces acting on the object. It is also a good idea to indicate the direction of the resulting acceleration.



Show the forces coming from the object's centre of gravity.

The vector on the right labelled "mā" is equal to the net force on the object.

In this scenario, the applied force is greater than the friction force because the acceleration is in the direction of the applied force.

$$\begin{aligned} \vec{F}_{applied} - \vec{F}_f &= m\vec{a} \\ \vec{F}_f &= \vec{F}_{applied} - m\vec{a} \\ &= 250 \text{ N} - 50.0 \text{ kg} \times 2.00 \text{ m/s}^2 \\ &= 150 \text{ N} \end{aligned}$$

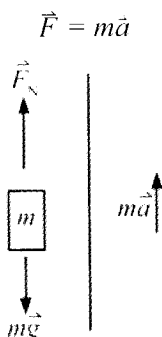
Therefore, the force of friction on the car is 150 N



### Example

Jasmine, who weighs 60 kg, enters an elevator to travel to the tenth floor in a building. As the elevator starts upward, Jasmine feels as though she is heavier for a moment. If Jasmine's apparent weight is 620 N, find the acceleration of the elevator.

First, draw the free-body diagram:



Notice that the normal force acting on Jasmine from the floor has to push harder upward than the force of gravity pulls downward in order for her to travel upward. This causes the net upward acceleration.

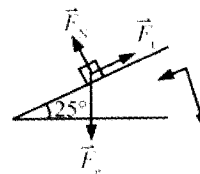
Your perception of weight is related to the force of gravity. However, you do not feel the force of gravity because it is balanced by the normal force exerted by the surface you stand on. Instead, you will sense any force that either prevents gravity from accelerating your body or causes your body to accelerate against gravity. For example, if someone jumps off a diving board, they would feel weightless from the time they left the board until they reached the water.

**Apparent weight** describes the sensation of heaviness that you feel. In this case, the apparent weight is equal to the normal force.

$$\begin{aligned} \vec{F}_N - m\vec{g} &= m\vec{a} \Rightarrow \vec{a} = \frac{\vec{F}_N - m\vec{g}}{m} \\ &= \frac{620 \text{ N} - 60.0 \text{ kg} \times 9.81 \text{ m/s}^2}{60.0 \text{ kg}} \\ &= 0.52 \text{ m/s}^2 \text{ upward.} \end{aligned}$$

### Example

Draw a free-body diagram for a 35-kg crate sliding with a uniform velocity on a ramp with a 25° incline and an unknown coefficient of friction,  $\mu_s$ . Find the coefficient of sliding friction.



The gravitational force has components perpendicular to and parallel to the ramp.

The perpendicular component of the gravitational force pushes the surface of the crate to the surface of the ramp. Therefore, it is equal to the normal force.

$$\begin{aligned} \vec{F}_N &= m\vec{g} \cos 25^\circ \\ &= 311 \text{ N} \end{aligned}$$

The component of gravity parallel to the ramp is the force that causes the object to slide down the ramp. If the force of friction is equal to this force, the object will not accelerate as it slides down the ramp.

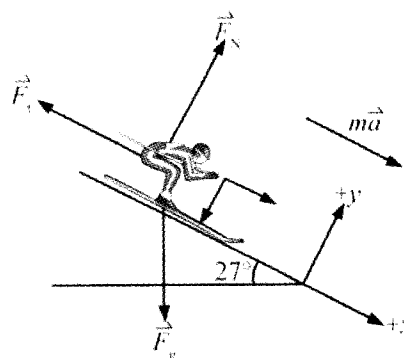
$$\begin{aligned} \vec{F}_f &= m\vec{g} \sin 25^\circ \\ &= 145 \text{ N} \end{aligned}$$

Use the formula for the friction force to find the ramp's coefficient of friction.

$$\vec{F}_f = \mu \vec{F}_N \Rightarrow \mu = \frac{\vec{F}_f}{\vec{F}_N} = \frac{145 \text{ N}}{311 \text{ N}} = 0.47$$

### Example

A 75-kg skier skis down a slope at an angle of 27°. If the coefficient of friction is 0.10, what is the skier's acceleration down the hill?





The component of gravity parallel to the slope is greater than the force of friction, so the acceleration is parallel to the slope.

$$\vec{F}_{g \text{ parallel}} - \vec{F}_f = m\vec{a}$$

$$\vec{a} = \frac{m\vec{g}\sin 27^\circ - \mu m\vec{g}\cos 27^\circ}{m}$$

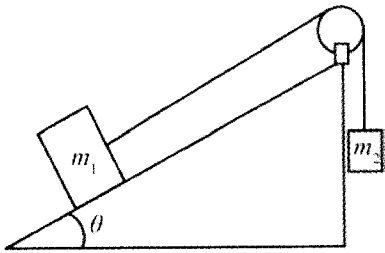
$$\vec{a} = \frac{75 \text{ kg} \times 9.81 \text{ m/s}^2 \times \sin 27^\circ}{75 \text{ kg}} - \frac{0.10 \times 75 \text{ kg} \times 9.81 \text{ m/s}^2 \times \cos 27^\circ}{75 \text{ kg}}$$

$$= 3.6 \text{ m/s}^2$$

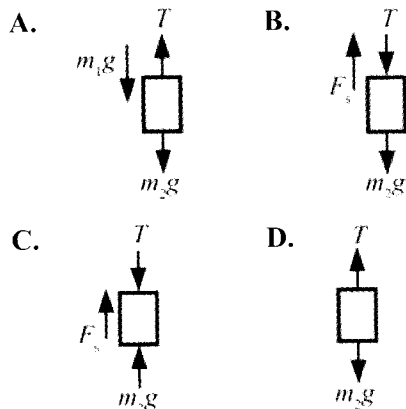
### Practice

Use the following information to answer the next question.

Two masses,  $m_1$  and  $m_2$ , are connected by a string that passes over a smooth pulley at the end of an incline. Mass  $m_1$  lies on a smooth incline and mass  $m_2$  hangs vertically.



3. If  $T$  is tension,  $F_s$  is the force of friction, and  $\vec{g}$  is the acceleration due to gravity, then which of the following diagrams **best** represents the free-body diagram for the forces acting on  $m_2$ ?

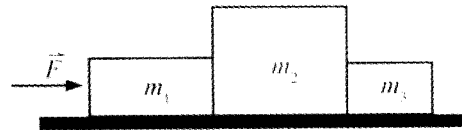


4. An elevator moves upward with an acceleration of  $2\vec{g}$  from the ground to the eighth floor of a building. The normal force,  $\vec{F}_N$ , exerted by the floor on a person of mass  $m$  is equivalent to
- A.  $3m\vec{g}$                       B.  $2m\vec{g}$   
C.  $m\vec{g}$                          D. 0

### Numerical Response

Use the following information to answer the next question.

A 75.0-N force is exerted on a group of three blocks resting on a frictionless surface. The blocks have masses  $m_1 = 10 \text{ kg}$ ,  $m_2 = 20 \text{ kg}$ , and  $m_3 = 8.0 \text{ kg}$ .



5. Calculate the force block  $m_2$  exerts on block  $m_3$ . \_\_\_\_ N

### Numerical Response

Use the following information to answer the next question.

Joe pushes a 45.0-kg dishwasher up a ramp into the back of his moving truck. The incline between the ramp and the ground forms an angle of  $33.0^\circ$ . The coefficient of friction between the dishwasher and the ramp is 0.200 and the dishwasher travels up the incline with a uniform velocity.

6. If Joe pushes with a force parallel to the incline, what is the magnitude of the force, in newtons, with which Joe pushes the dishwasher? \_\_\_\_ N

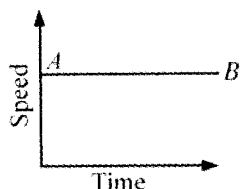


### Numerical Response

7. A student pushes a 12.5-kg box along a flat surface. If the force of friction acting on the box is 21.7 N, find the coefficient of friction for the surface. \_\_\_\_

### Open Response

Use the following information to answer the next question.



8. What can be concluded about the motion of the object in the given graph?

---



---



---



---

**12.1.1.3** *analyse and predict, in quantitative terms, and explain the motion of a projectile with respect to the horizontal and vertical components of its motion*

## TWO-DIMENSIONAL MOTION

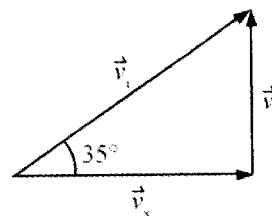
When an object is thrown into the air at slow to moderate speeds, the object's motion will make an arc through the air, similar in shape to a parabola. The motion is the result of a constant horizontal speed and a vertical speed accelerated at  $-9.81 \text{ m/s}^2$ .

This parabolic motion can be analyzed by dividing the motion into separate horizontal and vertical components. The time of the motion in the air is the same for both components.

### Example

A football is kicked from the ground at an angle of  $35^\circ$  with the horizontal at an initial velocity of  $25.0 \text{ m/s}$ .

1. Find the vertical and horizontal velocities.



**Vertical:**

$$\begin{aligned} \vec{v}_y &= \vec{v}_i \sin 35^\circ \\ &= 25.0 \text{ m/s} \sin 35^\circ \\ &= 14.3 \text{ m/s} \end{aligned}$$

**Horizontal:**

$$\begin{aligned} \vec{v}_x &= \vec{v}_i \cos 35^\circ \\ &= 25.0 \text{ m/s} \cos 35^\circ \\ &= 20.5 \text{ m/s} \end{aligned}$$

2. Find the time the ball was in the air.

Vertical:  $\vec{v}_{iy} = 14.3 \text{ m/s}$  (from previous calculation)

$\vec{a} = -9.81 \text{ m/s}^2$  (acceleration due to gravity)

$\vec{v}_y\text{---peak} = 0$  (when the football reaches its highest point)

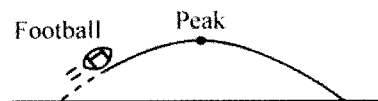
Therefore,  $\vec{v}_y\text{---peak} = \vec{v}_{iy} + \vec{a}t$

$$0 = 14.3 \text{ m/s} + (-9.81 \text{ m/s}^2)(t)$$

$$14.3 = 9.81t$$

$$t = 1.46 \text{ s}$$

This is the time for the ball to reach its peak from the ground. This is half of the total flight of the ball.



The total time of the flight is  $2(1.46) \text{ s} = 2.92 \text{ s}$ .

3. Find the maximum height of the ball.

$$\vec{d} = \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

$$= (14.3 \text{ m/s})(1.46 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(1.46 \text{ s})^2$$

$$= 10.4 \text{ m high}$$

4. Find the displacement

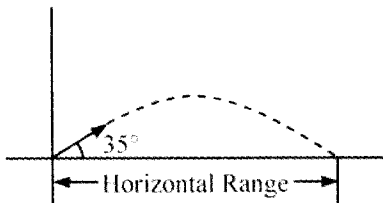


The distance from the initial point to the maximum height as the ball is rising is the same as the distance from the maximum height to the ground as the ball is falling. Therefore, the vertical displacement is zero. The horizontal displacement is:

$$\vec{d} = \bar{v}t = (20.5 \text{ m/s})(2.92 \text{ s}) = 59.9 \text{ m}$$

The total displacement is the vertical displacement plus the horizontal displacement.  
 $0 \text{ m} + 59.9 \text{ m} = 59.9 \text{ m}$

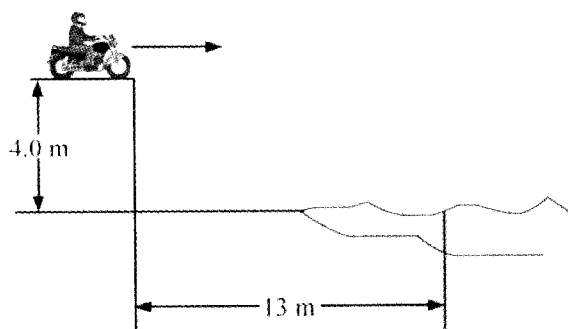
5. Find the range of the ball.



$$\begin{aligned}\vec{d} &= \bar{v}t \\ &= (20.5 \text{ m/s})(2.92 \text{ s}) \\ &= 59.9 \text{ m is the total range}\end{aligned}$$

### Example

A stunt driver wants to drive a motorcycle off a 4.0 m high parkade and into a lake. The stunt driver needs a horizontal range of 13 m in order to land in water deep enough to keep him safe. With what speed must the motorcycle leave the parkade?



Time in the air: The initial vertical velocity is zero since the motorcycle moves horizontally.

$$\begin{aligned}\vec{d} &= \bar{v}_i t + \frac{1}{2} \bar{a} t^2 \Rightarrow t = \sqrt{\frac{2\vec{d}}{\bar{a}}} \\ &= \sqrt{\frac{2 \times -40 \text{ m}}{-9.81 \text{ m/s}^2}} \\ &= 0.903 \text{ s}\end{aligned}$$

The horizontal velocity needed is

$$\begin{aligned}\bar{v} &= \frac{\vec{d}}{t} \\ &= \frac{13 \text{ m}}{0.903 \text{ s}} \\ &= 14.4 \text{ m/s}\end{aligned}$$

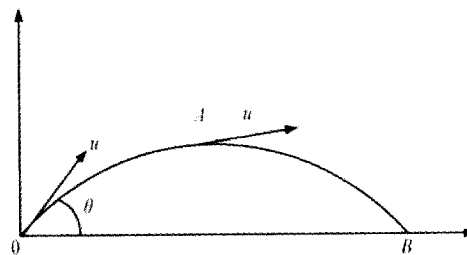
Since the rider needs to achieve at least 14.4 m/s, round the speed up instead of down when rounding for significant digits. This will ensure that the driver achieves a safe speed. Therefore, the stunt driver should leave the parkade at 15 m/s to be safe.

### Practice

9. Projectile motion is **best** described as
- two independent linear motions
  - two dependant linear motions
  - horizontal motion
  - vertical motion

Use the following information to answer the next question.

The given figure represents the motion of an object launched into the air.



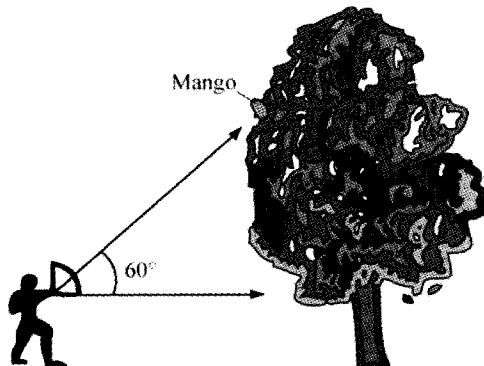
10. Which of the following expressions represents the horizontal component of the velocity of the object at any point on the curve in the given figure?
- $u \cos \theta$
  - $u \cot \theta$
  - $u \sin \theta$
  - $u \tan \theta$



**Numerical Response**

Use the following information to answer the next question.

Annemarie shot an arrow in the direction of an mango hanging from the branch of a tree with an initial velocity of 24 m/s. The arrow made an angle with the horizontal of 60°. The arrow reached the mango in 1.5 s.



11. The final velocity of the arrow as it reached the mango, expressed in scientific notation to three significant figure, is  $b \times 10^w$  m/s. What is the value of  $b$ ? \_\_\_\_ m/s

**Numerical Response**

Use the following information to answer the next question.

A stone is thrown horizontally with a speed of 8.00 m/s from the top of a 113 m high cliff.

12. The time that it takes the stone to reach the bottom of the cliff is \_\_\_\_ s.  
(Round and record your answer to three digits.)

**Open Response**

Use the following information to answer the next question.

Troy was playing with a ball on the balcony outside his apartment. He rolled the ball with a velocity of 5 m/s across the balcony. However, the ball accidentally rolled over the edge of the balcony. The balcony was 20 m above the ground.

13. What was the speed of the ball when it reached the ground?

---



---



---



---



---



---



**12.1.1.4** *analyse and predict, in quantitative terms, and explain uniform circular motion in the horizontal and vertical planes with reference to the forces involved*

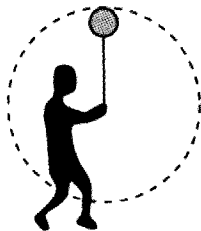
## UNIFORM CIRCULAR MOTION

A centripetal force is not a specific type of force. Instead, it is any force that acts to direct motion into a circular path. Many types of forces can be centripetal forces:

- Gravitational forces: When an object moves in a vertical circle, gravity, or some component of gravity, is part of the centripetal force.
- Normal forces: When an object travels in a vertical circle while in contact with a surface, the normal force is part of the centripetal force. For example, when a roller coaster travels in a vertical loop, the normal force from the rails is part of the centripetal force.
- Friction forces: When an object travels in a circular path while in contact with a surface, the friction between the object and the surface is part of the centripetal force. For example, when a car turns, the friction force from the road surface pushes the car toward the centre of the circular path.
- Tension forces: When an object travels in a circular path while attached by another object to the centre of the circle, the tension force in the object attached to the centre is a centripetal force. For example, the tension in the string causes a yo-yo to travel in a circular path.

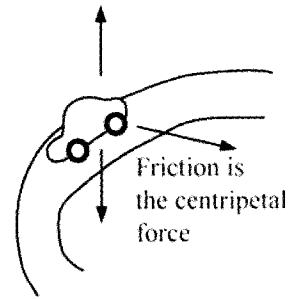
### Example

Identify the centripetal forces in each situation.



The centripetal forces when a yo-yo spins in a vertical circle are the tension in the string and gravity.

The centripetal force when a car turns a corner is the friction between the car's tires and the road.



## HORIZONTAL CIRCLES

### Example

Find the centripetal force required to keep a 1000-kg car moving in a uniform circle with a radius of 50.0 m at a speed of 50.0 km/h.

Convert km/h to m/s:

$$50 \text{ km/h} = \left( \frac{50 \cancel{\text{km}}}{\cancel{\text{h}}} \right) \left( \frac{1000 \text{ m}}{1 \cancel{\text{km}}} \right) \left( \frac{1 \cancel{\text{h}}}{3600 \text{ s}} \right) = 13.89 \text{ m/s}$$

$$\vec{F}_c = m\vec{a}_c$$

$$\vec{F}_c = \frac{m\vec{v}^2}{r}$$

$$\vec{F}_c = (1000 \text{ kg}) \frac{(13.89 \text{ m/s})^2}{50 \text{ m}} = 3.86 \times 10^3 \text{ N}$$



### Example

The centripetal force required to keep a 400-kg motorcycle moving in a uniform circular path of radius 75.0 m is 4000 N. What is the speed of the motorcycle, in km/h?

$$\vec{F}_c = \frac{m\vec{v}^2}{r}$$

$$4000 \text{ N} = (400 \text{ kg})\left(\frac{\vec{v}^2}{75 \text{ m}}\right)$$

$$\vec{v}^2 = 750 \text{ m}^2/\text{s}^2$$

$$\vec{v} = 27.4 \text{ m/s}$$

$$\vec{v} = \left(\frac{27.4 \text{ m}}{1 \text{ s}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)$$

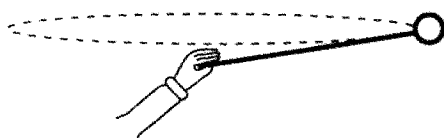
$$= 98.6 \text{ km/h}$$

It must be significant to alter the ball's path in a continuous, circular path.

### Example

A person swings a 100-g ball in a horizontal circle at the end of a string 55.0 cm long.

The string makes 2.50 revolutions in 1.00 s. Find its centripetal acceleration.



The net force on the ball must be non-zero. In this case, the centripetal force is the tension in the string.

$T = \frac{1}{f}$	$\vec{v} = \frac{2\pi r}{T}$
$= \frac{1}{2.5 \text{ s}^{-1}}$	$= \frac{2\pi(0.55 \text{ cm})}{0.40 \text{ s}}$
$= 0.4000 \text{ s}$	$= 8.639 \text{ m/s}$

$$\vec{F}_T = m\vec{a}_c$$

$$\vec{F}_T = \frac{m\vec{v}^2}{r} = \frac{0.100 \text{ kg} \times (8.639 \text{ m/s})^2}{0.55 \text{ m}}$$

$$\vec{F}_T = 13.6 \text{ N}$$

$$\therefore \vec{a}_c = \frac{\vec{F}_T}{m} = \frac{13.6 \text{ N}}{0.100 \text{ kg}} = 136 \text{ m/s}^2$$

This example could also be solved by using the formula for centripetal acceleration after finding the period.

$$\vec{F}_T = m \frac{4\pi^2 r}{T^2}$$

$$= (0.100 \text{ kg})\left(\frac{4 \times 3.14^2 \times 0.55 \text{ m}}{(0.4000 \text{ s})^2}\right)$$

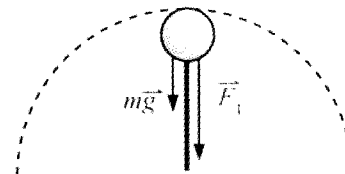
$$\vec{F}_T = 13.6 \text{ N}$$

## VERTICAL CIRCLES

### Example

A person swings a 100-g ball in a vertical circle at the end of a string 55.0 cm long. The string makes 2.50 revolutions in one second. Find the tension in the string at both the upper and lower ends of its arc.

- i. Draw a free-body diagram for the ball at the top of the circle:



The ball accelerates toward the centre of the circle (below the ball). The net force is the sum of the forces of gravity and tension.

$$\vec{F}_T + \vec{F}_g = \vec{F}_{\text{net}}$$

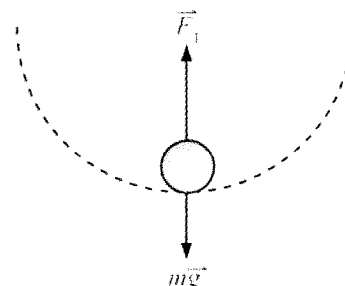
$$\vec{F}_T + m\vec{g} = \frac{4\pi^2 mr}{T^2} \Rightarrow \vec{F}_T = \frac{4\pi^2 mr}{T^2} - m\vec{g}$$

$$\vec{F}_T = \frac{4 \times 3.14^2 \times 0.100 \text{ kg} \times 0.55 \text{ m}}{(0.40 \text{ s})^2}$$

$$- 0.100 \text{ kg} \times 9.81 \text{ m/s}^2$$

$$\vec{F}_T = 12.6 \text{ N}$$

- ii. Now do the same at the bottom of the circle.





The ball accelerates toward the centre of the circle (above the ball). The tension must be larger than the gravitational force. The net force is the difference of the two forces.

$$\vec{F}_T - \vec{F}_g = \vec{F}_{\text{net}}$$

$$\vec{F}_T - m\vec{g} = \frac{4\pi^2 mr}{T^2} \Rightarrow \vec{F}_T = \frac{4\pi^2 mr}{T^2} + m\vec{g}$$

$$\vec{F}_T = \frac{4 \times 3.14^2 \times 0.100 \text{ kg} \times 0.55 \text{ m}}{(0.40 \text{ s})^2}$$

$$+ 0.100 \text{ kg} \times 9.81 \text{ m/s}^2$$

$$\vec{F}_T = 14.5 \text{ N}$$

### Practice

Use the following information to answer the next question.

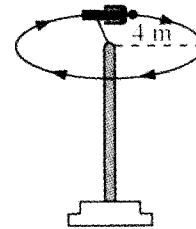
A particular ride at a carnival, called “The Rotor,” is a cylinder that revolves at high speeds. The riders stand with their backs against the inside wall of the cylinder.

When the cylinder reaches a frequency of 0.50 Hz, the floor drops away from the riders’ feet and they are stuck to the wall.

14. If the cylinder has a radius of 5.0 m, the minimum coefficient of friction needed so the riders will stick to the walls when the floor drops is \_\_\_\_.
- A. 0.020                      B. 0.20  
C. 0.50                        D. 5.0

Use the following information to answer the next question.

A magician who weighs 60 kg ties himself to one end of a rope 4.0 m long. He ties the other end of the rope to the top of an iron rod fixed vertically on a platform. The rod slowly starts to rotate about its axis, causing the magician to rotate in a horizontal circular motion as shown. The magician makes  $n$  revolutions per second.



15. Which of the following expressions represents the centripetal force produced in the string?
- A.  $120n^2\pi^2 \text{ N}$   
B.  $240n^2\pi^2 \text{ N}$   
C.  $480n^2\pi^2 \text{ N}$   
D.  $960n^2\pi^2 \text{ N}$

### Numerical Response

16. A 1.50-kg mass is swung in a vertical circle at the end of a 2.10-m long string. The string will break when it experiences a tension force of 160 N. What is the maximum speed with which the mass can swing without breaking the string? \_\_\_\_ m/s

### Numerical Response

17. Pat swings a bucket of water in a vertical circle of radius 0.66 m. What is the minimum velocity of the bucket at the top of the circle required to keep the water in the bucket? \_\_\_\_ m/s



### Open Response

18. A mass swinging in a vertical circle has a speed at the top of the circle of 3.14 m/s. This is the minimum speed needed to keep the mass swinging in a vertical circle. What is the radius of the circular motion?

---



---



---



---



---



---

**12.1.1.5** *distinguish between inertial and accelerating (non-inertial) frames of reference, and predict velocity and acceleration in a variety of situations*

### FRAMES OF REFERENCE

A frame of reference can be considered a coordinate system within the frame that is not moving with respect to an observer also within the frame of reference.

An inertial reference frame is one that has a constant velocity with respect to an observer outside the frame of reference. This velocity can be zero or some magnitude greater than zero. All physical laws are valid within any inertial frame of reference. This means that any physical interaction within an inertial frame of reference with a velocity will be identical to the same interaction occurring within an inertial frame of reference with a velocity of zero.

A non-inertial frame of reference is one that is moving with a changing velocity (i.e., accelerating). Fictitious forces might appear to act in non-inertial frames of reference because of the acceleration of that frame with respect to an observer in an inertial frame of reference. The Coriolis force and the centrifugal force are examples of fictitious forces that are the result of non-inertial frames of reference.

### Practice

19. Paige and Aiden each threw a rock over the edge of a bridge. Paige threw her rock vertically upward while Aiden threw his rock vertically downward. Paige and Aiden threw their rocks with the same speed and they released the rock at the exact same moment. Ignoring air resistance, Paige's rock will reach the water below the bridge with a velocity
- four times greater than the velocity of Aiden's rock
  - two times greater than the velocity of Aiden's rock
  - equal to the velocity of Aiden's rock
  - half the velocity of Aiden's rock

**12.1.1.6** *describe Newton's law of universal gravitation, apply it quantitatively, and use it to explain planetary and satellite motion*

### NEWTON'S LAW OF UNIVERSAL GRAVITATION

There are four fundamental types of forces. The strong and weak forces are involved in holding atoms together. The electromagnetic force acts between charged particles. The gravitational force acts between any two masses.

Isaac Newton tried to explain how two objects exert a force between each other without touching each other. He referred to this phenomenon as "action at a distance." He proposed that this force is directly proportional to the product of the two masses and inversely proportional to the square of the distance between them. This is expressed algebraically as:

$$\vec{F}_g = \frac{Gm_1m_2}{r^2}$$

in which  $G$  = the universal gravitational constant  
 $= 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$

**Example**

Find the force of gravity between a 40-kg person and 50-kg person when they are 2.0 m apart.

$$\begin{aligned}\vec{F}_g &= \frac{Gm_1m_2}{r^2} \\ &= \frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \times 40 \text{ kg} \times 50 \text{ kg}}{(2.0 \text{ m})^2} \\ &= 3.3 \times 10^{-8} \text{ N}\end{aligned}$$

This is an extremely small force that would not be noticed because the force of gravity between Earth and a 40-kg person is much greater.

**Example**

Find the force of gravity between Earth and a 40-kg person.

$$\begin{aligned}\vec{F}_g &= \frac{Gm_1m_2}{r^2} \\ &= \frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \times 40 \text{ kg} \times 5.98 \times 10^{24} \text{ kg}}{(6.37 \times 10^6 \text{ m})^2} \\ &= 393 \text{ N}\end{aligned}$$

In circular motion, the acceleration is directed toward the centre of the circular motion. This is also true for celestial bodies, such as Earth and the moon. Since no physical tie exists between the two bodies, gravity must be the force keeping them together.

**Practice**

Use the following information to answer the next question.

The gravitational field strength between masses  $M_1$  and  $M_2$  is  $F$ .

20. If the distance between masses  $M_1$  and  $M_2$  is doubled, then the gravitational field strength between them will be

- A.  $4F$                       B.  $2F$   
C.  $\frac{F}{2}$                         D.  $\frac{F}{4}$

21. Gravitational force is **not**

- A. an attractive force  
B. universal in nature  
C. directly proportional to the product of two masses  
D. directly proportional to the distance between two masses

**Numerical Response**

22. In the distant future, an interplanetary spaceship from Earth arrives at a planet orbiting the star 51 Pegasi. The spaceship orbits the planet with a uniform speed of  $3.00 \times 10^3$  m/s at a distance of  $7.47 \times 10^7$  m from the centre of the planet. The spaceship's crew calculates the mass of the planet as  $a.bc \times 10^{de}$  kg. The values of  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  in the form  $abcde$  are \_\_\_\_.

**Open Response**

Use the following information to answer the next question.

An artificial satellite orbits above Earth with an orbital radius of  $7.45 \times 10^6$  m.

23. Calculate the speed of the satellite and the period of its orbit.

---

---

---

---

---

---

---

---



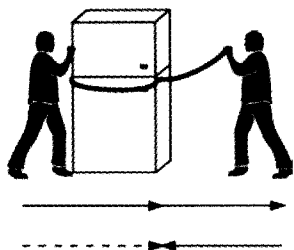
**12.1.2.1** analyse experimental data, using vectors, graphs, trigonometry, and the resolution of vectors into perpendicular components, to determine the net force acting on an object and its resulting motion

## CALCULATING NET FORCE

In order to calculate the net force acting upon an object, a free body diagram containing all of the forces acting upon that object will allow the student to clearly visualize the situation. Given the free body diagram, it is just a matter of adding the applicable vector forces together to determine the net force. Once the net force is acquired, the final motion of the object can be calculated.

### Example

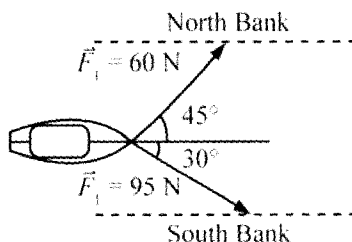
Tom pushes and Fred pulls a fridge to the right against friction. Tom pushes with a force of 150 N and Fred pulls with a force of 120 N. The force of friction between the fridge and the floor is 50 N. Find the net force on the fridge.



$$\vec{F}_{\text{net}} = 150 \text{ N} + 120 \text{ N} - 50 \text{ N} = 220 \text{ N to the right.}$$

### Example

A boat is pulled by two ropes, one from each side of a river. Determine the net force acting on the boat from the following diagram.

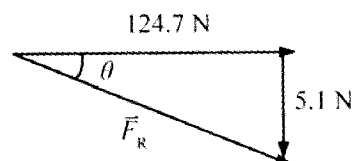


$\vec{F}_1$	$\vec{F}_2$
<b>Horizontal</b>	
$\begin{aligned}\vec{F}_x &= \vec{F}_1 \cos 45^\circ \\ &= 60 \cos 45^\circ \\ &= 42.4 \text{ N}\end{aligned}$	$\begin{aligned}\vec{F}_x &= \vec{F}_2 \cos (-30^\circ) \\ &= 95 \cos (-30^\circ) \\ &= 82.3 \text{ N}\end{aligned}$
<b>Vertical</b>	
$\begin{aligned}\vec{F}_y &= \vec{F}_1 \sin 45^\circ \\ &= 60 \sin 45^\circ \\ &= 42.4 \text{ N}\end{aligned}$	$\begin{aligned}\vec{F}_y &= \vec{F}_2 \sin (-30^\circ) \\ &= 95 \sin (-30^\circ) \\ &= -47.5 \text{ N}\end{aligned}$

### Sum

$$\text{Horizontal} = 42.4 + 82.3 = 124.7 \text{ N east}$$

$$\text{Vertical} = 42.4 - 47.5 = -5.1 \text{ N} = 5.1 \text{ N south}$$



$$\begin{aligned}\vec{F}_R &= \sqrt{(124.7)^2 + (5.1)^2} \\ &= 124.8 \text{ N} = 1.2 \times 10^2 \text{ N}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{5.1}{124.7}\right) \\ &= 2.3^\circ \text{ south of east.}\end{aligned}$$

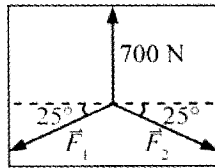
The net force is  $1.2 \times 10^2 \text{ N}$   $2.3^\circ$  S of E.



Practice

Numerical Response

Use the following information to answer the next question.



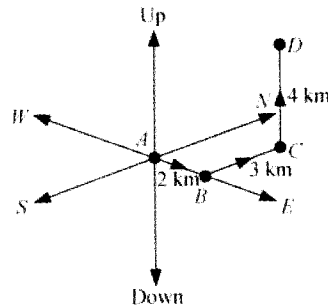
24. A motorboat travelling with a uniform velocity pulls two water-skiers behind it. Each rope makes an angle of 25.0° with respect to the back of the boat. If a force of 700 N propels the boat through the water, the tension in each rope is \_\_\_ N. (Round and record your answer to three significant digits.)

- 25. A vector can be resolved into
A. 2 components
B. 12 components
C. 100 components
D. an infinite number of components

Open Response

Use the following information to answer the next question.

A hiker has driven to a mountain to do some climbing. From the parking lot, point A, he hikes 2 km due east. From that point he turns north and begins to climb. As he climbs he moves 3 km north, while at the same time travelling 4 km vertically. Finally he arrives at point D.



26. Determine the magnitude of the displacement vector connecting points A and D, and determine the angle this vector makes with the horizontal.

Blank lines for writing the answer to question 26.



**12.1.2.2** *carry out experiments or simulations involving objects moving in two dimensions, and analyse and display the data in an appropriate form*

## FORCES AND MOTION

### EXPERIMENTS

Simple experiments can be set up involving two-dimensional motion. Such experiments can be used to calculate a variety of physical constants, such as the acceleration due to gravity. For example, the following experiment can be used to calculate the acceleration due to gravity.

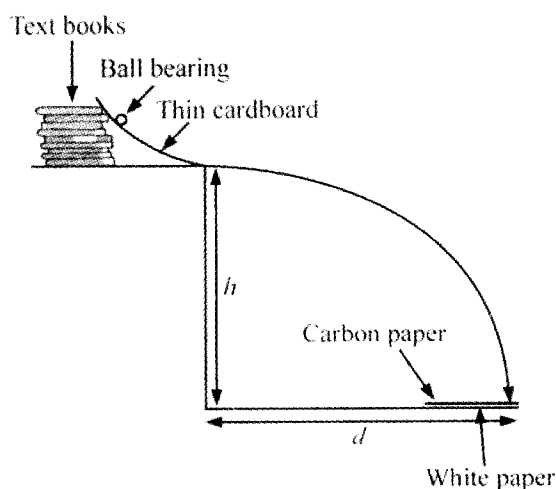
#### Example

##### Apparatus and Materials

- One table
- One stopwatch
- One roll of sticky tape
- One ball bearing or marble
- One sheet of carbon paper
- One thin sheet of cardboard
- One sheet of white drawing paper
- One metre stick or measuring tape
- Two text books of reasonable thickness

### Experimental Setup

1. Stack the two textbooks near the edge of the table.
2. Form a ramp by leaning the piece of cardboard against the textbooks so that the cardboard leads off of the edge of the table. Tape down the underside of the cardboard closest to the table edge and then tape the other end of the cardboard to the textbooks. The ramp should have a slight curve so that when the ball bearing leaves the table it will be projected horizontally.
3. Place the sheet of white paper on the floor beneath the ramp, then place the sheet of carbon paper on top of the white paper. Position the paper approximately where the ball bearing should land after leaving the ramp. Tape the pieces of paper down so that they do not move while you are taking measurements.
4. Mark a point somewhere near the top of the ramp as the release point for the ball bearing. This will ensure that the ball bearing will have a consistent velocity as it leaves the table for each successive trial.





### Experimental Procedure

1. Place the ball bearing at the release point on the ramp and release
2. As the ball bearing leaves the edge of the table start the stopwatch. Stop the stopwatch when the ball strikes the carbon paper. Record this time. (A sample data table is given below)
3. Measure the distance from the edge of the table to the mark on the white paper made by the landing ball bearing. Record this distance.
4. Repeat steps 1–3 for a total of 4 trials.
5. Measure the vertical distance from the edge of the table to the ground and record the value  $h$ .

	Trial 1	Trial 2	Trial 3	Trial 4	avg
$d(m)$					
$t(s)$					

$h =$

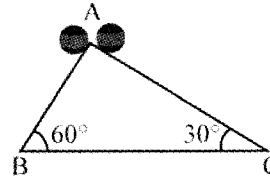
### Analysis

Given the height that the ball bearing fell from and the distance that it travelled horizontally, the following values can be calculated:

- The initial horizontal velocity of the ball as it leaves the ramp
  - The value of the acceleration due to gravity
- 

### Practice

Use the following information to answer the next question.



27. Two identical balls are released from position A: one along the inclined plane AB and the other along inclined plane AC. If the velocity of the ball moving down plane AB is  $v$  at point B, then the velocity of the ball moving down plane AC at point C is

- A.  $\frac{v}{3}$                       B.  $\frac{v}{\sqrt{3}}$
- C.  $\sqrt{3}v$                       D.  $v$

Use the following information to answer the next multipart question.

28. Fred left his 1850 kg car in neutral when he parked it on his driveway. The driveway has an incline of  $7.0^\circ$  from the horizontal, and the force of friction between the driveway surface and his car is 950 N.

Part A

### Open Response

Draw a free-body diagram to illustrate this scenario.

---



---



---



---



---



Part B

**Open Response**

Calculate the components for the force of gravity perpendicular and parallel to the driveway.

---

---

---

---

---

---

---

---

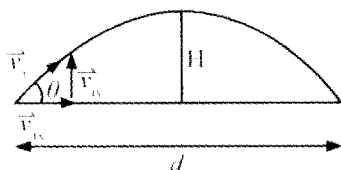
**12.1.2.3** *predict the motion of an object, and then design and conduct an experiment to test the prediction*

$$\vec{v}_{iy} = \vec{v}_i \cos \theta$$

**KINEMATIC EXPERIMENTS**

**Example**

A student wants to examine projectile motion experimentally. Using the formulae for kinematics, the student can derive equations for several values such as the maximum height that a projectile will reach ( $H$ ), the total time of flight ( $t$ ), and the total horizontal displacement of the projectile ( $d$ ).



$$\vec{v}_{ix} = \vec{v}_i \sin \theta = \frac{d}{t} \therefore \vec{v}_i = \frac{d}{t \sin \theta}$$

This formula will calculate the magnitude of the initial velocity of the projectile.

$$\Rightarrow d = (v_i)(t) \sin \theta$$

This formula will calculate the total horizontal distance that the projectile will travel.

$$\vec{v}_{iy} = \vec{v}_i \cos \theta$$

$$d = \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

$$d_{y \text{ max}} = H = \vec{v}_{iy} t + \frac{1}{2} \vec{g} t^2$$

This formula will calculate the maximum height of the projectile.

$$\vec{v}_f = \vec{v}_i + \vec{a} t$$

$$\vec{v}_{fy} = \vec{v}_{iy} + \vec{g} t$$

$$\therefore t = \frac{\vec{v}_{fy} - \vec{v}_{iy}}{g}$$

This formula will calculate the time that it took the projectile to reach its maximum height. When the projectile reaches the maximum height, the projectile will have a velocity of zero ( $\vec{v}_{fy} = 0$ ).

$$t = -\frac{\vec{v}_{iy}}{g}$$

To calculate the total time of the projectile's flight, multiply this value by 2.

$$t_{\text{flight}} = -\frac{2\vec{v}_{iy}}{g}$$

From these derived equations, you can compare the expected values with experimentally determined values by measuring the angle that the projectile was launched, the total horizontal distance travelled, and the maximum height reached.

**Practice**

29. A projectile is projected into the air at an angle  $\theta$  from the horizontal, and it has a velocity of  $u$  at point  $P$  in its path. What will be the horizontal and vertical components of the projectile's velocity at any particular instant?
- A.  $u \cos \theta, u \sin \theta$
  - B.  $u \cos \theta, u \sin (90 - \theta)$
  - C.  $u \cos (180 + \theta), u \sin (45 + \theta)$
  - D.  $u \cos (90 + \theta), u \sin (180 - \theta)$



**12.1.2.4** *investigate, through experimentation, the relationships among centripetal acceleration, radius of orbit, and the period and frequency of an object in uniform circular motion; analyse the relationships in quantitative terms; and display the relationships using a graph*

## CIRCULAR MOTION

Newton's first law states that an object will stay in motion unless acted upon by an outside force. This law demonstrates that an object in motion will continue moving straight in its current direction unless redirected by some force. In circular motion, the direction of the movement is always changing, both horizontally and vertically. Therefore, any object moving in a circle is continually accelerating, even if its speed is not changing.

The direction of the acceleration is toward the centre of the circle. The term centripetal means "directed toward the centre," so the acceleration of an object travelling in a circle is called centripetal acceleration. When a force acts on an object at a right angle to the object's velocity, and that force continues to act on that object at a right angle to the changing velocity, the object will travel along a circular path. The force is directed toward the centre of the circle, so it is a centripetal force.

The magnitude of the centripetal acceleration is found with the formula  $\vec{a}_c = \frac{\vec{v}^2}{r}$ .

### Example

Find the centripetal acceleration of a car turning in a circle with a radius of 12 m and at a speed of 10 m/s.

$$\vec{a}_c = \frac{\vec{v}^2}{r} = \frac{(10 \text{ m/s})^2}{12 \text{ m}} = 8.3 \text{ m/s}^2$$


---

## ORBITAL FREQUENCY AND PERIOD

The speed of the moving object is not always known for circular motion. Instead, the period of time that it takes for the object to travel around the circle once can be measured. This period of time is known as the period ( $T$ ) of the circular motion.

Remember that speed is the distance travelled divided by time. Therefore, the following formulae can be developed for circular motion:

$$\vec{v} = \frac{\vec{d}}{t} = \frac{2\pi r}{T} \qquad \vec{a}_c = \frac{4\pi^2 r}{T^2}$$

If an object travelling in circular motion is moving too fast, you might only be able to measure the frequency ( $f$ ) of the motion, the number of revolutions per second.

$$f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f}$$

### Example

A person swings a ball on the end of a string 40 cm long. Given that the period of the revolution is  $\frac{1}{3}$  s/rev, find the frequency and velocity of the ball's motion.

$$\begin{aligned} T &= \frac{1}{f} &= \frac{2\pi r}{T} \\ f &= \frac{1}{T} &= \frac{2\pi(0.40 \text{ cm})}{\frac{1}{3} \text{ s}} \\ &= \frac{1}{\frac{1}{3} \text{ s}} &= 7.5 \text{ m/s} \\ &= 3 \text{ rev/s} \end{aligned}$$

The unit rev/s is called a hertz (Hz).

---



### Example

A yo-yo swings on the end of a string. The frequency of the circular motion is 4.00 Hz, and the radius of the motion is 0.800 m. Find the centripetal acceleration of the yo-yo.

$$T = \frac{1}{f} = \frac{1}{4.00 \text{ Hz}} = 0.250 \text{ s}$$

$$\vec{a}_c = \frac{4\pi^2 r}{T^2} = \frac{4 \times 3.14^2 \times 0.800 \text{ m}}{(0.250 \text{ s})^2} = 505 \text{ m/s}^2$$

### Practice

30. What is the relationship between the linear acceleration,  $\vec{a}$ , and angular acceleration,  $\alpha$ , of a particle with respect to the radius of a circular path,  $r$ ?

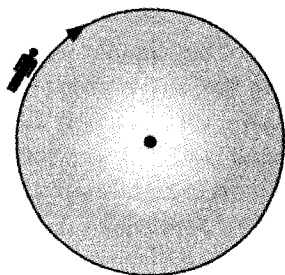
A.  $\vec{a} = \frac{r}{\alpha}$       B.  $\vec{a} = \frac{1}{\alpha \times r}$

C.  $\vec{a} = \alpha \times r$       D.  $\vec{a} = \frac{\alpha}{r - 1}$

### Numerical Response

Use the following information to answer the next question.

A child moves uniformly around a circular swimming pool that has a radius of 10 m. She completes 5.0 revolutions in 4.0 min.



31. What is the child's centripetal acceleration? \_\_\_\_ m/s<sup>2</sup>

12.1.3.1 describe, or construct prototypes, of technologies based on the concepts and principles related to projectile and circular motion

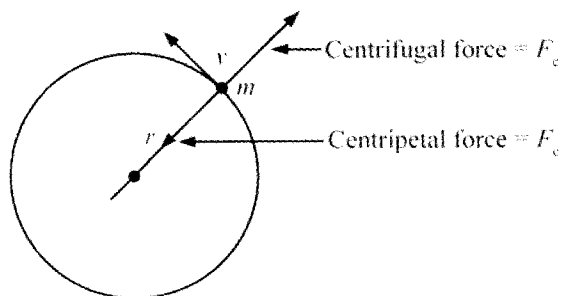
### TECHNOLOGIES BASED ON CIRCULAR MOTION

Astronauts use a centrifuge when they are in training to learn how their bodies will respond to the high gravitational forces involved in a space launch. An astronaut trainee will sit inside of a capsule located at the end of a long mechanical arm. The astronaut will sit with his back to the outside of the circular path the capsule takes. The mechanical arm then rotates the capsule at high velocities to simulate the launch force of the space shuttle.

The simulated launch force is the centripetal force on the astronaut. However, the astronaut experiences a centrifugal force that simulates the gravitational force that will pull the astronaut against the launching force.



The centrifugal force is a pseudo-force, because it does not actually exist. It appears to be an actual force to a person within the rotating reference frame of the centrifuge because they experience linear forces as viewed from an inertial frame of reference outside of the rotating system. It is simply the result of the centripetal force acting upon the rotating reference frame in order to keep it from continuing along a linear path. The acceleration that is applied to the rotating body will cause the fixed observer to experience the pseudo force directed the opposite direction to the centripetal force. This is a result of the rotating body (i.e., the astronaut) having some quantity of inertia which wants to continue moving in a straight line.



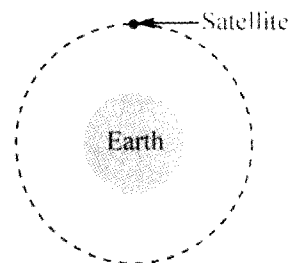
$$F_c = \frac{mv^2}{r}$$

centripetal force = radially inward  
centrifugal force = radially outward

The concept of the centrifuge is also used in medical science to separate the components of blood. Since the centrifugal force is related to the mass of the spinning object, the length of the rotating arm, and the angular velocity of the centrifuge, the components of the blood will experience forces of different magnitudes depending on the components' densities. This allows doctors to separate the white blood cells from the red blood cells and the plasma within the blood.

### Practice

Use the following information to answer the next question.



A satellite orbiting Earth does not fall to Earth's surface because the satellite travels with a velocity that balances out the centripetal force.

32. Which of the following forces acts on the satellite as the centripetal force?
- Gravitational force
  - Electrostatic force
  - Centrifugal force
  - Magnetic force

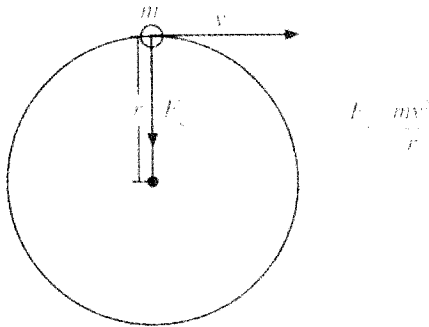
**12.1.3.2** *analyse the principles of dynamics and describe, with reference to these principles, how the motion of human beings, objects, and vehicles can be modified*

## DYNAMICAL PRINCIPLES OF THROWING A BASEBALL

The physics of throwing a baseball can be described as the conversion of the angular velocity of circular motion into linear velocity after the ball is released. As a baseball player throws a ball, the player grips the ball in the hand while the arm rotates the ball in circular motion at the end of the arm. This generates the angular velocity with which the ball moves while travelling around the arc formed by the player's arm.



When the player releases the ball from the hand, there is no longer any centripetal force holding the ball in the circular path. According to Newton's second law of motion, an object will continue travelling at a constant velocity unless acted upon by an external unbalanced force. Therefore, when the ball is released, the angular velocity becomes linear velocity in the direction of the tangent from the point the ball was released. After the ball is released, it then acts according to projectile motion depending on the direction and magnitude of the resultant velocity.



This model is a simplified explanation of all of the physics involved in throwing a ball. It neglects the relative motion of the elbow and the wrist, as well as the role of air friction, but it is a good model for explaining how velocity is generated. In order for a pitcher to increase the velocity of a pitch, the pitcher needs to increase the angular velocity of the ball before it is released. This can be accomplished by extending the length of the rotating arm by keeping the elbow straight, or by using more force to accelerate the ball around the same radius with a larger angular velocity.

**Practice**

33. A road has a banking angle of  $10^\circ$  at a circular bend, which has a radius of 10 m. At what speed should a vehicle travel around the bend so the normal contact force provides the necessary centripetal force to the vehicle ( $g = 9.81 \text{ m/s}^2$ )?
- A. 17.3 m/s      B. 9.9 m/s  
 C. 4.2 m/s      D. 1.3 m/s

**SOLUTIONS—FORCES AND MOTION**

1. B	8. OR	15. D	22. 1, 0, 1, 2, 5	Part B- OR
2. OR	9. A	16. 14.3	23. OR	29. A
3. D	10. A	17. 7.97	24. 828	30. C
4. A	11. 1.34	18. OR	25. D	31. 0.17
5. 15.8	12. 4.80	19. C	26. OR	32. A
6. 314	13. OR	20. D	27. D	33. C
7. 0.18	14. B	21. D	28. Part A- OR	

**1. B**

The newton is defined as the force that will cause a 1 kg mass to accelerate at 1 m/s<sup>2</sup>.

**2. Open Response**

The given graph shows a smooth curve. This suggests that the object's motion undergoes uniform acceleration. An object accelerating uniformly while moving in a straight line or falling from a bridge are both scenarios of uniform acceleration. Therefore, suggestions a. and c. are both plausible.

Suggestion b. is not plausible. The distance of the object from the centre of the circular path does not change with time. Only the direction of the motion changes, so the object undergoes uniform acceleration. Therefore, scenario b. is not a scenario for which the motion could be described by the given graph.

Since the frame is an accelerated frame of reference, it is non-inertial. In order to be an inertial frame of reference, the frame must be at rest or travelling at a constant velocity.

**3. D**

Two forces act on  $m_2$  in the given scenario:

- gravity ( $m_2\vec{g}$ ) acts vertically downward
  - tension in the string  $T$  acts vertically upward
- It can be assumed that there is no frictional force because the pulley and incline are both smooth. Therefore, the free-body diagram of  $m_2$  is

**4. A**

$$\begin{aligned} \text{During upward motion,} \\ \vec{F}_N &= m(\vec{g} + \vec{a}) \\ \vec{F}_N &= m(\vec{g} + 2\vec{g}) = 3m\vec{g} \end{aligned}$$

**5. 15.8**

The acceleration of the entire system is given by the formula

$$\vec{F} = m\vec{a}$$

In this case the system acts as a single mass with a magnitude of 38 kg.

The acceleration of the entire system is then

$$\vec{a} = \frac{\vec{F}}{m} = \frac{75.0 \text{ N}}{38 \text{ kg}} = 1.97 \text{ m/s}^2$$

Since the entire system accelerates at this rate, the force that block  $m_2$  exerts on block  $m_3$  can be calculated as follows:

$$\vec{F} = m\vec{a} = 8.0 \text{ kg} \times 1.97 \text{ m/s}^2 = 15.8 \text{ N}$$

**6. 314**

$$\begin{aligned} \vec{F}_g &= m\vec{g} \\ &= (45.0 \text{ kg})(9.81 \text{ m/s}^2) \\ &= 441 \text{ N} \\ \vec{F}_f &= \mu\vec{F}_g \cos \theta \\ &= (0.200)(441 \text{ N})(\cos 33.0^\circ) \\ &= 74.0 \text{ N} \\ (\vec{F}_g)_x &= \vec{F}_g \sin \theta \\ &= (441 \text{ N})(\sin 33.0^\circ) \\ &= 240 \text{ N} \\ \vec{T} &= \vec{F}_f + (\vec{F}_g)_x \\ &= 74 \text{ N} + 240 \text{ N} \\ &= 314 \text{ N} \end{aligned}$$

**7. 0.18**

$$\begin{aligned} \vec{F}_g &= \vec{F}_N \\ \vec{F}_N &= mg \\ &= (12.5 \text{ kg})(9.81 \text{ m/s}^2) \\ &= 123 \text{ N} \\ \vec{F}_f &= \mu\vec{F}_N \\ \mu &= \frac{\vec{F}_f}{\vec{F}_N} \\ &= \frac{21.7 \text{ N}}{123 \text{ N}} \\ &= 0.18 \end{aligned}$$



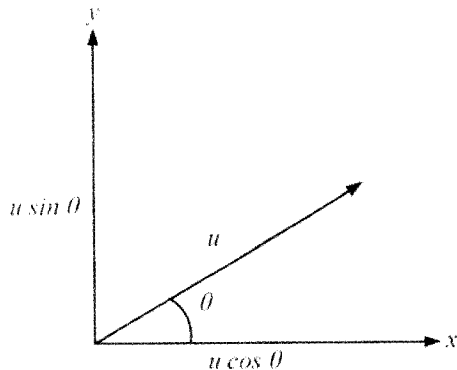
### 8. Open Response

The object is moving in uniform motion. It has moved from point *A* to point *B*, so it has displacement, and the speed is constant between the two points.

### 9. A

Projectile motion consists of a uniform horizontal motion and an accelerated vertical motion. Therefore, projectile motion is best described as two independent linear motions.

### 10. A



The velocity  $u$  of the object at any point along its path can be reduced into its two components—the horizontal and vertical components. The vertical component of the velocity is given by  $u \sin \theta$  while the horizontal component is given by  $u \cos \theta$ .

### 11. 1.34

$$\vec{v}_{\text{net}} = \sqrt{\vec{v}_{f_x}^2 + \vec{v}_{f_y}^2}$$

$$\vec{v}_{f_x} = \vec{v}_{i_x} + \vec{a}t$$

$$\vec{v}_{f_x} = \vec{v} \cos 60^\circ + 0 \times t = 24 \text{ m/s} \times \frac{1}{2}$$

$$\vec{v}_{f_x} = 12 \text{ m/s}$$

(Since the acceleration in the horizontal direction is 0)

$$\vec{v}_{f_y} = \vec{v}_{i_y} - \vec{a}_g t = \vec{v} \sin 60^\circ - gt$$

$$= (24 \text{ m/s}) \times \frac{\sqrt{3}}{2} - 9.81 \text{ m/s}^2 \times 1.5 \text{ s} = 6.07 \text{ m/s}$$

$$\vec{v}_{\text{net}} = \sqrt{\vec{v}_{f_x}^2 + \vec{v}_{f_y}^2}$$

$$\vec{v}_{\text{net}} = \sqrt{\vec{v}_{f_x}^2 + \vec{v}_{f_y}^2}$$

$$\vec{v}_{\text{net}} = \sqrt{(12 \text{ m/s})^2 + (6.07 \text{ m/s})^2}$$

$$= 13.44 \text{ m/s}$$

$$= 1.34 \times 10^1 \text{ m/s}$$

### 12. 4.80

Vertical motion:

$$\vec{d} = \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

$$\vec{v}_i = 0$$

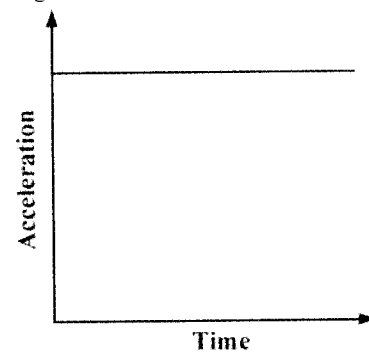
so

$$t = \sqrt{\frac{2\vec{d}}{\vec{a}}} = \sqrt{\frac{2 \times (-113 \text{ m})}{(-9.81 \text{ m/s}^2)}}$$

$$t = 4.80 \text{ s}$$

### 13. Open Response

The horizontal and vertical components of projectile motion are independent of one another. In the given case, the ball was initially moving on a horizontal surface with a horizontal velocity of 5 m/s. The vertical component of the ball's initial velocity was zero. Therefore, time for the ball to fall from the roof to the ground is the same as the time for a ball to reach the ground if it were dropped from the same height.



$$(\vec{v}_{y_f})^2 = (\vec{v}_{y_i})^2 + 2(\vec{g})d_y$$

$$2(9.8)20 \text{ m}^2/\text{s}^2 = (\vec{v}_{y_f})^2$$

$$\vec{v}_{y_f} = 19.8 \text{ m/s}$$

$$(\vec{v}_f)^2 = (\vec{v}_{y_f})^2 + (\vec{v}_{x_f})^2$$

$$(\vec{v}_f)^2 = (19.7 \text{ m/s})^2 + (5 \text{ m/s})^2$$

$$\vec{v}_f = 20.3 \text{ m/s} \approx 20 \text{ m/s}$$

Therefore, the speed of the ball when it reached the ground was approximately 20 m/s.

### 14. B

The normal force from the wall is the net centripetal force here so it is necessary to calculate the force due to centripetal motion.

First it is necessary to calculate the centripetal acceleration in order to calculate the normal force on a body within this amusement ride.

It is known that

$$a_c = \frac{v^2}{r}, \text{ and } v = \frac{2\pi r}{T} = 2\pi r f.$$

$$\therefore a_c = 4\pi^2 r f^2 = 4\pi^2 (5.0 \text{ m}) (0.5 \text{ s}^{-1})^2$$

$$a_c = 49.3 \text{ m/s}^2$$



The force of friction must equal the force of gravity so a person does not slide down the wall.

$$\vec{F}_N = m\vec{a}_c = 75 \text{ kg} \times 49.3 \text{ m/s}^2$$

$$= 3698 \text{ N}$$

$$\vec{F}_f = m\vec{g} = 75 \text{ kg} \times 9.81 \text{ m/s}^2$$

$$= 736 \text{ N}$$

$$\vec{F}_f = \mu \vec{F}_N$$

$$\mu = \frac{\vec{F}_f}{\vec{F}_N} = \frac{736 \text{ N}}{3698 \text{ N}} = 0.20$$

**15. D**

Since the magician moves in a horizontal plane, his weight is ignored.

Let  $m$  be the magician's mass,  $v$  be his velocity,  $r$  be the radius of his circular path, and  $F_c$  be the centripetal force in the string.

$$v = (\text{circumference})(n) \text{ m/s}$$

$$\therefore v = 8\pi n \text{ m/s}$$

$$F_c = \frac{mv^2}{r}$$

$$\frac{m(8\pi n \text{ m/s})^2}{r}$$

$$\frac{(60 \text{ kg})(8\pi n \text{ m/s})^2}{4.0 \text{ m}}$$

$$= 960n^2\pi^2 \text{ N}$$

The centripetal force produced in the string is

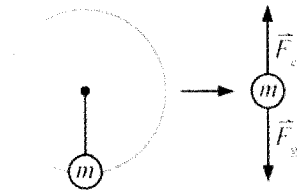
$$960n^2\pi^2 \text{ N.}$$

**16. 14.3**

$$\vec{F}_g = m\vec{g}$$

$$(1.50 \text{ kg})(-9.81 \text{ m/s}^2)$$

$$= -14.7 \text{ N}$$



$$\vec{F}_c = \vec{T} + \vec{F}_g$$

$$= (160 \text{ N}) + (-14.7 \text{ N})$$

$$= 145.3 \text{ N}$$

$$F_c = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{F_c r}{m}}$$

$$= \sqrt{\frac{(145.3 \text{ N})(2.10 \text{ m})}{1.50 \text{ kg}}}$$

$$= 14.3 \text{ m/s}$$

**17. 7.97**

$$\vec{F}_T = 0$$

$$\vec{F}_c = \vec{F}_g$$

$$\frac{m\vec{v}^2}{r} = m\vec{g}$$

$$\vec{v} = \sqrt{r\vec{g}}$$

$$= \sqrt{(0.66 \text{ m})(9.81 \text{ m/s}^2)}$$

$$= 7.97 \text{ m/s}$$

**18. Open Response**

$$\vec{v} = \sqrt{r\vec{g}}$$

$$r = \frac{\vec{v}^2}{g}$$

$$\frac{(3.14 \text{ m/s})^2}{9.81 \text{ m/s}^2}$$

$$= 1.01 \text{ m}$$

**19. C**

Both rocks would arrive at the surface of the water with the same velocity because the acceleration on both would be the same. Paige's rock will rise to a maximum height, where the velocity of the rock would equal zero. The rock will then fall from the maximum height and return to the height from which Paige threw her rock with the same speed it was thrown into the air. However, the velocity would be directed downward, so its velocity at that height would then be equal to the velocity of Aiden's rock at that same height. Therefore, the velocities of both rocks as they reached the water surface would be equal.

**20. D**

According to Newton's universal law of gravitation, the gravitational force between two bodies is given by the relation

$$F = \frac{GM_1M_2}{r^2}$$

This relation implies that the gravitational force is inversely proportional to the square of the distance between the two bodies. Therefore, when the distance between the bodies is doubled, the gravitational field strength between them will be

$$F = \frac{GM_1M_2}{(2r)^2} = \frac{GM_1M_2}{4r^2} = \frac{1}{4} \frac{GM_1M_2}{r^2} = \frac{F}{4}$$

**21. D**

The universal law of gravitation implies that the gravitational force is inversely proportional to the square of the distance between two masses, not directly proportional.

The law of gravitation is a universal law that applies to any two bodies possessing mass. The universal law of gravitation states that an attractive force exists between two bodies of masses  $m_1$  and  $m_2$  that are separated by a distance  $r$ . The magnitude of this force is given as

$$F = \frac{Gm_1m_2}{r^2}$$

**22. 1, 0, 1, 2, 5**

$$\bar{v} = \sqrt{\frac{Gm_p}{r}}$$

$$\bar{v}^2 = \frac{Gm_p}{r}$$

$$m_p = \frac{r\bar{v}^2}{G}$$

$$\begin{aligned} m_p &= \frac{(7.47 \times 10^7 \text{ m})(3.00 \times 10^3 \text{ m/s})^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} \\ &= 1.01 \times 10^{25} \text{ kg} \end{aligned}$$

**23. Open Response**

To calculate the speed of the satellite, use  $\bar{v} = \sqrt{\frac{Gm_c}{r}}$

$$\begin{aligned} \bar{v} &= \sqrt{\frac{Gm_c}{r}} \\ &= \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{7.45 \times 10^6 \text{ m}}} \\ &= 5.35 \times 10^7 \text{ m/s} \end{aligned}$$

To calculate the period of the satellite's orbit, use Kepler's

$$\text{third law } T = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{Gm_c}}$$

$$T = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{Gm_c}}$$

$$\begin{aligned} &= \frac{2\pi(7.45 \times 10^6 \text{ m})^{\frac{3}{2}}}{\sqrt{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}} \\ &= 6.40 \times 10^3 \text{ s} \end{aligned}$$

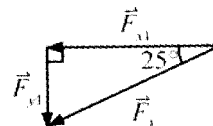
Therefore, the speed of the orbiting satellite is  $5.35 \times 10^7$  m/s and the period of its orbit is  $6.40 \times 10^3$  s.

**24. 828**

$\vec{F}_{\text{net}} = 0$  because  $\vec{a} = 0$  (constant velocity)

- i. The components of  $\vec{F}_1$  and  $\vec{F}_2$  have equal magnitudes. Use the Pythagorean theorem to determine the component magnitudes of  $\vec{F}_1$ .

These magnitudes will be equal to the magnitudes of  $\vec{F}_2$ . The  $y$ -components will be parallel to the motion of the boat.



- ii. The  $y$ -components must add up to 700 N. The  $y$ -component can be calculated using  $\sin 25^\circ$  in this case.

$$\vec{F}_1 \sin 25.0^\circ + \vec{F}_2 \sin 25.0^\circ = 700 \text{ N}$$

$$2\vec{F}_1 \sin 25.0^\circ = 700 \text{ N}$$

$$\vec{F}_1 = \frac{350 \text{ N}}{\sin 25.0^\circ}$$

$$\vec{F}_1 = 828 \text{ N}$$

$$\vec{F}_2 = \vec{F}_1 = 828 \text{ N}$$

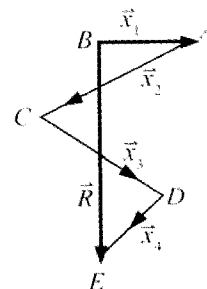
Therefore, the tension for each of the ropes is 828 N.

**25. D**

A vector can be resolved into an infinite number of components. Resolving a vector into its components means to determine a set of vectors that forms a resultant equal to the given vector. Each vector in that set is then a component of the given vector.

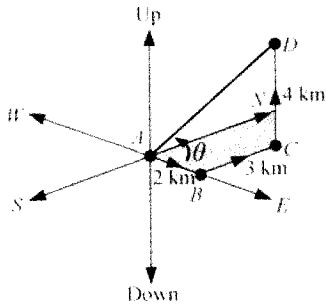
$$\vec{R} = \vec{x}_1 + \vec{x}_2 + \vec{x}_3 + \vec{x}_4$$

$$\therefore \vec{R} = \sum_{i=1}^n \vec{x}_i$$





**26. Open Response**



To determine the magnitude of the resultant displacement vector we need to break this question into right triangles that are perpendicular to each other. Let us label the vertices of the diagram as follows.

First we will determine the magnitude of the vector that points from  $A$  to  $C$  using the Pythagorean Theorem.

$$|\vec{AC}| = \sqrt{2^2 + 3^2}$$

$$|\vec{AC}| = 3.6 \text{ km}$$

Knowing this we can determine the length of the vector joining  $A$  to  $D$ , by applying the Pythagorean Theorem to the right triangle  $ACD$ .

$$|\vec{AD}| = \sqrt{3.6^2 + 4^2}$$

$$|\vec{AD}| = 5.4 \text{ km}$$

This gives us the magnitude of the displacement vector. To determine the angle that this vector makes with the horizontal we only need to use basic trigonometry. We can use the inverse tangent function on the data we calculated for triangle  $ACD$  as follows

$$\theta = \tan^{-1}\left(\frac{4}{3.6}\right)$$

$$\theta = 48^\circ$$

Thus the total displacement vector from  $A$  to  $D$  is 5.4 km at an angle of elevation of  $48^\circ$ .

**27. D**

The first step here is to calculate the component of the acceleration directed down each balls respective slope.

$$a_{AB} = g \sin 60^\circ = \frac{\sqrt{3}g}{2}$$

$$a_{AC} = g \sin 30^\circ = \frac{g}{2}$$

The vertical height that both balls are starting from is  $d$ . From this, it is necessary to calculate the distance down each slope that each ball travels.

$$\sin 60^\circ = \frac{d}{d_{AB}} \Rightarrow d_{AB} = \frac{d}{\sin 60^\circ} = \frac{2d}{\sqrt{3}}$$

$$\sin 30^\circ = \frac{d}{d_{AC}} \Rightarrow d_{AC} = \frac{d}{\sin 30^\circ} = 2d$$

$$\text{Now, } v_f^2 = v_i^2 + 2ad, \text{ but } v_i = 0,$$

$$\therefore v_f^2 = 2ad \Rightarrow v_f = \sqrt{2ad}$$

$$v_{f(AB)} = \sqrt{2a_{AB}d_{AB}} = \sqrt{(2)\left(\frac{\sqrt{3}g}{2}\right)\frac{2d}{\sqrt{3}}}$$

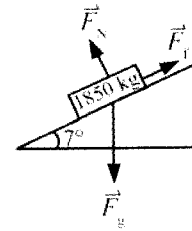
$$v_{f(AB)} = \sqrt{2gd}$$

$$v_{f(AC)} = \sqrt{2a_{AC}d_{AC}} = \sqrt{(2)\left(\frac{g}{2}\right)(2d)}$$

$$v_{f(AC)} = \sqrt{2gd}$$

$$\therefore v_{f(AB)} = v_{f(AC)} = v$$

**28. Part A – Open Response**



**Part B – Open Response**

$$\vec{F}_\perp = mg \cos \theta$$

$$= 1850 \text{ kg} \times 9.81 \text{ m/s}^2 \times \cos 7^\circ$$

$$= 18\,013 \text{ N} = 18 \text{ kN}$$

The perpendicular component for the force of gravity is 18 kN.



$$\vec{F}_\parallel = mg \sin \theta$$

$$= 1850 \text{ kg} \times 9.81 \text{ m/s}^2 \times \sin 7^\circ$$

$$= 2212 \text{ N} = 2.2 \text{ kN}$$

The parallel component for the force of gravity is 2.2 kN.

**29. A**

The velocity  $u$  of any particle at a point  $P$  in its path can be resolved into the horizontal and vertical components. The horizontal component is equal to  $u \cos \theta$ , and the vertical component is equal to  $u \sin \theta$ .

**30. C**

Linear acceleration,  $\vec{a}$ , is the product of angular acceleration,  $\alpha$ , and the radius of a circular path,  $r$ . The direction of the linear acceleration is in the direction of the motion of the particle on its circular path.

$$\vec{a} = \alpha \times r$$

**31. 0.17**

Use the equation for centripetal acceleration.

$$a_c = \frac{v^2}{r} \text{ and } v = \frac{2\pi r}{T}$$

Substitute the given values.

$$\begin{aligned} a_c &= \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} \\ &= \frac{4\pi^2 \times 10 \text{ m}}{(48 \text{ s})^2} = \frac{394.384 \text{ m}}{2304 \text{ s}^2} \\ &= 0.17 \text{ m/s}^2 \end{aligned}$$

**32. A**

When an object moves in a circular path, a force acts on the object directed toward the centre of the circle that constantly adjusts the path of the object around the path. This keeps the object on the circular path. This is called the centripetal force.

When a satellite orbits around Earth, the gravitational force of Earth attracts the satellite and providing it the centripetal force necessary to maintain its circular motion.

Without Earth's gravity, the satellite would move in a straight line.

**33. C**

Let  $\bar{v}$  be the actual speed of the vehicle,  $\theta$  be the banking angle, and let  $r$  be the radius of the bend.

$$\begin{aligned} \tan \theta &= \frac{\bar{v}^2}{rg} \\ \bar{v} &= \sqrt{rg \tan \theta} \\ &= \sqrt{10 \times 9.81 \times \tan 10^\circ} \\ &= 4.2 \text{ m/s} \end{aligned}$$



# Unit Test



1. Which of the following scenarios is **not** an example of an inertial frame of reference?
- A car travels north at 38 km/h.
  - A chair is at rest on a kitchen floor.
  - A hockey puck slides across a smooth ice surface (consider ice surface friction free).
  - A stone falls toward a river after being dropped from a bridge.

*Use the following information to answer the next question.*

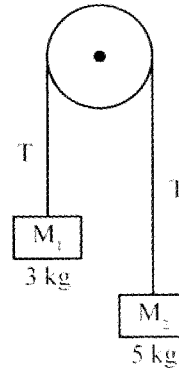
A 2-kg block is placed onto an inclined plane that makes an angle of  $30^\circ$  with the horizontal plane. The block is initially at rest on the inclined plane. The coefficient of friction between the block and the surface is  $\frac{\sqrt{3}}{2}$ .

2. Which of the following statements about the block is **true**?
- The weight of the block will not cause it to slide down the plane.
  - The weight of the block will not cause it to slide up the plane.
  - The weight of the block will cause it to slide down the plane.
  - The weight of the block will cause it to slide up the plane.

### Numerical Response

*Use the following information to answer the next question.*

Two masses are hung on two sides of a massless pulley as shown in the diagram. Assume that air resistance is negligible and the force of friction on the pulley is 8.0 N.



3. What is the acceleration on mass  $M_1$ ? \_\_\_  $\text{m/s}^2$

### Numerical Response

4. A 5.5-kg object is pushed along a horizontal surface. If the coefficient of friction between the object and the surface is 0.15, what is the magnitude of the force of friction? \_\_\_ N
5. A bomber airplane, travelling horizontally with a speed of 200 m/s, released a bomb at a height of 1500 m from the ground. If the airplane continued to fly at a constant speed, the bomb would have landed
- below the jet
  - ahead of the jet
  - to the left of the jet
  - to the right of the jet

**Numerical Response**

6. A helicopter rises with a uniform velocity at an angle of  $50^\circ$  to the horizontal. The helicopter travelled a distance of 1.00 km in 33.3 s. How long would it take for the helicopter to reach an altitude of 1.00 km? \_\_\_\_ s
7. A 100-g block moves with a uniform speed in a horizontal circular groove with a radius of 25 cm. If it takes 2.0 seconds for the block to complete a single revolution in the groove, what is the normal contact force provided by the vertical walls of the groove, in newtons?
- A. 0.25 N                      B. 0.79 N  
 C. 1.70 N                      D. 2.50 N

**Numerical Response**

8. A 2.28-kg object is swung in a vertical circle with a radius of 1.75 m. If the period for one revolution is 1.3 s, what is the tension in the string when it is at the bottom of its swing? \_\_\_\_ N  
 (Assume uniform speed.)

**Open Response**

9. A 1.95 kg object swings from the end of a 0.87 m string in a vertical circle. If the time for one revolution of the object is 0.92 s, what is the tension in the string when the object is at the top of the circle? (Assume uniform speed.)

---

---

---

---

---

---

---

---

10. Which of the following events is an example of an ideal inertial frame?
- A. A constantly accelerating car  
 B. A rocket drifting in deep outer space  
 C. A frame rotating with constant velocity  
 D. A satellite orbiting a planet in outer space

*Use the following information to answer the next question.*

A geostationary satellite remains motionless with respect to Earth's surface.

11. Which of the following expressions **best** represents the radius of the satellite's circular orbit if  $T$  is the period of the orbit, and  $R$  is Earth's mean radius?

- A.  $\left(\frac{gR^2T^2}{2\pi^2}\right)^{\frac{1}{2}}$   
 B.  $\left(\frac{gR^2T^2}{2\pi^2}\right)^{\frac{1}{3}}$   
 C.  $\left(\frac{gR^2T^2}{4\pi^2}\right)^{\frac{1}{2}}$   
 D.  $\left(\frac{gR^2T^2}{4\pi^2}\right)^{\frac{1}{3}}$

**Numerical Response**

Use the following information to answer the next question.

A space shuttle launches upward from Earth's surface. The shuttle's mission is to approach the sun until it reaches the location at which the astronauts within the shuttle experienced absolute weightlessness.

Mass of the sun ( $m_s$ ) =  $6 \times 10^{30}$  kg

Mass of Earth ( $m_E$ ) =  $6 \times 10^{24}$  kg

Distance between Earth and the sun ( $r$ ) =  $15 \times 10^{10}$  m

12. The distance from Earth at which the astronauts would experience weightlessness, expressed in scientific notation, is  $b \times 10^w$  m. The value of  $b$  is \_\_\_\_.

**Open Response**

Use the following information to answer the next question.

Sputnik-1 was the world's first Earth-orbiting artificial satellite. The Soviet Union launched Sputnik-1 into orbit on October 4, 1957.

13. If Sputnik-1 orbited Earth with a uniform speed of  $8.06 \times 10^3$  m/s, what was it's orbital radius?

---

---

---

---

---

---

---

---

**Numerical Response**

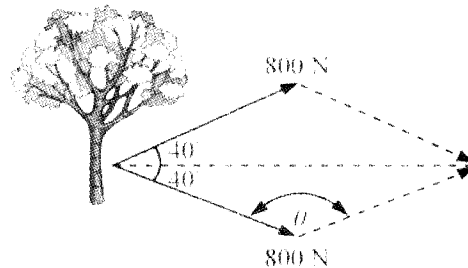
14. Chung pulls his 20 kg wagon behind him, accelerating it at  $1.5 \text{ m/s}^2$ . A force of friction of 5.0 N opposes acts between the sidewalk and the wagon, and the wagon's handle makes an angle of  $45^\circ$  with the ground. What is the magnitude of the force that Chung exerts along the handle? \_\_\_\_ N.

(Round and record your answer to two digits.)

**Open Response**

Use the following information to answer the next question.

Two horses are being used to pull a fallen tree off of a path. The tree will require 1200 N of force to move it. Each one pulls with a force of about 800 N as represented in the diagram below.



15. Using the parallelogram method of vector addition, determine the value of  $\theta$ , the magnitude of the resultant, and determine whether or not the horses are able to move the tree from off the path.

---

---

---

---

---

---

---

---

Use the following information to answer the next question.

Towers AB and CD stand a certain distance apart. Tower AB is 20 m tall, and tower CD is 30 m tall.

An object with a mass of  $m$  is thrown horizontally toward tower CD from the top of tower AB with a velocity of 10 m/s. Simultaneously, an object with a mass of  $2m$  is thrown downward from the top of tower CD at an angle of  $60^\circ$  from the horizontal toward tower AB with a velocity of the same magnitude as the velocity of the first object. The two objects collide in the air between the two buildings and stick together.

16. What is the distance between the two towers, in metres?
- A.  $10\sqrt{3}$                       B.  $12\sqrt{3}$   
 C.  $14\sqrt{3}$                       D.  $15\sqrt{3}$

Use the following information to answer the next question.

A moving target travelled away from a shooter at 72 km/h. When the shooter fired at an angle of  $45^\circ$  from the horizontal, the target is at a distance of 500 m from the gun.

17. If the bullet hit the target in the bull's-eye, how long to the nearest second does it take for the bullet to reach the moving target?
- A. 8 s                                  B. 12 s  
 C. 15 s                                D. 21 s

### Numerical Response

Use the following information to answer the next question.

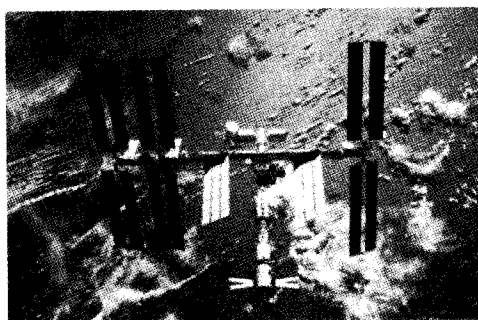
The moon orbits Earth with an average period of 27.3 days at an average orbital radius of  $3.84 \times 10^5$  km. (The radius of the orbit includes both the radii of Earth and the moon.)

18. The centripetal acceleration of the moon in its orbit around Earth is  $a.bc \times 10^{-d}$ . The values of  $a$ ,  $b$ ,  $c$ , and  $d$  are \_\_\_\_, \_\_\_\_, \_\_\_\_, and \_\_\_\_.

### Open Response

Use the following information to answer the next question.

The International Space Station (ISS) orbits Earth at an altitude of 390 km and completes one orbit every 92 min.



19. Calculate the speed of the space station in its orbit.

---



---



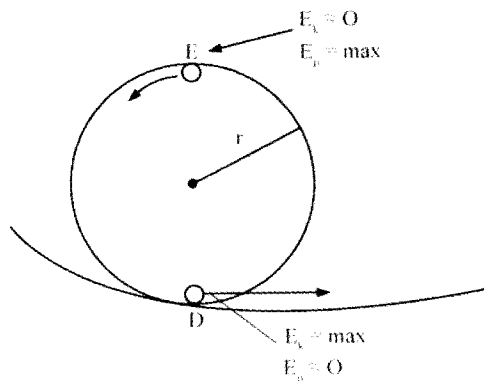
---



---

Use the following information to answer the next multipart question.

20. The given diagram represents the track for a toy car. The car has a mass of 1.20 kg.



Part A

**Open Response**

The car ascends the loop coming almost to a stop at point E before hurtling down toward point D. If the velocity at point E was 0 m/s and at point D it was 2.3 m/s. If the time taken to get from point E back to point D was 1.3 s what was the car's acceleration?

---



---



---



---

Part B

**Open Response**

The toy car had no kinetic energy at the top of the loop and a lot of kinetic energy at the bottom. Energy cannot be created or destroyed, so how did the car gain kinetic energy?

---



---



---



---

Use the following information to answer the next question.

To make repairs to the Hubble Space Telescope, NASA astronauts used the Canadarm to grasp the telescope and bring it closer to the shuttle. When the arm applied a force on the telescope, the telescope also applied a force on the arm.

21. If the two forces cancelled each other out, it would be impossible for the Canadarm to bring the telescope closer to the shuttle. Which of the following statements explains the reason that the two forces do **not** cancel each other out?
- A. The two forces act on different bodies.
  - B. Newton's laws do not apply in a vacuum.
  - C. The two forces act in the same direction.
  - D. The two forces are not equal in magnitude.



## SOLUTIONS

1. D	6. 43.5	11. D	16. A	Part B- OR
2. A	7. A	12. 1.50	17. B	21. A
3. 2	8. 116	13. OR	18. 2,7,2,4	
4. 8.1	9. OR	14. 49	19. OR	
5. A	10. B	15. OR	20. Part A- OR	

### 1. D

An inertial frame of reference is a reference frame that moves with constant velocity. In other words, the speed and direction of the frame never change. The stone falling toward a river after being dropped from a bridge experiences acceleration due to gravity. As a result, the stone's frame of reference is accelerating, not inertial.

### 2. A

The free-body diagram of the whole system is as follows:



The weight of the block,  $W$ , acts vertically downward. It can be resolved into two components, one downward along the inclined plane,  $W \sin 30^\circ$ , and the other acting normal to the plane,  $W \cos 30^\circ = F_N$ .

The force acting on the block down along the plane is  $W \sin 30^\circ = W \frac{1}{2}$

$$\begin{aligned} F_s &= \mu F_N \\ &= \mu W \cos 30^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right)W \left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{3}{4}W \end{aligned}$$

$\left(\frac{W}{2}\right) < \left(\frac{3}{4}W\right)$ , so the force of friction will keep the block from sliding down the plane due to its own weight.

### 3. 2

The acceleration of the system ( $M_1$  and  $M_2$  will have the same acceleration) is calculated using Newton's second law.

$$\begin{aligned} F_{\text{net}} &= M_1 g - M_2 g - F_f \\ F_{\text{net}} &= (5 \text{ kg} - 3 \text{ kg}) \times 9.81 \text{ m/s}^2 - 8.0 \text{ N} = 11.6 \text{ N} \\ a &= \frac{F_{\text{net}}}{m_{\text{total}}} \\ &= \frac{11.6 \text{ N}}{7 \text{ kg}} = 1.7 \text{ m/s}^2 = 2 \text{ m/s}^2 \end{aligned}$$

### 4. 8.1

$$\begin{aligned} \vec{F}_N &= \vec{F}_g \\ \vec{F}_g \vec{F}_g &= m\vec{g} \\ &= (5.5 \text{ kg})(9.81 \text{ m/s}^2) \\ &= 54 \text{ N} \\ \vec{F}_f &= \mu \vec{F}_N \\ &= (0.15)(54 \text{ N}) \\ &= 8.1 \text{ N} \end{aligned}$$

### 5. A

In projectile motion, the horizontal motion is independent of the vertical motion. The bomb is released with the same horizontal velocity as the bomber airplane. Since the airplane continued to travel with the same constant speed, both the bomb and the airplane travelled the same horizontal distance before the bomb struck the ground. Therefore, the bomb would have landed below the airplane.

### 6. 43.5

The velocity of the helicopter is

$$\begin{aligned} \vec{v} &= \frac{\vec{d}}{t} = \frac{1000 \text{ m}}{33.3 \text{ s}} \text{ } 50^\circ \text{ above horizontal} \\ &= 30 \text{ m/s at } 50^\circ \text{ above horizontal} \end{aligned}$$

The time to reach an altitude of 1.00 km can be determined from the helicopter's vertical velocity.

The vertical component of this velocity is:

$$\vec{v}_y = (30.0 \text{ m/s})(\sin 50^\circ) = 23.0 \text{ m/s}$$

$$t = \frac{\vec{d}}{\vec{v}_y} = \frac{1000 \text{ m}}{23.0 \text{ m/s}} = 43.5 \text{ s}$$

Therefore, the helicopter will reach an altitude of 1.00 km after 43.5 s.

### 7. A

$$\begin{aligned} \text{The speed of the block } v &= \frac{2\pi r}{T} = \frac{2\pi \times 25}{2} = 78.5 \text{ cm/s} \\ &= 0.785 \text{ m/s} \end{aligned}$$

where  $v$  = speed,  $r$  = radius, and  $T$  = time  
Therefore, the centripetal

$$\text{acceleration of the block} = \frac{v^2}{r} = \frac{(0.785)^2}{0.25} = 2.5 \text{ m/s}^2$$

This centripetal acceleration acts toward the centre of the circle.

The only force operating in this direction is the normal contact force provided by the vertical walls of the groove. Therefore, this normal force =  $0.1 \times 2.5 = 0.25 \text{ N}$

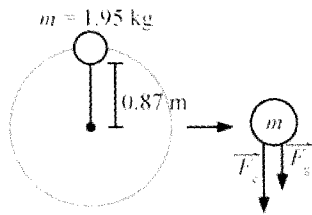


8. 116

$$\begin{aligned} \vec{F}_c &= \frac{4\pi^2 rm}{T^2} \\ &= \frac{(4\pi^2)(1.75 \text{ m})(2.28 \text{ kg})}{(1.3 \text{ s})^2} \\ &= 93.2 \text{ N} \\ \vec{F}_g &= mg \\ &= (2.28 \text{ kg})(-9.81 \text{ m/s}^2) \\ &= -22.4 \text{ N} \\ \vec{F}_c &= \vec{F}_T + \vec{F}_g \\ \vec{F}_T &= \vec{F}_c - \vec{F}_g \\ &= 93.2 \text{ N} - (-22.4 \text{ N}) \\ &= 116 \text{ N} \end{aligned}$$

9. Open Response

The following free-body diagram represents the given scenario.



Using the information from the free-body diagram, calculate the centripetal force on the object.

$$\begin{aligned} \vec{F}_c &= \frac{4\pi^2 rm}{T^2} \\ &= \frac{(4\pi^2)(0.87 \text{ m})(1.95 \text{ kg})}{(0.92 \text{ s})^2} \\ &= 79.1 \text{ N} \end{aligned}$$

The centripetal force is directed radially inward, so the centripetal force at the top of the circle will be  $-79.1 \text{ N}$ .

$$\begin{aligned} \vec{F}_g &= mg \\ &= (1.95 \text{ kg})(9.81 \text{ m/s}^2) \\ &= -19.1 \text{ N} \\ \vec{F}_c &= \vec{F}_T - \vec{F}_g \\ \vec{F}_T &= \vec{F}_c + \vec{F}_g \\ &= -79.1 \text{ N} + (-19.1 \text{ N}) \\ &= -98.2 \text{ N} \end{aligned}$$

10. B

A rocket drifting in deep outer space is an example of an ideal inertial frame of reference because the frame of reference is not accelerating. When either the magnitude of the velocity increases, such as a constantly accelerating car, or the direction of the motion changes, such as objects travelling along circular paths, the frame of reference is non-inertial.

11. D

A geostationary satellite appears motionless when its period of revolution is the same as Earth's period of revolution. This means that the satellite is motionless within the frame of reference of Earth's surface. In other words, the period of the geostationary satellite is  $T = 24 \text{ h}$ .

Let  $r$  be the radius of the orbit from Earth's centre. The centripetal force balances the gravitational force.

$$\begin{aligned} \vec{F}_c &= \vec{F}_g \\ m\left(\frac{2\pi r}{T}\right)^2 &= \frac{GMm}{r^2} \\ \left(\frac{2\pi}{T}\right)^2 r^3 &= GM \\ r^3 &= \frac{GMT^2}{4\pi^2} \end{aligned}$$

Recall that  $g = \frac{GM}{R^2}$ , where  $R$  is Earth's average radius.

$$\begin{aligned} \text{so } GM &= gR^2 \\ r^3 &= \frac{GMT^2}{4\pi^2} = \frac{gR^2T^2}{4\pi^2} \\ \therefore r &= \left(\frac{gR^2T^2}{4\pi^2}\right)^{\frac{1}{3}} \end{aligned}$$

Therefore, the radius of the circular orbit of the satellite is equal to  $\left(\frac{gR^2T^2}{4\pi^2}\right)$ .

12. 1.50

The astronauts will experience weightlessness when the gravitational forces of attraction on the shuttle are equal. Assume that the only significant objects affecting the shuttle are Earth and the sun.

$$F_{g \text{ Earth-rocket}} = F_{g \text{ sun-rocket}}$$

Let the rocket's distance from Earth's centre be  $x$ , and let the sun's distance from Earth's centre be  $r$ .

Therefore, weightlessness occurs when

$$\begin{aligned} \frac{Gm_E m_r}{x^2} &= \frac{Gm_S m_r}{(r-x)^2} \\ \Rightarrow \frac{m_E}{x^2} &= \frac{m_S}{(r-x)^2} \end{aligned}$$



It is now necessary to put this into a form that we can use to enter into the quadratic equation

$$m_E(r-x)^2 = m_S x^2$$

$$m_E(r^2 - 2rx + x^2) = m_S x^2$$

$$\frac{m_S}{m_E} = \frac{r^2 - 2rx + x^2}{x^2} = \frac{r^2}{x^2} - 2\frac{r}{x} + 1$$

$$\frac{m_S}{m_E} - 1 = \frac{r^2}{x^2} - 2\frac{r}{x}$$

$$\left(\frac{m_S}{m_E} - 1\right)x^2 - 2rx + r^2 = 0$$

$$\therefore \left(\frac{m_S}{m_E} - 1\right)x^2 + 2rx - r^2 = 0$$

So now it is a simple matter to plug the values for  $a$ ,  $b$ , and  $c$  into the quadratic formula.

$$a = \left(\frac{m_S}{m_E} - 1\right) = 10^6 - 1 \approx 10^6$$

$$b = 2r = 30 \times 10^{10} \text{ m}$$

$$c = -r^2 = -2.25 \times 10^{22} \text{ m}^2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Plugging the values of  $a$ ,  $b$ , and  $c$  into the quadratic equation gives.

$$x = \frac{(-30 \times 10^{10} \text{ m}) \pm \sqrt{9.0 \times 10^{28} \text{ m}^2}}{2 \times 10^6}$$

$$x = -15 \times 10^4 \text{ m} \pm 1.50 \times 10^8 \text{ m}$$

$$x \approx 1.50 \times 10^8 \text{ m}$$

The positive value for  $x$  is chosen since the situation states that the rocket is travelling from Earth to the sun.

The negative value for  $x$  would represent a situation where the rocket ship travelled away from both Earth and the sun.

### 13. Open Response

You must rearrange this equation  $v = \sqrt{\frac{Gm_c}{r}}$  to solve for radius.

$$v^2 = \frac{Gm_c}{r}$$

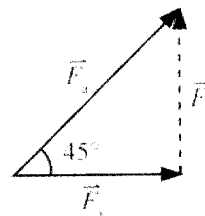
$$r = \frac{Gm_c}{v^2}$$

$$r = \frac{(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(5.98 \times 10^{24} \text{ kg})}{(8.06 \times 10^3 \text{ m/s})^2}$$

$$r = 6.14 \times 10^6 \text{ m}$$

Therefore, the orbital radius for Sputnik-1 would have been  $6.14 \times 10^6 \text{ m}$  if its orbital speed was  $8.06 \times 10^3 \text{ m/s}$ .

### 14. 49



$$\vec{F}_{\text{net}} = m\vec{a}$$

$$= 20 \text{ kg}(1.5 \text{ m/s}^2)$$

$$= 30 \text{ N}$$

$\vec{F}_x$  is responsible for the waggon's horizontal motion. Therefore, only the  $x$ -component of  $\vec{F}_a$  is part of the net force. The  $y$ -component,  $\vec{F}_y$ , is not needed because the object does not move vertically.

$$\vec{F}_{\text{net}} = \vec{F}_x + \vec{F}_y$$

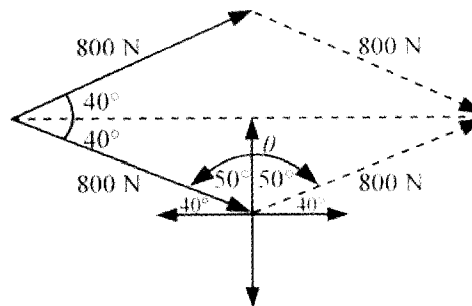
$$30 \text{ N} = \vec{F}_a (\cos 45^\circ) + (-5.0 \text{ N})$$

$$\vec{F}_a = \frac{35 \text{ N}}{\cos 45^\circ}$$

$$\vec{F}_a = 49 \text{ N}$$

### 15. Open Response

To use the parallelogram method, we need only fill in the information about the angles and the magnitude of the vectors that is missing from the diagram that is already provided.



Now we need to apply the Cosine Law to determine the magnitude of the resultant vector as follows.

$$|\vec{R}| = \sqrt{800^2 + 800^2 - 2(800)(800)\cos 100^\circ}$$

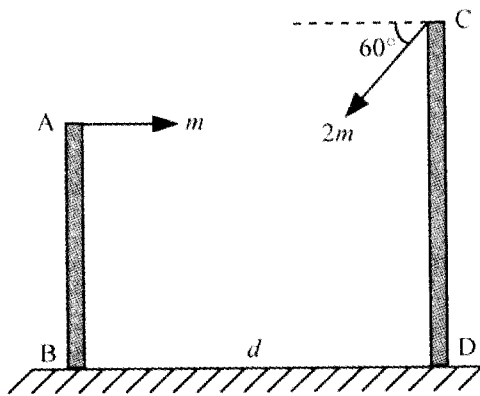
$$|\vec{R}| = 1225.7 \text{ N}$$

Due to the symmetry of the scenario, the resultant vector will be directly along the horizontal line indicated in the illustration.

Because the tree requires 1200 N of force to move it, the horses will indeed be able to move the tree with an excess of 25.7 N.



16. A



Let  $d$  be the distance between towers AB and CD.  
 Let the objects collide at point E after time  $t$ .  
 Let the initial speed of each object be  $u = 10 \text{ m/s}$ .  
 The difference in the vertical displacement =  $30 - 20 = 10 \text{ m}$

$$\Delta h = v_{2m} y^t - v_m y^t$$

$$10 \text{ m} = \left( u(\sin 60^\circ)t + \frac{1}{2}gt^2 \right) - \left( \frac{1}{2}gt^2 \right)$$

$$10 \text{ m} = u(\sin 60^\circ)t$$

$$10 = 10 \times \frac{\sqrt{3}}{2} \times t$$

$$\therefore t = \frac{2}{\sqrt{3}} \text{ s}$$

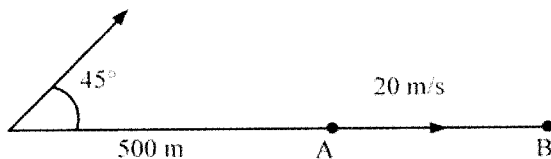
The sum of the magnitudes of the horizontal displacements =  $d$

$$d = ut + u(\cos 60^\circ)t$$

$$d = 10 \times \frac{2}{\sqrt{3}} + 10 \times \frac{1}{2} \times \frac{2}{\sqrt{3}}$$

$$= \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$

17. B



Let point O be the position of the bullet when it was fired, and point A be the position of the target when the bullet was fired. The distance  $d_{OA} = 500 \text{ m}$ .

Let point B be the position of the target and the bullet when they make contact.

Let  $v$  be the initial velocity of the bullet, which made an angle of  $45^\circ$  with the horizontal.

Let  $u$  be the velocity of the target =  $72 \text{ km}$ .

Let  $t$  be the time elapsed.

The motion of the bullet can be resolved into horizontal and vertical components. The vertical component of its motion can be written using the expression for time of flight,

$$\vec{v}_{y \text{ final}} = \vec{v}_{y \text{ initial}} - \frac{1}{2}gt$$

Since the vertical velocity of the projectile at the end of its flight will be the negative of the initial vertical velocity

$$\vec{v} \sin 45^\circ = -\frac{(\vec{g})(t)}{2}$$

$$\Rightarrow \vec{v} = -\frac{(\vec{g})(t)}{\sqrt{2}} = \frac{(9.8 \text{ m/s})(t)}{\sqrt{2}}$$

The horizontal motion the equation can be written as

$$\vec{v}_x t = d_{AB} + d_{OA}$$

$$d_{AB} = \vec{u}t$$

$$\vec{u} = 72 \text{ km/h} = \frac{72 \times 1000}{3600} \text{ m/s} = 20 \text{ m/s}$$

$$\therefore d_{AB} = 20t \text{ m}$$

$$d_{\text{total}} = \vec{v} \cos 45^\circ \times t = (20t + 500) \text{ m}$$

Substitute for  $v$ ,

$$\frac{9.8t}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times t = 20t + 500$$

$$\Rightarrow 4.9t^2 - 20t + 500 = 0$$

$$\Rightarrow t = 12.35 \text{ s}$$

$$t = 12 \text{ s}$$

18. 2,7,2,4

$$\vec{a}_c = \frac{\vec{v}^2}{r} \text{ and } \vec{v} = \frac{2\pi r}{T}$$

Substitute,

$$\vec{a}_c = \frac{\left( \frac{2\pi r}{T} \right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$\vec{a}_c = \frac{4\pi^2 (3.84 \times 10^7 \text{ m})}{(27.3 \text{ d} \times 24 \text{ h/d} \times 3600 \text{ s/d})^2}$$

$$= 2.72 \times 10^{-4} \text{ m/s}^2$$

19. Open Response

$$\vec{v} = \frac{2\pi r}{T}$$

$$= \frac{2\pi (3.90 \times 10^5 \text{ m} + 6.37 \times 10^6 \text{ m})}{(92 \text{ min})(60 \text{ s/min})}$$

$$= 7.7 \times 10^3 \text{ m/s}$$

20. Part A – Open Response

$$\vec{a} = \frac{\vec{v}}{t}$$

$$= \frac{2.3 \text{ m/s}}{1.3 \text{ s}}$$

$$\vec{a} = 1.8 \text{ m/s}^2$$

Part B – Open Response

Although energy cannot be created or destroyed, it can be converted from one form into another. At the top of the loop, the car had no kinetic energy, but it had potential energy. As the car moved through the loop, this potential energy converted into kinetic energy.



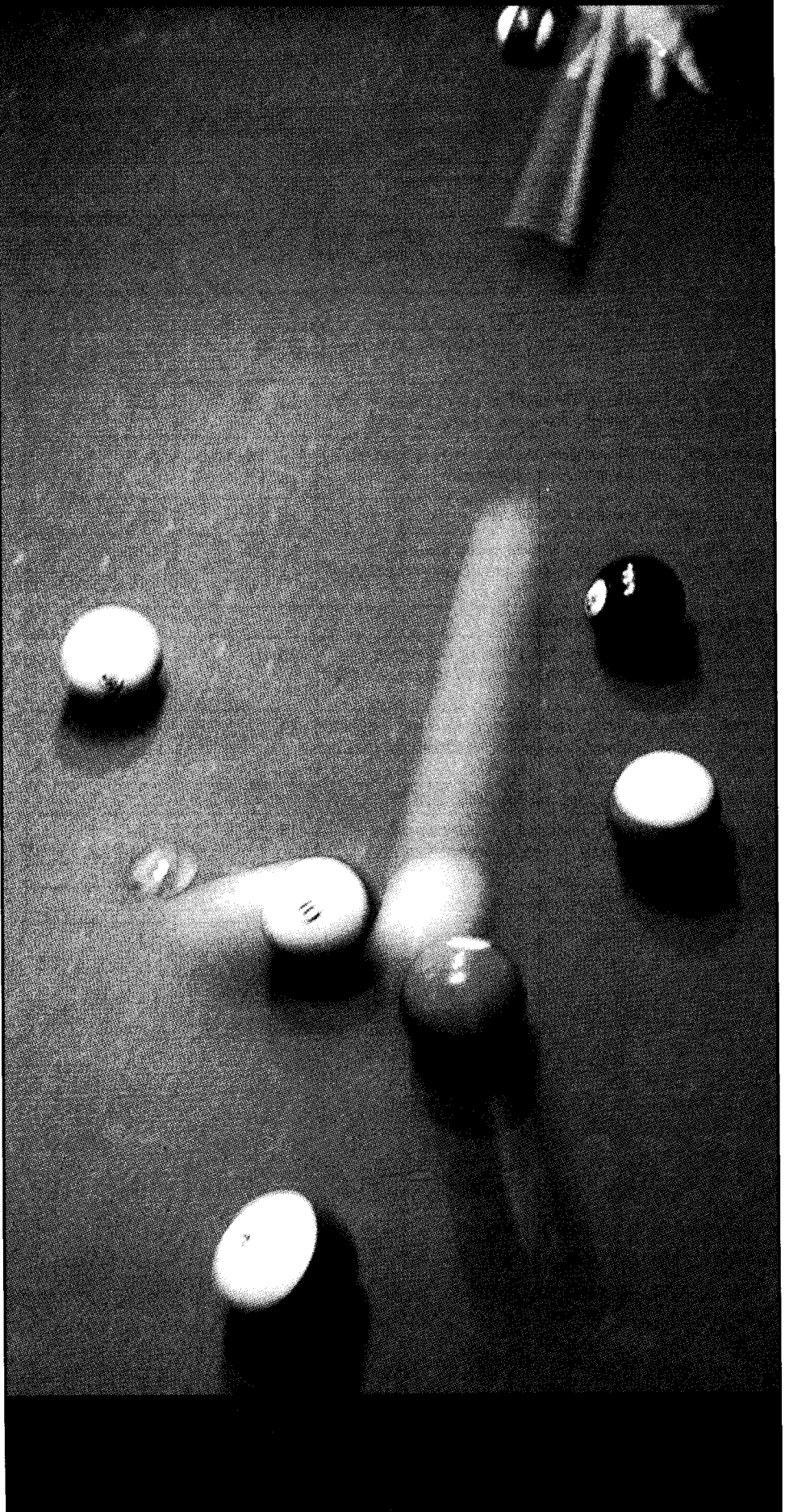
**21. A**

Newton's third law explains that for every action (force), there is an equal and opposite reaction (force). The force on the telescope is the action, and the resulting force on the arm is the reaction. The forces are equal in magnitude, but they act in the opposite directions.

Therefore, the telescope moves because the two forces act on different bodies with different masses.



# Energy and Motion





## Energy and Motion

### Table of Correlations

Specific Expectation	Practice Questions	Unit Test Questions
<b>12.2.1</b> Understanding Basic Concepts		
<b>12.2.1.1</b> <i>define and describe the concepts and units related to momentum and energy</i>	1	2
<b>12.2.1.3</b> <i>analyse situations involving the concepts of mechanical energy, thermal energy and its transfer (heat), and the laws of conservation of momentum and of energy</i>	2, 3	3
<b>12.2.1.4</b> <i>distinguish between elastic and inelastic collisions</i>	4, 5	4
<b>12.2.1.5</b> <i>analyse and explain common situations involving work and energy, using the work-energy theorem</i>	9, 10	5
<b>12.2.1.6</b> <i>analyse the factors affecting the motion of isolated celestial objects, and calculate the gravitational potential energy for each system, as required</i>	11, 12	6, 7
<b>12.2.1.7</b> <i>analyse isolated planetary and satellite motion and describe it in terms of the forms of energy and energy transformations that occur</i>	13, 14, 15, 16, 17	8, 9
<b>12.2.1.8</b> <i>state Hooke's law and analyse it in quantitative terms</i>	18, 19	10, 11
<b>12.2.1.2</b> <i>analyse, with the aid of vector diagrams, the linear momentum of a collection of objects, and apply quantitatively the law of conservation of linear momentum</i>	6, 7, 8	1
<b>12.2.2</b> Developing Skills of Inquiry and Communication		
<b>12.2.2.1</b> <i>investigate the laws of conservation of momentum and of energy in one and two dimensions by carrying out experiments or simulations and the necessary analytical procedures</i>	20, 21	12
<b>12.2.2.2</b> <i>design and conduct an experiment to verify the conservation of energy in a system involving kinetic energy, thermal energy and its transfer (heat), and gravitational and elastic potential energy</i>	22, 23, 24	13, 14
<b>12.2.3</b> Relating Science to Technology, Society and the Environment		
<b>12.2.3.1</b> <i>analyse and describe, using the concepts and laws of energy and of momentum, practical applications of energy transformations and momentum conservation</i>	25, 26	15
<b>12.2.3.2</b> <i>identify and analyse social issues that relate to the development of vehicles</i>	27, 28	



**12.2.1.1** *define and describe the concepts and units related to momentum and energy*

**12.2.1.3** *analyse situations involving the concepts of mechanical energy, thermal energy and its transfer (heat), and the laws of conservation of momentum and of energy*

**12.2.1.4** *distinguish between elastic and inelastic collisions*

## CONSERVATION OF MOMENTUM

An isolated system is defined as a system that is not influenced by its surroundings. An isolated system is considered an ideal system because the conservation laws (mass, energy, momentum) can be perfectly demonstrated. While a perfectly isolated system cannot be created, physicists attempt to approximate isolated mechanical systems by reducing friction. Thus, in the study of mechanics, an isolated system is often defined as a system in which no net force acts from outside the system.

In an isolated system, momentum is perfectly conserved. In reality, a system is never isolated, and there may appear to be a loss of momentum in the system, even after measurement errors are considered. Two colliding objects transfer some momentum to the air, some to the surface on which they move, and some to the atoms of which they are composed. These losses can be minimized, but they can never be eliminated.

Nevertheless, well-designed experiments have confirmed that momentum is conserved in all interactions. That is, in an isolated system, the total momentum of the system before some interaction will be the same as the total momentum of the system after this interaction.

## CONSERVATION OF MOMENTUM IN AN ISOLATED SYSTEM

Vector quantities are described in terms of both magnitude and direction. Momentum is a vector quantity that is the product of an object's mass and velocity. The direction of an object's momentum is the same direction as the object's velocity. Momentum can be calculated using the formula  $\vec{p} = m\vec{v}$ , where  $\vec{p}$  is the object's momentum (measured in  $\text{kg} \cdot \text{m/s}$  or  $\text{N} \cdot \text{s}$ ),  $m$  is the object's mass, and  $\vec{v}$  is the object's velocity.

### Example

What is the momentum of a 1500 kg mass travelling at 20 m/s to the west?

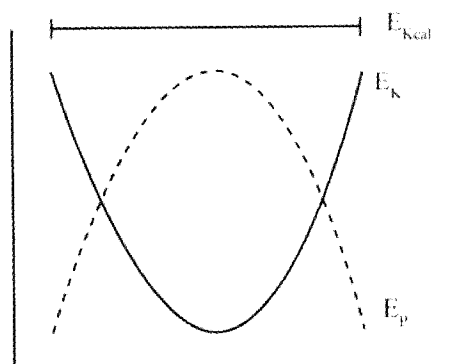
$$\begin{aligned}\vec{p} &= m\vec{v} \\ \vec{p} &= (1500 \text{ kg})(20 \text{ m/s west}) \\ \vec{p} &= 3.0 \times 10^4 \text{ kg} \cdot \text{m/s west}\end{aligned}$$

Consider when a stone ricochets off of a wall. The stone's momentum has a definite direction both before and after the collision with the wall. The angle and speed with which the stone strikes the wall determines the angle and speed with which it leaves the wall.

## CONSERVATION OF ENERGY

Suppose that a ball is thrown upward and then caught. The ball has a combination of kinetic and gravitational potential energy at all points of its trajectory. Ignoring friction, you will notice that one kind of energy changes to another, while the total energy is constant.

This can be shown graphically:





In the graph, the potential and kinetic energies change from one to the other, and the total mechanical energy remains constant throughout.

Energy is a scalar quantity. This means that it does not have direction, only size. The actual energy of a system is neither negative nor positive. Instead, only the change in energy has a negative or positive value. An energy change is positive when the system gains energy, and it is negative when the system has lost energy.

## ELASTIC AND INELASTIC COLLISIONS

An elastic collision occurs when both the kinetic energy and momentum are conserved. An inelastic collision occurs when the momentum is conserved but some of the kinetic energy is lost as other forms of energy, such as heat, light, or sound.

A completely inelastic collision occurs when two objects stick together after they collide. In other words, if the total kinetic energy of a system before a collision is not equal to the system's total kinetic energy after the collision, it is said to be an inelastic collision.

### Example

A 1.0-kg ball travelling with a velocity of 2.0 m/s to the right collided head-on with a stationary 2.0-kg ball. After the collision, the 2.0-kg ball travelled to the right with a velocity of 1.2 m/s, while the 1.0-kg ball travelled in the opposite direction at 0.40 m/s. Is this an elastic collision?

$$E_k = E_k'?$$

$$E_k = \frac{1}{2}(1.0 \text{ kg})(2.0 \text{ m/s})^2 + 0 = 2.0 \text{ J}$$

$$E_k' = \frac{1}{2}(2.0 \text{ kg})(1.2 \text{ m/s})^2 + \frac{1}{2}(1.0 \text{ kg})(0.40 \text{ m/s})^2$$

$$E_k' = 1.5 \text{ J}$$

Since  $E_k \neq E_k'$ , this collision is not elastic.

Note: Energy is a scalar quantity, so the directions of the moving objects are not considered in this solution.

### Practice

Use the following information to answer the next question.

The gravitational potential at a point is defined as the work done in bringing a unit mass from infinity to that point. It is given that the gravitational constant,  $G$ , is equal to  $6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-1}$ .

- What is the gravitational potential of a 10-kg body at a distance of 10 m?
  - $-9.27 \times 10^{-12} \text{ J/kg}$
  - $-6.67 \times 10^{-11} \text{ J/kg}$
  - $-5.27 \times 10^{-11} \text{ J/kg}$
  - $6.67 \times 10^{-11} \text{ J/kg}$

Use the following information to answer the next question.

Pendulums are used in grandfather clocks as timing devices. The period of a pendulum, the time required for it to complete one swing, is always the same. The time required for one complete swing changes only when the length of the pendulum changes. A pendulum swing is an example of periodic motion.

- If the end of a pendulum is held stationary at the top of one of its swings, what type of energy does it have?
  - Kinetic.
  - Thermal.
  - Potential.
  - Chemical.



3. Which of the following energy conversions occurs in an internal combustion engine?
- Heat energy is converted to mechanical energy.
  - Chemical energy is converted to mechanical energy.
  - Wind energy is converted to heat energy, which in turn is converted to mechanical energy.
  - Chemical energy is converted to heat energy, which in turn is converted to mechanical energy.

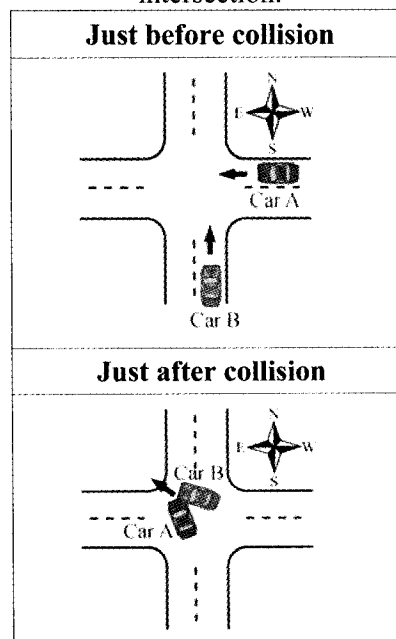
*Use the following information to answer the next question.*

A sphere with a mass of 500 g moving with a velocity of 200 cm/s collides centrally with another sphere with a mass of 100 g and a velocity of 100 cm/s coming from the opposite direction

4. If the collision is assumed to be perfectly elastic, what is the velocity of the 500-g sphere after the collision?
- 100 cm/s
  - 103 cm/s
  - 108 cm/s
  - 112 cm/s

*Use the following information to answer the next question.*

Two cars collide at an unmarked intersection.



5. Which of the following statements **best** describes the inelastic collision in the given diagram?
- Both momentum and kinetic energy are conserved.
  - Neither momentum nor kinetic energy are conserved.
  - Momentum is not conserved, but kinetic energy is conserved.
  - Momentum is conserved but, kinetic energy is not conserved.



**12.2.1.2** analyse, with the aid of vector diagrams, the linear momentum of a collection of objects, and apply quantitatively the law of conservation of linear momentum

## CONSERVATION OF LINEAR MOMENTUM

Momentum in a system is conserved as a vector quantity during any interactions between two or more objects.

$$\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$$

$$\vec{p}_1 + \vec{p}_2 + \dots + \vec{p}_n = \vec{p}'_1 + \vec{p}'_2 + \dots + \vec{p}'_n$$

### Example

A 1.0-kg ball moving with a velocity of 2.0 m/s to the right collides head-on with a stationary 2.0-kg ball. After the collision, the 2.0-kg ball moves off to the right with a velocity of 1.2 m/s. What is the velocity of the 1.0-kg ball after the collision?

Assuming vectors to the right are positive,

$$\vec{P}_{\text{total}} = \vec{P}'_{\text{total}}$$

$$(1.0 \text{ kg})(2.0 \text{ m/s}) + 0$$

$$= (1.0 \text{ kg})(\vec{v}') + (2.0 \text{ kg})(1.2 \text{ m/s})$$

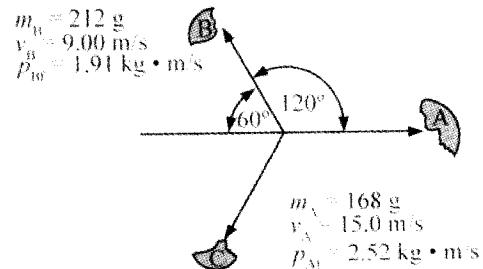
$$\vec{v}' = -0.40 \text{ m/s}$$

### Practice

Use the following information to answer the next question.

A 575-g glass ornament sitting on a table experienced a resonant frequency of 440 Hz.

The frequency caused the ornament to break into three pieces which then travelled horizontally across the frictionless tabletop.



Fragment A had a mass of 168 g, and fragment B had a mass of 212 g.

6. The magnitude of the momentum of fragment C immediately after the ornament broke into pieces was
- 5.19 kg · m/s
  - 3.85 kg · m/s
  - 2.28 kg · m/s
  - 0.610 kg · m/s

### Numerical Response

Use the following information to answer the next question.

An astronaut fires a 50-g bullet from a gun with a velocity of 400 m/s. The total mass of the astronaut, his spacesuit, and his gun is 120 kg.

7. What is the velocity of the astronaut's recoil? \_\_\_\_ m/s



### Open Response

Use the following information to answer the next question.

A student sits on a chair fixed to a cart that is initially at rest on frictionless rails. Answer the following questions, using what you know about the conservation of momentum and isolated systems. You may introduce equations to help with your explanation.

8. If the student throws a sandbag off the side of the cart, will the cart move? Is momentum conserved?

---



---



---



---



---



---

#### 12.2.1.5 analyse and explain common situations involving work and energy, using the work-energy theorem

Work is a force applied through a distance against a resistance. Power is the rate of doing work.

$$W = Fd$$

$$P = \frac{W}{t}$$

In accordance with Newton's three laws of motion, you need to consider friction, force, and work when you calculate the conservation of mechanical energy.

This is because the system will respond by changing its total energy when work is done on a system.

This is expressed algebraically as:

$$W = \Delta E$$

This can be rewritten in many different forms.

For example,

$$W = \Delta E = E_f - E_i$$

If the system gains energy, the work done is positive, and if the system loses energy, the work done is negative.

### Example

A 1500 kg car moving at 20.0 m/s over level ground slows to 10.0 m/s while travelling 400 m. Find the work done and the force of friction.

First, since the road surface is level, the potential energy does not change. Thus,  $E_p = 0$  in this case.

That way, you have no potential energy to track.

$$W = \Delta E = E_f - E_i$$

$$W = E_{k \text{ (final)}} - E_{k \text{ (initial)}}$$

$$W = \frac{1}{2} m \bar{v}_f^2 - \frac{1}{2} m \bar{v}_i^2$$

$$W = \frac{1}{2} \times 1500 \text{ kg} \times (10.0 \text{ m/s})^2$$

$$- \frac{1}{2} \times 1500 \text{ kg} \times (20.0 \text{ m/s})^2$$

$$= -2.25 \times 10^5 \text{ J}$$

The negative value indicates a loss of energy in the system. This is typical with friction.

Next:

$$W = Fd \Rightarrow \vec{F}_f = \frac{W}{d} = \frac{-2.25 \times 10^5 \text{ J}}{400 \text{ m}}$$

$$\vec{F}_f = -562.5 \text{ N} = -563 \text{ N}$$

**Note:** The negative sign on the friction force means that it is in the opposite direction from the motion of the car.

---

**Example**

A 20.0 kg box slides 2.00 m down a ramp that is inclined at 30.0°. If there is a 40.0 N force of friction acting on it, then what will the speed of the box be at the bottom of the ramp?

The change in the height of the box is  $2.00 \text{ m} \times \sin 30^\circ = 1.00 \text{ m}$

$$W = \Delta E = E_f - E_i$$

$$F_f d = E_{k(\text{bottom})} - E_{p(\text{top})}$$

$$F_f d = \frac{1}{2} m \bar{v}_f^2 - m \bar{g} h$$

$$\frac{1}{2} m \bar{v}_f^2 = F_f d + m \bar{g} h$$

$$\bar{v}_f^2 = \frac{2(F_f d + m \bar{g} h)}{m}$$

$$\bar{v}_f = \sqrt{\frac{2F_f d + m \bar{g} h}{m}}$$

$$= \sqrt{\frac{2(-40.0 \text{ N} \times 2.00 \text{ m}) + 20.0 \text{ kg} \times 9.81 \text{ m/s}^2 \times 1.00 \text{ m}}{20.0 \text{ kg}}}$$

$$\bar{v}_f = 3.41 \text{ m/s}$$

This question can also be broken down into small steps:

Work done (system loses this energy):

$$W = Fd = 40.0 \text{ N} \times 2.00 \text{ m} = 80.0 \text{ J}$$

Initial energy (all potential):

$$E_p = m \bar{g} h = 20.0 \text{ kg} \times 9.81 \text{ m/s}^2 \times 1.00 \text{ m}$$

$$E_p = 196.2 \text{ J}$$

Final energy (all kinetic):

$$E_k = 196.2 \text{ J} - 80.0 \text{ J} = 116.2 \text{ J}$$

$$E_k = \frac{1}{2} m \bar{v}^2 \Rightarrow \bar{v} = \sqrt{\frac{2E_k}{m}}$$

$$= \sqrt{\frac{2 \times 116.2 \text{ J}}{20.0 \text{ kg}}} = 3.41 \text{ m/s}$$

**Practice**

Use the following information to answer the next question.

A person lifts a 20-kg bag from the floor and puts it on a shelf 2 m above the floor.

9. What is the work done by the force of gravity in the given situation?
- A. -392 J                      B. -200 J  
C. -100 J                      D. 0 J
10. Which of the following statements about work done is **true**?
- A. Work is done moving a positive charge inside a positively charged metallic sphere.  
B. No work is done inside a positively charged metallic sphere.  
C. Outside the sphere, work is done by the charged particle.  
D. The electric field is not zero inside the sphere.

**12.2.1.6** analyse the factors affecting the motion of isolated celestial objects, and calculate the gravitational potential energy for each system, as required

## GRAVITY AND THE MOTION OF CELESTIAL OBJECTS

Gravitation, which is one of the four fundamental forces in nature, is an attractive force acting between any two masses, proportional in strength to the masses involved. Because of the large masses of systems of celestial objects, the main force affecting their motion is gravity.

The gravitational force between a pair of masses is described to high accuracy by Newton's famous law of universal gravitation. The force between a pair of masses,  $m_1$  and  $m_2$ , separated by a distance,  $r$ , is given by

$$\vec{F}_g = \frac{Gm_1m_2}{r^2}$$



## NUMERICAL DESCRIPTION OF A CIRCULAR ORBIT

If the pair of masses is isolated, the law of conservation of momentum implies that the centre of mass is at rest in some inertial frame. Often, the pair of masses is such that one of them,  $M$ , is much larger than the other,  $m$ , as for example is the case with the sun and Earth. In cases such as this, the centre of mass of the system is almost the same as the centre of the larger mass. A simple numerical relation describing the orbit can be obtained if the orbit is circular. This is approximately true for the planets and planetoids in our solar system, with the exception of Pluto.

The gravitational force provides the centripetal force in this case, so

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \therefore v = \sqrt{\frac{GM}{r}},$$

where  $v$  is the speed of the smaller mass. Since the circumference of the orbit is given as  $2\pi r$ , the time,  $T$ , taken to complete one revolution is

$$T = \frac{2\pi r}{v} = 2\pi \sqrt{\frac{r^3}{GM}}.$$

This shows that the time period does not depend on the smaller mass, and it is proportional to  $r^{\frac{3}{2}}$ . In general, a closed, stable orbit around a single large mass is elliptical in shape, however, the relationship obtained above turns out to be true even if the orbit is not circular. It is important to note that the orbit does not depend on the small mass,  $m$ , when the other mass is much larger.

The equation is valid when the sizes of the two masses are negligible compared to the distance  $r$ . Each of the masses pulls the other towards itself with a force of the above magnitude, in the direction of the other mass along the line joining the two. The constant  $G$  is called the gravitational constant and has a value of  $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$ .

## STABLE ORBITS UNDER GRAVITY

If two massive objects, far away from all other masses, were placed at rest at some distance from each other, they would start accelerating towards one another and eventually collide. To get a stable orbit under gravity, the two masses must initially have some component of relative velocity perpendicular to the line joining them. Then the gravitational force, though acting along the line joining them, only changes their velocities, so that they end up “falling around” one another.

## GRAVITATIONAL POTENTIAL ENERGY IN A UNIFORM GRAVITATIONAL FIELD

When a number of objects produce gravitational effects in some region of space, it is useful to group their combined effect into a gravitational field. This is a position dependant vector defined as the gravitational force per unit mass acting on a small mass placed in that region.

## GRAVITATIONAL POTENTIAL ENERGY OF A PAIR OF MASSES

As gravity is an attractive force, external work is required to separate a pair of masses against their mutual attraction. This leads to the concept of gravitational potential energy,  $U$ , of a massive object at a given location. This is equal to the minimum work required to remove the mass infinitely far from the given location against the gravitational pull of its neighbouring objects. It is important to note that  $U$  depends on the position of the mass.

For a small mass,  $m$ , at a distance,  $r$ , from a much larger mass,  $M$ , with no other masses nearby, the gravitational potential energy is

$$U = -\frac{GMm}{r}.$$

The negative sign is a reminder that the system is bound by an attractive force.



$$\vec{g} = \frac{\vec{F}_g}{m}$$

For example, when there is a single big mass,  $M$ , producing the field, Newton's law of gravitation implies that it has a gravitational field around it of the magnitude

$$\vec{g} = \frac{GM}{r^2},$$

and is directed radially inward towards the centre of the big mass.

When the field is nearly uniform in a region of space, the gravitational potential energy in that region is of the form  $U = -m\vec{g}x$ , where  $x$  is the coordinate toward which the field points. This form is commonly used for objects at a location close to the surface of Earth, where the potential energy of a mass,  $m$ , is given by  $E_p = m\vec{g}h$ .

### Practice

11. Which of the following statements about gravitational potential energy is **true**?
- A. Gravitational potential energy is independent of the configuration of a body.
  - B. Gravitational potential energy depends on the path a body takes.
  - C. Gravitational potential energy gained by a body depends on the magnitude of its velocity.
  - D. Gravitational potential energy may be negative.

### Numerical Response

Use the following information to answer the next question.

A planet of mass  $8.4 \times 10^{25}$  kg moves in a region of space where the gravitational field strength has an approximately constant value of  $1.5 \times 10^{-4}$  N/kg, directed along the positive  $x$ -axis. Over a period of time, the location of the planet changes from the point  $(4.21, 5.58) \times 10^{12}$  m to the point  $(3.91, 4.52) \times 10^{12}$  m.

12. Calculate the change in this planet's gravitational potential energy, correct to two significant digits in the form  $a.b \times 10^{cd}$  J. Give your answer in the form  $a, b, c,$  and  $d$ .  
\_\_\_\_, \_\_\_\_, \_\_\_\_, and \_\_\_\_.

**12.2.1.7** *analyse isolated planetary and satellite motion and describe it in terms of the forms of energy and energy transformations that occur*

### SATELLITE MOTION

In circular motion, acceleration is directed towards the centre of the circle. This must also be true for celestial bodies, such as Earth and the moon. Even though no physical tie exists between the two, gravity is the force keeping them together.

Isaac Newton worked out the shapes of the planets' orbits mathematically by assuming that the force of gravity was acting as the centripetal force. Although planetary orbits are elliptical, they are extremely close to perfect circles.



Algebraically, Newton's argument is:

$$\vec{F}_g = m\vec{a}_c$$

Depending upon whether the speed of a satellite, moon, or planet in its orbit is known, or whether the period of its orbit is known, the algebra takes on this form:

$$\vec{F}_g = m\vec{a}_c$$

$$\frac{Gm_1m_2}{r^2} = \frac{m_2\vec{v}^2}{r}$$

or

$$\frac{Gm_1m_2}{r^2} = \frac{4\pi^2m_2r}{T^2}$$

### Example

Find the speed of Jupiter's moon Io in its orbit.

The orbital radius is  $4.22 \times 10^8$  m, and the mass of Jupiter is  $1.90 \times 10^{27}$  kg.

$$\vec{F}_g = m\vec{a}_c$$

$$\frac{Gm_{\text{Jupiter}}m_{\text{Io}}}{r^2} = \frac{m_{\text{Io}}\vec{v}^2}{r}$$

Notice that the mass of Io is on both sides of the equation. This can be divided out of the equation. This is true for all orbit calculations—the mass of the body in orbit cancels out. As well, the radius cancels out on the right side of the equation.

As a result, the equation becomes  $\frac{Gm_{\text{Jupiter}}}{r} = \vec{v}^2$ .

Therefore, solving for the speed of Io's orbit is as follows:

$$\vec{v} = \sqrt{\frac{Gm_{\text{Jupiter}}}{r}}$$

$$= \sqrt{\frac{6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \times 1.90 \times 10^{27} \text{ kg}}{4.22 \times 10^8 \text{ m}}}$$

$$\vec{v} = 1.73 \times 10^4 \text{ m/s} = 17.3 \text{ km/s}$$

### Example

Find the speed of the space station in its orbit 360 km above Earth.

To answer this question, the orbital radius is needed.

Earth's radius is 6370 km ( $6.37 \times 10^6$  m) and the space station is 360 km (360 000 m) above the ground. Thus, its radius of orbit is 6730 km, or  $6.73 \times 10^6$  m.

$$\frac{Gm_{\text{E}}m_{\text{ss}}}{r^2} = \frac{m_{\text{ss}}\vec{v}^2}{r}$$

$$\vec{v} = \sqrt{\frac{Gm_{\text{E}}}{r}}$$

$$= \sqrt{\frac{6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \times 5.98 \times 10^{24} \text{ kg}}{6.73 \times 10^6 \text{ m}}}$$

$$\vec{v} = 7.698 \text{ m/s}$$

$$= 7.70 \text{ km/s}$$

### Example

Earth revolves around the sun at an average distance of  $1.50 \times 10^{11}$  m once each year. Find the mass of the sun.

First, find the time, one year, in seconds.

$$1\text{y} = 365.25 \text{ d} \times 24 \text{ h/d} \times 3600 \text{ s/h}$$

$$= 3.156 \times 10^7 \text{ s}$$

$$\frac{Gm_{\text{s}}m_{\text{E}}}{r^2} = \frac{4\pi^2m_{\text{E}}r}{T^2} \Rightarrow Gm_{\text{s}}T^2 = 4\pi^2r^3$$

$$\Rightarrow m_{\text{s}} = \frac{4\pi^2r^3}{GT^2}$$

$$= \frac{4 \times 3.14^2 \times (1.50 \times 10^{11} \text{ m})^3}{6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \times (3.156 \times 10^7 \text{ s})^2}$$

$$= 2.01 \times 10^{30} \text{ kg}$$



## ESCAPE VELOCITY CALCULATIONS

The gravitational potential energy of a small mass,  $m$ , at a distance,  $R$ , from the centre of a single massive spherical mass,  $M$ , is given by the relation

$$U = -\frac{GMm}{R}$$

When the small mass moves nearer to the large mass, its potential energy decreases (keeping in mind the negative sign), and, simultaneously, it speeds up, increasing its kinetic energy by the same amount. Similarly, when the small mass moves away from the large mass, it gains potential energy at the expense of the kinetic energy. As long as the system is isolated from other influences, the sum of the kinetic and potential energies remains constant.

If the small object has sufficient kinetic energy, it may be able to escape altogether from the gravitational field of the large object. The minimum velocity it requires to escape from a given position near the large mass is called the escape velocity ( $v_e$ ) at that position. The escape velocity can be found using the idea of interconversion of kinetic energy and potential energy, as described above.

Kinetic Energy + Potential Energy  
(at a finite distance from  $M$ ) = Kinetic Energy  
+ Potential Energy (far away from  $M$ ).

It is clear that the minimum kinetic energy is required if the small object ends up with negligible velocity after escaping from the gravitational field of  $M$  (i.e., far away from it). On the other hand, the potential energy far away from  $M$  is defined through the fact that “far away” is mathematically represented by the value of  $R$  approaching infinity. In this limiting case  $U = 0$ .

Minimum Kinetic Energy To Escape  
+ Potential Energy Near  $M = 0 + 0$ .

Using this relation, the minimum escape velocity can be derived.

$$\begin{aligned} E_{k \min} + U(R) &= 0 \\ \frac{1}{2}mv_e^2 + -\frac{GMm}{R} &= 0 \\ \Rightarrow v_e^2 &= \frac{2GM}{R} \\ \therefore v_e &= \sqrt{\frac{2GM}{R}} \end{aligned}$$

From this, it can be seen that the escape velocity is independent of the mass of the small object  $m$ .

The mass of Earth is approximately  $6.0 \times 10^{24}$  kg, while its mean radius is  $6.4 \times 10^6$  m. Thus, the escape velocity on the surface of Earth is

$$v_e = \sqrt{\frac{2 \times 6.7 \times 10^{-11} \times 6.0 \times 10^{24}}{6.4 \times 10^6}} \text{ m/s}$$

$$v_e = 1.1 \times 10^4 \text{ m/s}$$

So to launch a rocket into space from the surface of Earth, the rocket must be accelerated to a speed of about 11 km/s at blast off. The chemical energy stored in the rocket’s fuel is rapidly used up to provide the kinetic energy of the exhaust gases. This ejection of exhaust gases from the rocket increases the kinetic energy of the rocket itself. This kinetic energy is gradually converted to gravitational potential energy as the rocket moves away from Earth.

### Practice

13. A shuttle is launched into a circular orbit close to Earth’s surface. If the acceleration due to gravity is  $9.8 \text{ m/s}^2$  and the radius of Earth is  $6.4 \times 10^6$  m, then what additional velocity should be imparted to the shuttle in orbit for it to overcome the gravitational pull of Earth?

- A. 3.0153 km/s      B. 3.2805 km/s  
C. 3.3851 km/s      D. 3.4562 km/s

Use the following information to answer the next question.

A satellite is moving in a circular orbit around Earth with a speed equal to half of the magnitude of the escape velocity from Earth. Assume that Earth’s radius is 6400 km and acceleration due to gravity,  $g$ , is  $9.8 \text{ m/s}^2$ .

14. What is the height of the satellite above Earth’s surface?
- A. 6180 km      B. 6400 km  
C. 7150 km      D. 8300 km



**Numerical Response**

*Use the following information to answer the next question.*

Escape velocity is the specific value of velocity that a body must have in order to break the bonds of gravity and escape Earth's gravitational pull.

The mass of the moon equals  $7.4 \times 10^{22}$  kg, and its radius is equal to 1740 km.

15. What is the escape velocity of the moon? \_\_\_\_ km/s

**Numerical Response**

*Use the following information to answer the next question.*

Communication satellites require rocket thrusters that must be periodically fired in short bursts, to keep the satellites from drifting out of their orbits. Usually, a gas such as ammonia is heated using electrodes. The expanding hot gas is allowed to escape, which provides the thrust. Unfortunately, the ammonia erodes the electrodes, eventually rendering them useless.

An alternative method to heat the ammonia uses microwaves. A  $1.00 \times 10^3$  W microwave generator is used.

The microwaves in the thrusters heat the gas to tens of thousands of degrees. A satellite has a mass of 172 kg. To correct its orbit, a thruster is fired for 2.27 s, which changes the velocity of the satellite by  $5.86 \times 10^{-3}$  m/s. The force generated by the thrusters, expressed in scientific notation, is  $b \times 10^{-w}$  N.

16. The value of  $b$  is \_\_\_\_\_. (Record your answer to three digits.)

**Open Response**

*Use the following information to answer the next question.*

The movies *Deep Impact* and *Armageddon* revived concerns about the possible impact of an asteroid on Earth. If an



asteroid appears to be on a collision course with Earth, it will be essential to determine the mass and velocity of the asteroid before any action is taken. One possible way to obtain this information would be to send an unmanned space probe to approach and land on the asteroid.

17. The probe would require a certain escape velocity in order to leave the asteroid. This would be determined by setting the gravitational potential energy of the probe (relative to the centre) equal to its kinetic energy when taking off. Using a value of 0.25 N/kg for the gravitational field and 370 km for the radius, calculate the necessary escape velocity of the probe.

---



---



---



---



**12.2.1.8** state Hooke's law and analyse it in quantitative terms

## ELASTIC BEHAVIOUR AND HOOKE'S LAW

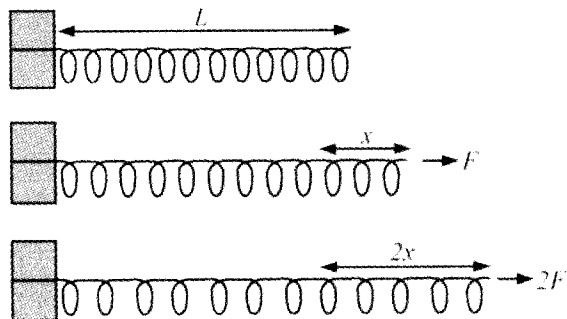
It is a familiar fact that springs can be stretched or compressed by external forces. As systems of springs and masses are extremely common in mechanics, the relation between the force applied and the change in length is important. Indeed, the length of a spring is the property used in the simplest force-measuring device, the spring balance.

If the forces on a spring are small enough, the changes in the length produced by them are reversible and simple to describe.

Let  $L$  be the length of a spring clamped at one end when there is no external force to stretch or compress it. Then, if a small external force,  $\vec{F}$ , is applied to change the length to  $l = L + x$ , it is observed that

$$\vec{F} = kx.$$

The fact that the elongation or compression,  $x$ , is directly proportional to the external force,  $\vec{F}$  applied, is referred to as Hooke's law. The value  $k$ , called the spring constant, is fixed for a given spring, but varies from one spring to another. Given any two springs, the one with the larger spring constant,  $k$ , will require a larger force to produce the same change in length. The spring attains equilibrium because of an internal restoring force that opposes any further change in length.

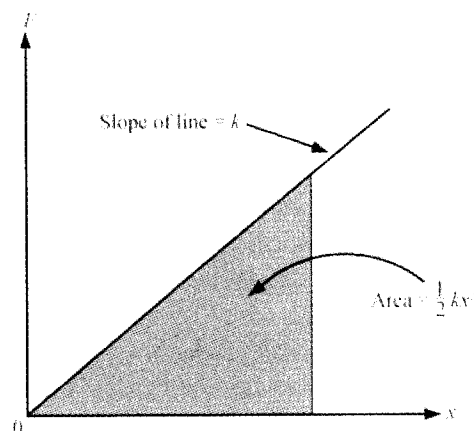


If the external force acts to compress the spring, the change in  $x$  is negative. It is important to note that the force is proportional to the change in length, not to the length itself.

### Example

If a spring of intrinsic length 15 cm with a spring constant of 500 N/m is to be stretched by 2 cm = 0.02 m, the external force required is  $\vec{F} = kx = 500 \text{ N/m} \times 0.02 \text{ m} = 10 \text{ N}$ . When the spring is stretched by 2 cm, an internal force of 10 N develops in it, preventing any further elongation.

Because of the proportional relationship between the two variables, a graph of  $\vec{F}$  against  $x$  results in a straight line passing through the origin, with a slope of  $k$ . A graph of  $\vec{F}$  against the total length,  $l$ , also has the same slope, though it does not pass through the origin. Instead, it passes through the  $x$ -axis at the unstretched length,  $L$ .



Stretching or compressing a spring both clearly involve a displacement by the external force against the internal force. Thus, work must be done in the process. The work done is stored as the elastic potential energy of the spring, which may be utilized later to do work.

As the force is not necessarily constant, the work done in stretching the spring must be found using the area under the  $\vec{F}$  vs.  $x$  graph. A simple calculation shows that the elastic energy for elongation,  $x$ , to be  $W = \frac{1}{2}kx^2$ .



## STRESSES AND STRAINS IN SOLIDS

Solids may be stretched or compressed even when they are not in the form of a spring; however, for realistic values of the external force, the change of length is too small to be noticed. This may be explained by the fact that solids can be thought of as an array of atoms interacting with each other through strong, spring-like forces.

For a wire or a rod, Hooke's law is often stated in terms of the tensile stress  $S = \frac{\vec{F}}{A}$ , which is a measure of the internal force per unit cross-sectional area, and the strain  $e = \frac{x}{L}$ , which is the fractional change in length.

$$S = Ye \text{ or } \frac{\vec{F}}{A} = Y \frac{x}{L}.$$

The constant  $Y$ , known as Young's modulus, varies from one material to another, and determines how hard it is to change the length of a given solid.

### Practice

Use the following information to answer the next question.



18. A spring with a spring constant of 16 464 N/m is attached to the ground. A ball that has a mass of 2 kg is dropped on the spring from a height of 1 m above the top of the spring, as shown in the given figure. When the ball is dropped on the spring, how much will the spring decrease in length?
- A. 2 cm                      B. 3 cm  
C. 4 cm                      D. 5 cm

### Numerical Response

19. A compressed spring has a potential energy of 10 J. The maximum energy it can impart to a 1.2-kg ball is \_\_\_ J. (Round and record your answer to two digits.)

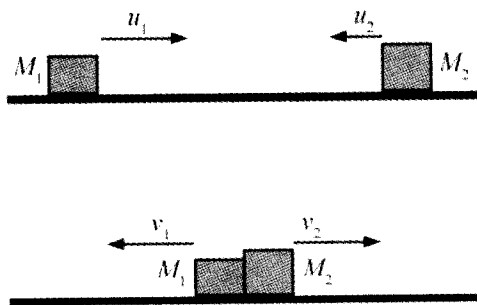


**12.2.2.1** investigate the laws of conservation of momentum and of energy in one and two dimensions by carrying out experiments or simulations and the necessary analytical procedures

### ONE-DIMENSIONAL COLLISIONS

If two masses moving in a straight line collide so that the common surface of contact is perpendicular to their velocities, the collision is said to be head-on or one-dimensional.

Let the masses  $M_1$  and  $M_2$  have initial velocities  $\vec{u}_1$  and  $\vec{u}_2$  directed along the  $x$ -axis. Let  $\vec{v}_1$  and  $\vec{v}_2$  be the final velocities. The velocities are considered positive if they are directed rightward and negative if they are directed leftward.



If the collision is elastic, the final velocities can be obtained by applying the law of conservation of momentum, and equating the total kinetic energy before and after the collision.

$$\vec{v}_1 = \frac{2M_2\vec{u}_2 + (M_1 - M_2)\vec{u}_1}{M_1 + M_2}$$

$$\vec{v}_2 = \frac{2M_1\vec{u}_1 + (M_2 - M_1)\vec{u}_2}{M_1 + M_2}$$

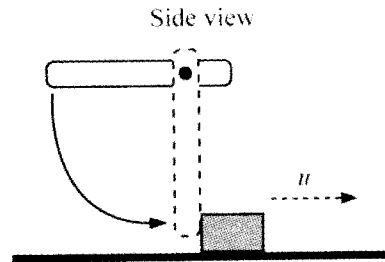
If the two masses are equal, the velocities are simply exchanged in the collision:

$$\vec{v}_1 = \vec{u}_2; \vec{v}_2 = \vec{u}_1$$

### AN EXPERIMENT TO DETERMINE WHETHER A COLLISION IS ELASTIC

Take two rigid, identical disc-shaped masses with flat bases.

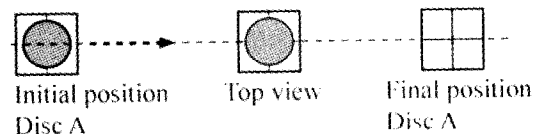
You can easily set up a handmade arrangement to get one of the discs moving with a consistent velocity. For example, you could let the end of a pivoted rod fall from a fixed position so that it strikes the disc at the end of its fall.



Fix a large sheet of chart paper to a flat board. Mark a square to fix the initial position of the “striker” disc A. Note the final position of the disc on the same sheet by marking another square. This position depends on the amount of kinetic friction acting on the disc.

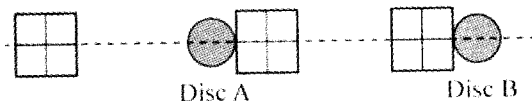


Now place the other disc B, midway between these two positions so that disc A will collide head-on with it after being struck.



If the collision is elastic, discs A and B will just exchange velocities during the collision. This is roughly true for metal discs. Since B is at rest before the collision, this implies that A should come to rest just after the collision, while B carries on with the velocity of A at the position of collision.

If the collision were perfectly elastic, the final positions of the two discs would be as shown below.

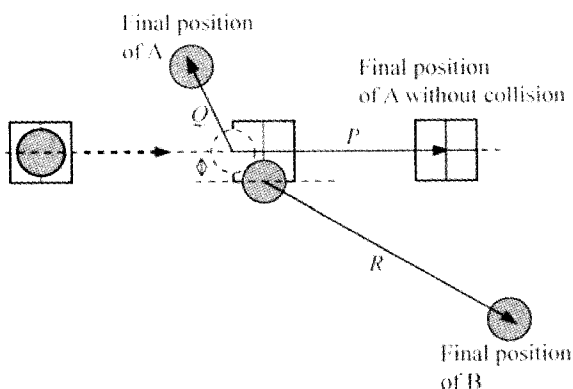




In practice, there will be deviations because of any asymmetry in the “striker” mechanism or small errors in placement. A small amount of energy will be lost as sound during the collision.

If possible, use discs of softer material, like rubber. See if you observe systematically larger deviations from the elastic case.

Note that the conservation of momentum holds true during the collision, because the force of kinetic friction does not have enough time to act, but not before or after the collision.

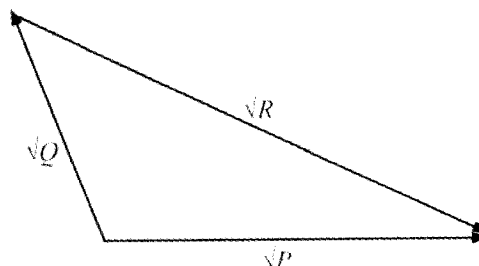


## VERIFYING CONSERVATION OF MOMENTUM IN TWO DIMENSIONS

You can modify the first experiment to quantitatively verify conservation of momentum in two dimensions. Just shift the initial position of B from the straight line on which the centre of A moves.

Measure the vectors  $\vec{P}$ ,  $\vec{Q}$ , and  $\vec{R}$  after marking the final positions of the two discs. Measure the angles with respect to  $\vec{P}$ .

On a separate piece of paper, draw to scale a vector diagram showing three vectors whose magnitudes are  $\sqrt{P}$ ,  $\sqrt{Q}$ ,  $\sqrt{R}$ , and with the same orientation as the original  $\vec{P}$ ,  $\vec{Q}$ ,  $\vec{R}$  vectors. Theoretically, the conservation of momentum in the collision implies that the three vectors form a triangle.



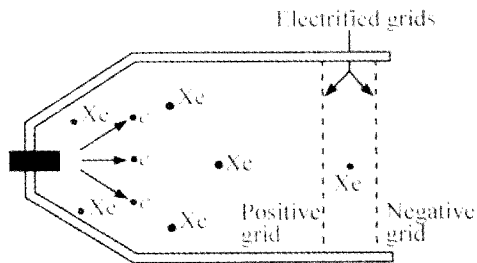


### Practice

Use the following information to answer the next question.

The Deep Space 1 mission (DS1) uses an ion propulsion system (IPS) on the DS1 capsule. The IPS involves the ionizing of atoms of xenon, accelerating these ions through an electric field produced by electrified grids, and ejecting these ions into space behind the capsule.

In the IPS chamber, high-speed electrons collide with xenon atoms. These collisions can ionize the xenon atoms. The electric field then accelerates the ions and ejects them from the IPS chamber, propelling the DS1 capsule forward.



IPS Operating Specifications for DS1

<b>propellant ions</b>	Xe <sup>+</sup>
<b>total mass of propellant</b>	81.5 kg
<b>mass of empty DS1 capsule</b>	489.5 kg

The energy required to ionize a xenon atom = 12.1eV

The mass of a single

xenon atom =  $2.18 \times 10^{-25}$  kg

The average exit speed of the xenon ions

= 43.0 km/s

20. If all of the xenon propellant could be expelled in a single short burst, the change in the speed of the DS1 capsule after all the fuel was expelled would be

- A. 6.14 m/s
- B. 7.16 m/s
- C.  $6.14 \times 10^3$  m/s
- D.  $7.16 \times 10^3$  m/s

### Open Response

Use the following information to answer the next question.

A student sits on a chair fixed to a cart that is initially at rest on frictionless rails. Answer the following questions, using what you know about the conservation of momentum and isolated systems. You may introduce equations to help with your explanation.

21. If the student throws a sandbag off the front of the cart, will the cart move? Is momentum conserved?

---



---



---



---



**12.2.2.2** design and conduct an experiment to verify the conservation of energy in a system involving kinetic energy, thermal energy and its transfer (heat), and gravitational and elastic potential energy

## CONSERVATION OF ENERGY IN MECHANICS

The law of conservation of energy finds repeated application in almost every area of physics. In mechanical systems where friction is negligible, it amounts to the fact that the sum of the potential energy and the kinetic energy remains constant. For vertical motion under gravity, this gives the equation:

$$m\vec{g}y + \frac{1}{2}mv^2 = \text{constant},$$

or equivalently,

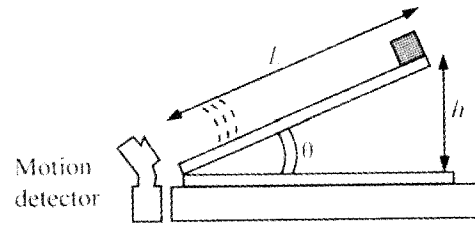
$$\vec{v}^2 + 2\vec{g}y = \text{constant}.$$

A straightforward experiment to verify this can be set up if an ultrasonic motion detector with a data logging interface is available for use. A light object made of plastic or rubber can be dropped from some height above the ultrasonic detector. The detector measures the vertical position,  $y$ , of the object at different times, and a computer can estimate the velocity,  $\vec{v}$ , of the object at those times. A graph of  $\vec{v}^2$  against  $y$  for the data collected must be a straight line with a slope of  $-2\vec{g}$  if the equation above is true.

### VERIFYING CONSERVATION OF ENERGY WITH FRICTIONAL EFFECTS

A block sliding down an inclined plane provides a simple means of verifying the conservation of energy when friction is present. The coefficient of kinetic friction  $K$  between the block and the plane can be measured separately, and is assumed to be known. The mass of the block is denoted as  $m$ .

The block is arranged so that it starts sliding from rest from some height  $h$  above the base. The angle  $\theta$  is increased gradually until this happens spontaneously. An ultrasonic motion detector is placed at the minimum distance of detection from the base.



The motion detector is adjusted so that it tracks the position of the sliding block as it changes with time. A computer can calculate the velocities from the timed position data. The point where the block slips off the plane will be marked by an abrupt change in velocity, so the final velocity  $\vec{v}$  of the block on the plane will be the value just before this point.

The normal force of the plane on the block is given by:

$$\vec{F}_N = m\vec{g} \cos \theta;$$

therefore, the frictional force on the block is given as  $\vec{F}_f = \mu_K \vec{F}_N = \mu_K m\vec{g} \cos \theta$ .

As the block slides down the plane, its potential energy is partly converted to kinetic energy, while the remainder is used to do work against the frictional force. The work done is lost as heat and sound energy and can be calculated using the equation

$$W = \vec{F}_N L = \mu_K m\vec{g} L \cos \theta.$$

Initially, the block has no kinetic energy, and has potential energy of

$$E_p = m\vec{g}h.$$

Just before it slips off the plane, the block has kinetic energy given by:

$$E_k = \frac{1}{2}m\vec{v}^2$$

and potential energy of 0.

The conservation of energy predicts that

$$m\vec{g}h = \frac{1}{2}m\vec{v}^2 + \mu_K m\vec{g}L \cos \theta$$

or,

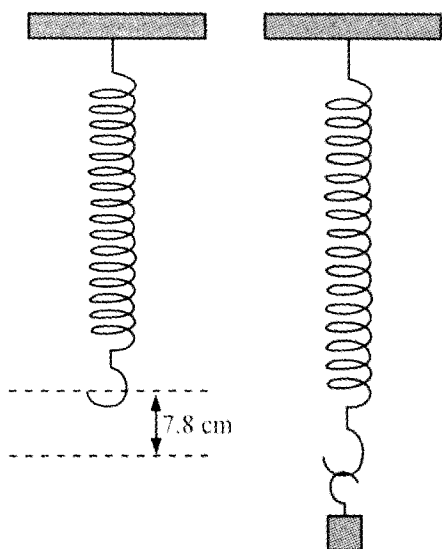
$$\vec{g}h = \frac{1}{2}\vec{v}^2 + \mu_K \vec{g}L \cos \theta.$$

By checking that this equation holds true for the set of experimentally measured values of  $h$ ,  $L$ ,  $\theta$ , and  $\vec{v}$ , the conservation of energy is verified.

Conversely, the same setup can be used to estimate the coefficient of kinetic friction, assuming the conservation of energy.



## INTERCONVERSION OF GRAVITATIONAL POTENTIAL, ELASTIC POTENTIAL, AND KINETIC ENERGY



It is possible to verify the conservation of mechanical energy in a system involving both elastic and gravitational potential energy by using an oscillating spring-mass system. A long spring is loaded with a mass, and then a motion detector is placed directly under the mass. The equilibrium position  $y_0$  is noted.

The system is then set oscillating by pulling the mass below its equilibrium position and releasing it. The mass,  $m$ , and the spring constant,  $k$ , are chosen so that the oscillations are as slow as possible.

The motion detector records the position  $y$  of the mass at various times, and a computer estimates the velocity  $\vec{v}$  at each point in the timed position data.

At any position, the elastic potential energy, the gravitational potential energy, and the kinetic energy of the system are  $\frac{1}{2}k(y - y_0)^2$ ,  $m\vec{g}y$ , and  $\frac{1}{2}m\vec{v}^2$ , respectively.

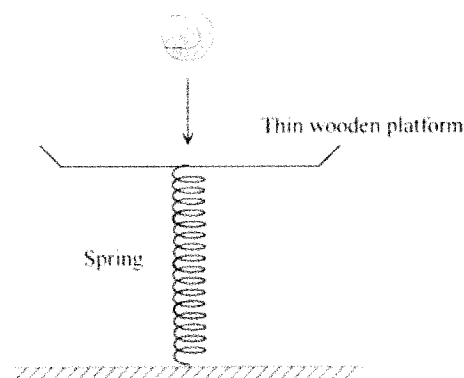
The conservation of mechanical energy for this system implies that if the frictional forces acting are negligible,

$$\frac{1}{2}k(y - y_0)^2 + m\vec{g}y + \frac{1}{2}m\vec{v}^2 = \text{constant.}$$

In practice, the spring-mass oscillations will gradually die down, but for moderate intervals of time, the above equation can be verified approximately using experimental data.

### Practice

Use the following information to answer the next question.



22. A spring is attached to a thin wooden platform, as shown in the given figure. A heavy ball is dropped from a certain height onto the wooden platform. What is the transfer of energy that takes place during this action?
- From the elastic potential energy of the spring to the elastic potential energy of the ball.
  - From the gravitational potential energy of the spring to the elastic potential energy of the ball.
  - From the gravitational potential energy of the ball to the elastic potential energy of the spring.
  - From the gravitational potential energy of the ball to the gravitational potential energy of the spring.



### Numerical Response

Use the following information to answer the next question.

Adeline sets up an experiment using a spring-mass system undergoing vertical oscillations, and an ultrasonic motion detector with a resolution of 1 mm connected to a computer interface. She uses a long spring with a spring constant of 100 N/m and a mass of 0.80 kg. The motion detector measures the vertical position,  $y$ , of the mass at fixed intervals, and a computer calculates the velocity  $v$  from the position readings.

Adeline notes that the spring stretches by 0.078 m when the mass is loaded gently and brought to rest. At this position of the mass, the reading of the ultrasonic detector is 0.211 m.

She then sets the mass oscillating.

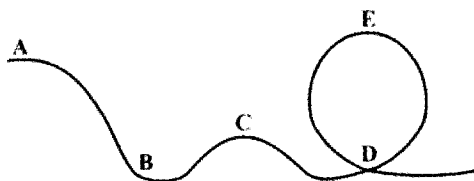
When  $y = 0.240$  m, the computed estimate of the speed is 0.640 m/s.

23. Assuming that the total energy of the spring is conserved, the expected value of the speed when  $y = 0.150$  m is \_\_\_ m/s.

### Open Response

Use the following information to answer the next question.

The following diagram shows the track of a toy roller coaster. The car has a mass of 1.20 kg.



24. The roller coaster car had no kinetic energy at the top of the loop and a lot of kinetic energy at the bottom. Energy cannot be created or destroyed, so how did the car gain kinetic energy?

---



---



---



---



---

**12.2.3.1** analyse and describe, using the concepts and laws of energy and of momentum, practical applications of energy transformations and momentum conservation

### SAFETY DEVICES BASED ON MOMENTUM TRANSFER

As acceleration is just the rate of change of velocity, Newton's second law,  $\vec{F} = m\vec{a}$ , implies that if a constant force acts for a time  $t$ , then the force applied over some period of time is just the change in momentum, or impulse.

$$\vec{F}t = m\vec{a}t = m\Delta\vec{v} = \Delta\vec{p}$$

The equation is valid even when the force is not constant if  $\vec{F}$  denotes the average force.



Conversely, when there is an impulse or momentum transfer  $\Delta\vec{p}$  in a collision where the forces are otherwise unknown, this leads to the equation

$$\vec{F} = \frac{\Delta\vec{p}}{t}. \text{ The shorter the duration of the collision,}$$

the larger the average force involved. Usually, collisions between rigid objects involve a quick transfer of momentum, resulting in large forces.

Many safety devices, such as airbags, are based on the idea of making the momentum transfer to the objects they protect slower. This is achieved by using deformable objects which extend the time of collision as they deform. This reduces the forces brought into play and hence the chances of physical damage.

## **MECHANICAL APPLICATIONS BASED ON ENERGY TRANSFORMATIONS**

Almost all devices that people use are based on the transformation of energy. Perhaps the three most important kinds of devices involving the interconversion of mechanical energy with other forms of energy are engines, dynamos, and motors. Engines convert chemical or heat energy to mechanical energy. Dynamos or generators convert kinetic energy to electrical energy, while motors do the opposite.

Apart from these, there are also a number of commonly used devices that are based on the interconversion of kinetic and potential energy. Mechanical clocks, such as the types that use pendulums, or springs, are based on the conversion of the potential energy, stored in the pendulum or spring, back into the kinetic energy of the clock's moving parts. The same principle is used in key wound toys.

Shock absorbers in vehicles are based on the conversion of mechanical energy into heat energy. This heat is the result of some form of damping medium converting stored elastic potential energy into some form of energy other than kinetic.

Motor vehicles include a suspension system to isolate the frame of the vehicle from the forces acting on the wheels. The two active parts of the suspension are the springs and the so-called shock absorbers. When a vehicle passes over a bump or depression, the springs in the suspension contract or expand. This ensures that the change in height is not immediately transferred to the frame of the vehicle. If the springs were used by themselves, they would oscillate for a period of time making the ride uncomfortable. The role of the shock absorber is to reduce this mechanical energy from the oscillating spring and convert it to heat. As the spring loses its mechanical energy, its oscillations slow down (i.e., are damped).



### Practice

Use the following information to answer the next question.

The Deep Space 1 mission (DS1) uses an ion propulsion system (IPS) on the DS1 capsule. The IPS involves ionizing atoms of xenon, accelerating them through an electric field produced by electrified grids, and ejecting the ions into space behind the capsule.

#### IPS Chamber of the DS1 Capsule

In the IPS chamber, high-speed electrons collide with xenon atoms. These collisions can ionize xenon atoms. The electric field then accelerates the ions and ejects them from the IPS chamber, which propels the DS1 capsule forward.

#### IPS Operating Specifications for DS1

propellant ions	Xe <sup>+</sup>
total mass of propellant	81.5 kg
mass of DS1 capsule	489.5 kg

#### (without propellant)

energy required to ionize a xenon atom	12.1 eV
mass of a single xenon atom	$2.18 \times 10^{-25}$ kg
exit speed of xenon ions	43.0 km/s

25. The physics principle that best describes the propulsion of the DS1 capsule is the law of conservation of \_\_\_\_
- charge.
  - energy.
  - momentum.
  - nucleon number.

### Open Response

26. Briefly explain the purpose and mechanism of a telescopic-type hydraulic shock absorber in a car.

---



---



---



---



---



---

**12.2.3.2** identify and analyse social issues that relate to the development of vehicles

## VEHICULAR SAFETY AND NEWTON'S LAWS

The use of certain passive safety devices in vehicles (e.g., seat belts or airbags) has been mandated by law in many parts of the world. Ontario law requires the use of seat belts and child safety seats in cars. Although many drivers and passengers view the use of seat belts as an inconvenience, they are indisputably effective in reducing injury in the event of a crash or sudden stop. They spread the forces over a larger area of the body and limit the chances of a violent impact between a vehicle's occupants and the interior of the car.

Additional passive safety features including energy absorbing windshields and crumple zone technologies are designed to minimize injury if a crash occurs. On the other hand, active safety features such as anti-lock braking systems, intelligent speed adaptation, and electronic stability control help prevent crashes from occurring in the first place.



One issue arising in connection with car safety is the legal allocation of liability and responsibility.

In many countries, the responsibility for car safety lies with the manufacturers rather than the users. Another major issue with additional safety devices in automobiles is that their research and development costs can be extremely high and can push up the cost of the automobile. So car models with added safety features may not be as affordable as similar models without them. As sales are the primary concern for car manufacturers, it may be favourable to their business interests to keep only the minimum safety features as required by regional law. On the other hand, it is in the interest of insurance companies to ensure the maximum car safety possible.

Crash tests with dummies to simulate the presence of human occupants are frequently used by automobile manufacturers to evaluate car safety. Many countries now have laws requiring a certified degree of safety, as measured by a crash test. Physical crash tests are expensive and are not always easily reproducible. On the other hand, computer simulations are not considered to be sufficiently authentic.

The evaluation of safety features and safety ratings for cars can also turn out to be contentious, as adverse evaluations may affect the commercial interests of individual manufacturers. There have been cases of legal conflict between car manufacturers and safety activists in the past.

The effectiveness of certain classes of safety features is also debatable. For example, some statistics indicate that the role of airbags in saving lives has so far been much less than that of seat belts, despite the high costs of airbags, which may cost more than \$500. Further, as an airbag inflates, it may cause injuries to occupants of the vehicle due to its rapid rate of expansion in the initial moments of a crash.

### Practice

27. Which of the following statements regarding airbags in vehicles is **false**?
- A. They can cause a loss of hearing upon deployment.
  - B. They inflate at a rate that allows them to be inflating as the occupants make initial contact.
  - C. They can cause fatal injuries if, while inflating, they come in contact with a person's head or chest.
  - D. They can prevent fatal injuries by cushioning the head and upper torso in a high-speed collision.

### Open Response

28. Discuss the usage, cost, and effectiveness of airbags as safety devices in automobiles.

---



---



---



---



---



---



## SOLUTIONS—ENERGY AND MOTION

1. B	7. -0.167	13. B	19. 10	25. C
2. C	8. OR	14. B	20. D	26. OR
3. D	9. A	15. 2.4	21. OR	27. B
4. A	10. B	16. 4.44	22. C	28. OR
5. D	11. D	17. OR	23. 0.24	
6. C	12. 3,8,3,3	18. D	24. OR	

### 1. B

The mass of the body = 10 kg

$$\text{Gravitational potential } U = -\frac{GM}{r}$$

$G$  = universal gravitational constant

$M$  = the mass of the body

$r$  = the distance

$$U = -\frac{GM}{r}$$

$$= -\frac{6.67 \times 10^{-11} \times 10}{10} = -6.67 \times 10^{-11} \text{ J/kg}$$

$\therefore$  The gravitational potential is  $-6.67 \times 10^{-11} \text{ J/kg}$

### 2. C

In this question, the energy that the stationary pendulum has at the top of its swing must be classified as either kinetic, mechanical, potential, or thermal.

Kinetic energy is energy due to motion. Since the pendulum is stationary, it has no motion, and therefore no kinetic energy, therefore A is incorrect.

Thermal energy is energy due to the random motion of molecules (also known as heat). The temperature of the pendulum is not due to its swing, therefore the pendulum does not have thermal energy. Answer B is incorrect.

Potential energy is energy due to position or condition. The potential energy of a system increases as its height increases, and decreases as its height decreases. Because the pendulum at the top of its swing is at its greatest height, it has potential energy. Answer C is correct.

Chemical energy is the energy stored in the bonds of molecules. The energy of the pendulum, is not due to chemical energy, therefore D is incorrect.

### 3. D

The chemical energy stored in the fuel is transformed into heat energy as the fuel is burned in the engine. This heat energy is transformed into mechanical energy by the engine. The mechanical energy is used to do mechanical work.

### 4. A

When a collision is perfectly elastic, the following relation holds true:

$$\vec{u}_1 - \vec{u}_2 = \vec{v}_2 - \vec{v}_1 \quad (1),$$

where  $\vec{u}_1, \vec{u}_2 \rightarrow$  initial velocity of the bodies before the collision;

$\vec{v}_1, \vec{v}_2 \rightarrow$  velocity of the bodies after the collision.

$\therefore$  From equation (1),

$$\vec{v}_2 - \vec{v}_1 = (200 \text{ cm/s}) - (-100 \text{ cm/s}) = 300 \text{ cm/s} \quad (2)$$

By the principle of conservation of momentum,

$$500\vec{v}_1 + 100\vec{v}_2 = 9 \times 10^4$$

$$5\vec{v}_1 + \vec{v}_2 = 900 \quad (3)$$

From equations (1) and (2),

$$\vec{v}_1 = 100 \text{ cm/s}$$

The velocity of the first sphere after the collision is 100 cm/s.

### 5. D

Momentum is conserved in all collisions, but kinetic energy is not conserved in an inelastic collision.

The statement "momentum is conserved, but kinetic energy is not conserved", is the best description for this collision.

### 6. C

Since the ornament was stationary before it broke, its momentum was zero. Therefore, the total momenta of the three pieces after the break must also add to zero.

Since the total momentum of the system must be zero, the three momentum vectors can be added tail-to-tip to see the solution more clearly. In this case, the tail of momentum A can move to touch the tip of momentum B; therefore, momentum C will complete a closed triangle. The cosine law can be used to find the magnitude of momentum C.

$$\begin{aligned} C &= \sqrt{A^2 + B^2 - 2AB\cos 60^\circ} \\ &= \sqrt{(2.52)^2 + (1.91)^2 - 2(2.52)(1.91)\cos 60^\circ} \\ &= 2.28 \text{ kg} \cdot \text{m/s} \end{aligned}$$



OR, using vector components:

$$[p_x = p \cos \theta, p_y = p \sin \theta]$$

$$p_{Af} = [2.52 \text{ kg} \cdot \text{m/s}, 0]$$

$$p_{Bf} = [-0.955 \text{ kg} \cdot \text{m/s}, 1.65 \text{ kg} \cdot \text{m/s}]$$

$$p_{Cf} = [1.57 \text{ kg} \cdot \text{m/s}, 1.65 \text{ kg} \cdot \text{m/s}]$$

momentum of piece

$$p_{Cf} = \sqrt{1.57^2 + 1.65^2}$$

$$= 2.28 \text{ kg} \cdot \text{m/s}$$

7. **-0.167**

Before the bullet is fired, the momentum of the system comprised of the gun and bullet is 0.

Let the recoil velocity be  $\bar{v}$  m/s.

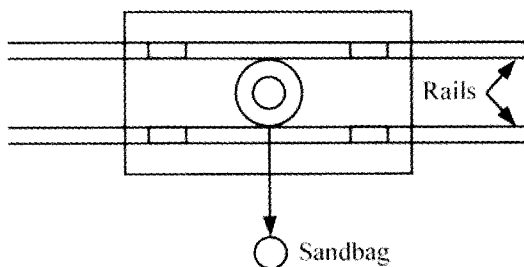
Applying the principle of conservation of linear momentum,

$$0 = 0.05 \times 400 + 120 \times \bar{v}$$

$$\bar{v} = -\frac{20}{120} = -0.167 \text{ m/s}$$

The negative sign indicates that the astronaut recoils in a direction opposite to the velocity of the bullet.

8. **Open Response**



Momentum is still conserved, but the cart is prevented from moving perpendicular to the rails by the way the wheels meet the rails. The student, and perhaps the chair, will recoil in the opposite direction of the sand bag's motion, thus conserving the momentum of the system.

9. **A**

The work done by the force of gravity is negative, compared to the work done by the person.

∴ the work done by the force of gravity = -392 J

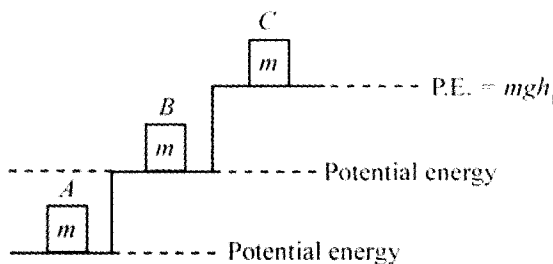
10. **B**

Inside the charged metallic sphere, every point on the outer surface of the sphere has the same electric potential; hence, the electric field is zero. No work is done inside a positively charged metallic sphere. Outside the charged metallic sphere there exists a potential gradient, and therefore, there is an electric field at every point.

11. **D**

When a body is located at any point above the reference level for potential energy, it has positive potential energy. At points below the reference level, it has negative potential energy.

In the figure given below, position B is the reference level.



When a mass,  $m$ , is raised to the point C, its gravitational potential energy is given by potential =  $mgh_1$ .

When at A, its potential energy (P.E.) is  $(-mgh_1)$ .

∴ The gravitational potential energy may be negative.

12. **3,8,3,3**

The gravitational potential energy of the planet is of the form  $U = -mgx$ , which depends only upon the  $x$ -coordinate. When the position changes from  $x_1$  to  $x_2$ , the change of the potential energy is

$$\Delta U = U_{\text{final}} - U_{\text{initial}} = -mg(x_f - x_i)$$

$$\Delta U = mg(x_1 - x_2)$$

$$\Delta U = (8.4 \times 10^{25} \text{ kg})(1.5 \times 10^{-4} \text{ N/kg})(0.3 \times 10^{12} \text{ m})$$

$$\Delta U = 3.8 \times 10^{33} \text{ J}$$

13. **B**

The orbital velocity of the given shuttle in its circular orbit close to Earth's surface is given by  $v_o = \sqrt{\frac{GM}{r}}$  where  $v_o$  is the orbital velocity of the shuttle,  $G$  is the gravitational constant,  $M$  is Earth's mass, and  $r$  is the radius of the circular orbit.

Since the circular orbit of the shuttle is close to Earth's surface,  $r = R$  in the above equation.

$$v_o = \sqrt{\frac{GM}{R}}$$

According to Newton's universal gravitational law, the acceleration due to gravity is given as  $g = \frac{GM}{R^2}$ .

$$\therefore GM = gR^2;$$

therefore, the orbital velocity of the shuttle is  $v_o = \sqrt{gR}$ ;

$$v_o = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$v_o = 7919.5 \text{ m/s}$$

$$v_o = 7.9195 \text{ km/s}$$

The escape velocity is given as  $v_e = \sqrt{2Rg}$

$$v_e = \sqrt{2} \times 7.9195 \text{ km/s}$$

$$v_e = 11.2 \text{ km/s}$$

A velocity equal to the difference between the escape velocity and the orbital velocity of the shuttle should be added to its orbital velocity for it to overcome the gravitational pull of Earth.



The additional velocity required = the escape velocity – the orbital velocity of the shuttle.  
 = 11.2 km/s – 7.9195 km/s = 3.2805 km/s

**14. B**

Let  $R$  = Earth's radius, and  $h$  = the height of the satellite above Earth's surface.

The orbital velocity =  $v_o = \sqrt{\frac{gR^2}{R+h}}$ .

The escape velocity =  $v_e = \sqrt{2gR}$ .

When

$$v_o = \frac{1}{2}v_e \Rightarrow \sqrt{\frac{gR^2}{R+h}} = \frac{1}{2}\sqrt{2gR}$$

$$\Rightarrow \frac{gR^2}{R+h} = \frac{1}{2}gR$$

$$\frac{R}{R+h} = \frac{1}{2}$$

$$h = R = 6400 \text{ km}$$

The height of the satellite is 6400 km above Earth's surface.

**15. 2.4**

Escape velocity  $v_e = \sqrt{\frac{2GM}{R}}$

[where  $G$  = gravitational constant,  $M$  = mass, and  $R$  = radius]

$$\therefore v_e = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2 \times 7.4 \times 10^{22} \text{ kg}}{1740 \times 10^3 \text{ m}}}$$

$$= 2.38 \times 10^3 \text{ m/s} = 2.4 \text{ km/s}$$

**16. 4.44**

Use the equation for momentum:

$$\vec{F}\Delta t = \Delta\vec{p} = m\Delta\vec{v}$$

Insert the values provided by the question:

$$\vec{F}(2.27 \text{ s}) = (172 \text{ kg})(5.86 \times 10^{-3} \text{ m/s})$$

$$\vec{F} = 4.44 \times 10^{-1} \text{ N}$$

**17. Open Response**

$$E_p = E_k \frac{Gm_1m_2}{r} = \frac{1}{2}mv^2$$

The calculation requires the asteroid's mass, so

$$g = \frac{Gm}{r^2} \text{ yields}$$

$$m = \frac{(0.25 \text{ N/kg})(370\,000 \text{ m})^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2}$$

$$m = 5.13 \times 10^{20} \text{ kg}$$

and then gives

$$v = \sqrt{\frac{2Gm}{r}}$$

$$\sqrt{\frac{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(5.13 \times 10^{20} \text{ kg})}{(370\,000 \text{ m})}}$$

$$= 430 \text{ m/s} = 4.3 \times 10^2 \text{ m/s}$$

**18. D**

Assume that the length of the spring decreases by  $x$  m. According to the law of conservation of energy, elastic potential energy of the spring = gravitational potential energy of the ball.

Let  $k$  be the spring constant of the spring,  $x$  be the distance the spring is compressed,  $m$  be the mass,  $g$  be the acceleration due to gravity, and  $h$  be the height.

$$\frac{1}{2}kx^2 = mg(h+x)$$

Inserting the values,

$$\frac{1}{2} \times (16\,464) \times x^2 = 2 \times 9.8(1+x)$$

$$x = 0.05 \text{ m}$$

The length of the spring decreases by 5 cm.

**19. 10**

Because energy is conserved, the maximum energy (any form) that the spring can impart to the 1.2-kg ball is 10 J. [Note: In reality, the energy imparted would be less than the full 10 J due to heat losses etc.]

**20. D**

The propulsion that occurs can be described using the law of conservation of momentum.

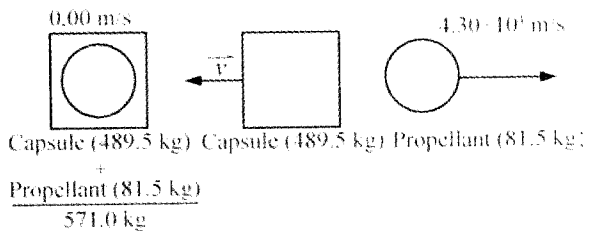
$$\vec{p}_{\text{before}} = \vec{p}_{\text{after}}$$

$$\vec{p} = m\vec{v}$$

$$(571.0 \text{ kg})(0.00 \text{ m/s}) = (489.5 \text{ kg})(\vec{v})$$

$$+ 81.5 \text{ kg}(-4.30 \times 10^4 \text{ m/s})$$

$$\vec{v} = 7.16 \times 10^3 \text{ m/s}$$



**21. Open Response**

Since the student, the chair, the cart ( $c$ ) and the sand bag ( $s$ ) are initially stationary, the initial momentum is zero. Since momentum is conserved in an isolated system, it will remain zero.

$$\vec{p}_i = \vec{p}_f$$

$$m_{c+s}\vec{v}_{c+s} = m_c\vec{v}_c + m_s\vec{v}_s$$

$$(m_{c+s})(0) = m_c\vec{v}_c + m_s\vec{v}_s$$

$$0 = m_c\vec{v}_c + m_s\vec{v}_s$$

$$m_c\vec{v}_c = -m_s\vec{v}_s$$

$$\vec{v}_c = -\frac{m_s\vec{v}_s}{m_c}$$

Thus, the cart-student-chair will move in the opposite direction the sandbag is thrown, as indicated above.



22. C

Gravitational potential energy is the energy possessed by an object by virtue of its height above the surface of Earth. The ball initially possesses gravitational potential energy. When it is dropped onto the wooden platform, it compresses the spring, hence, its gravitational potential energy is transferred into the elastic potential of the spring.

23. 0.24

The conservation of energy in this case takes the form:

$$\frac{1}{2}k(y - y_0)^2 + mgy + \frac{1}{2}mv^2 = E(\text{constant}).$$

The position  $y_0$  refers to the situation where the spring is at its original length. Note that this position is not necessarily within the range of the oscillating mass.

According to the information given,  
 $y_0 = (0.211 + 0.078) \text{ m} = 0.289 \text{ m}$ .

When  $y = 0.240 \text{ m}$ , then  $v = 0.640 \text{ m/s}$ .

Putting these values along with those of the known constants into the equation, you get the constant total mechanical energy of the system to be

$$E = \frac{1}{2}k(y - y_0)^2 + mgy + \frac{1}{2}mv^2$$

$$E = \left( \frac{1}{2} \times 100 \times (0.24 - 0.289)^2 \right.$$

$$\left. + 0.8 \times 9.8 \times 0.24 + \frac{1}{2} \times 0.8 \times 0.64^2 \right) \text{ J}$$

$$E = 2.166 \text{ J}.$$

Once the constant  $E$  is known, you can use the same equation to find the unknown value of  $v$  when

$$y = 0.150 \text{ m}.$$

$$\frac{1}{2}mv^2 = E - \frac{1}{2}k(y - y_0)^2 - mgy$$

$$v = \sqrt{\frac{2E}{m} - \frac{k(y - y_0)^2}{m} - gv}$$

$$= \sqrt{\left( \frac{2 \cdot 2.166}{0.8} - \frac{100(0.15 - 0.289)^2}{0.8} - 9.8 \cdot 0.15 \right)} \text{ m/s}$$

$$= 0.24 \text{ m/s}$$

24. Open Response

Although energy cannot be created or destroyed, it can be converted from one form to another. Potential energy that the car had at the top of the loop was converted to kinetic energy at the bottom of the loop.

The car started above ground, which gives the car gravitational potential energy. This energy was converted into kinetic energy. At the bottom, there was no potential energy because it had all been converted to kinetic energy.

25. C

As the xenon ions leave the capsule, their momentum is converted to momentum of the capsule. Momentum is conserved.

26. Open Response

The purpose of a shock absorber is to stop the oscillation of the suspension spring it is connected to.

A telescopic-type shock absorber has two tubes, one of which can move inside the other. The inner tube is filled with liquid. The far ends of the tube are attached to the ends of the suspension spring. If the spring expands or contracts, the two tubes have to move relative to one another. The far end of the outer tube drives a piston inside the inner tube.

When the piston moves, it displaces the liquid in the inner tube through small openings attached to a fluid well.

This creates a high resistance to the motion of the piston, which in turn slows the oscillations of the suspension spring. The energy of the spring is lost in overcoming this resistance, causing the liquid to heat up. This heat is then gradually dissipated to the surroundings.

27. B

Airbags are designed to deploy in less than one-tenth of a second after receiving a signal from a sensor that detects a high deceleration exceeding some limiting value.

The airbag needs to be fully inflated before any contact with the occupants of the vehicle occurs in order to slow the occupants deceleration due to the collision. If the airbag were still inflating at the time of contact, it would not slow the rate of deceleration as desired to prevent injury. It is their speed of inflation which makes them useful while also posing a safety hazard.

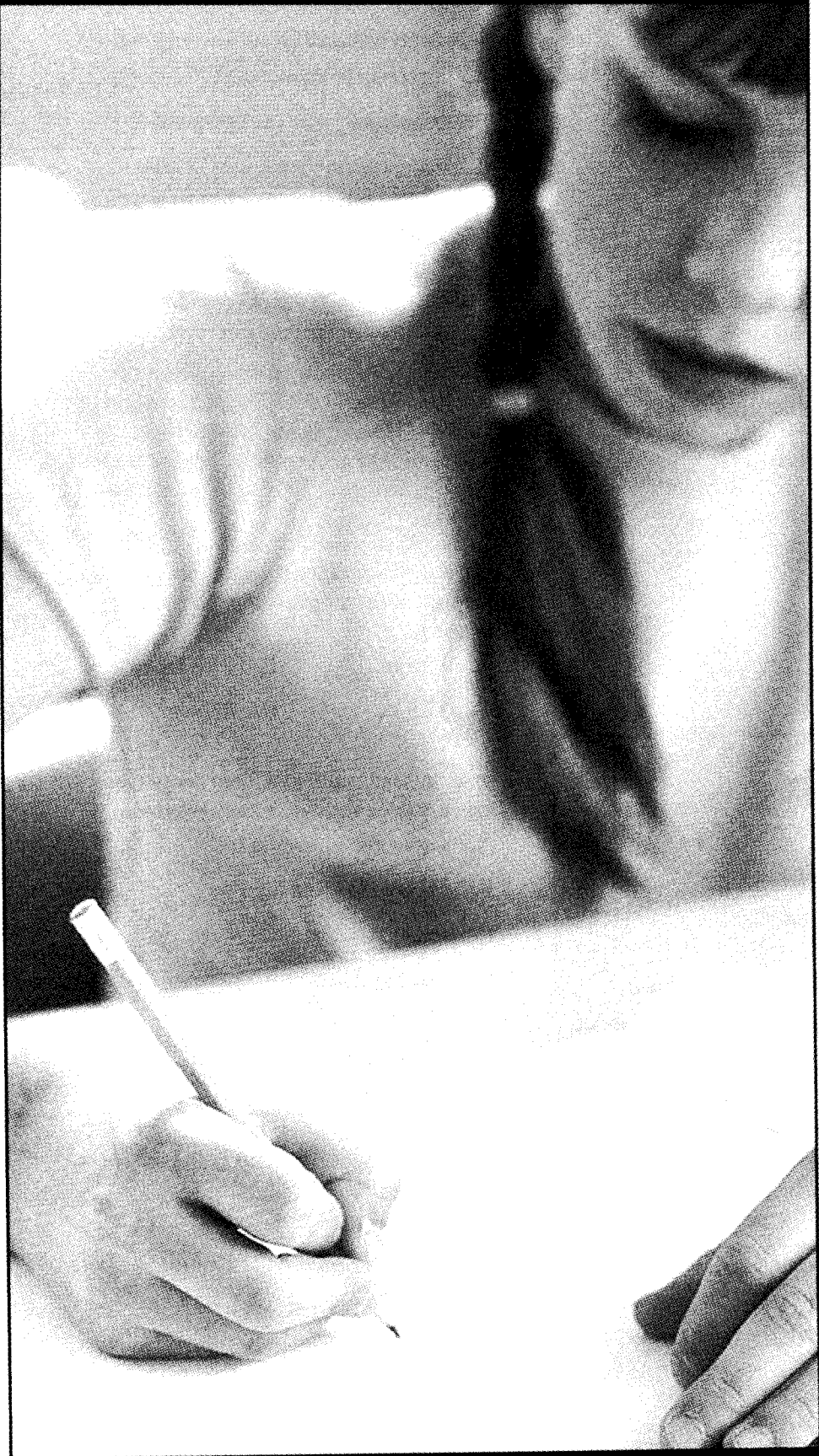
28. Open Response

While airbags are increasingly becoming standard safety features in automobiles, their usage is not mandated by law in Canada. US law does require front airbags. Most new cars come with at least two front airbags. Airbags are still expensive, with costs on the order of \$500 per vehicle.

The actual effectiveness of airbags in saving lives is still controversial, though statistics suggest a steady improvement over the last decade.

Though airbags are meant to be protective devices, they can themselves cause injury and hearing impairment due to their rapid inflation by chemically generated hot gases such as nitrogen. Usually, the injuries that result from airbag deployment are minor, but several cases of fatalities have been recorded. Typically, airbags are likely to be more effective in high-speed collisions, while introducing a safety hazard in low-speed collisions. Airbags need to be handled very carefully when being installed or replaced: they may cause severe injuries if they are accidentally triggered at such times.

# Unit Test

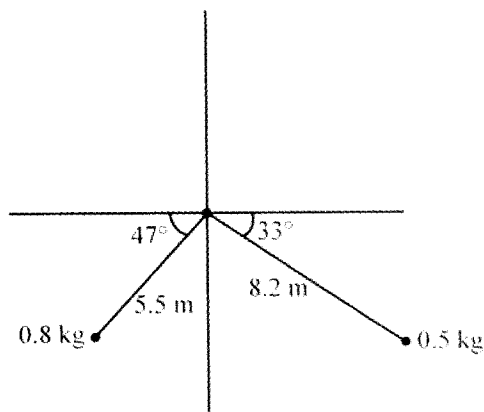


**Open Response**

*Use the following information to answer the next question.*

A small sphere containing an explosive mixture with mass  $M = 2.5$  kg, initially at rest, bursts into three significant fragments of masses 0.5 kg, 0.8 kg, and 1.2 kg, respectively. There are no external forces acting on the sphere or its fragments. The fragments do not exert forces on each other.

The coordinates of the fragments are tracked in a system of coordinates with the origin at the initial position of the sphere. After 1.2 s, the positions of the 0.5-kg fragment and the 0.8-kg fragment are shown in the diagram below.



1. What is the position of the 1.2-kg fragment 1.2 s after the explosion?

---



---



---



---



---



---

2. Flywheels are used in railway engines to?
  - A. increase the momentum
  - B. concentrate mass at that point
  - C. decrease the moment of inertia
  - D. assure the smooth working of the engine

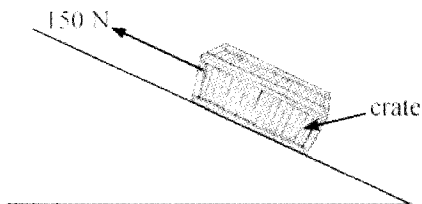
*Use the following information to answer the next question.*

A skier slides down an ice-covered slope that has a length of 3.00 m and a height of 2.00 m. The skier then moves over a horizontal ice path until he comes to rest. The coefficient of friction of the path is 0.04.

3. What is the distance the skier covers from the foot of the slope to when he comes to a stop?
 

A. 17.8 m	B. 27.8 m
C. 37.8 m	D. 47.8 m
4. In an inelastic collision, the energy that appears to be missing is converted into
  - A. sound and momentum
  - B. force and momentum
  - C. sound and heat
  - D. heat and force

Use the following information to answer the next question.



A crate with a mass of 70 kg is pulled over a distance of 12 m up an inclined plane, and in the process, its centre of gravity is raised by 2.0 m. In order to do this, a force of 150 N is applied to the crate in a direction parallel to the inclined plane. [ $g = 10 \text{ m/s}^2$ ]

5. In this situation, how much work is done?
- A. 1.6 kJ                      B. 1.8 kJ  
C. 2.0 kJ                      D. 2.2 kJ

Use the following information to answer the next question.

In 1977, two Voyager spacecraft were launched to explore the outer planets of the solar system. These spacecraft took advantage of a rare planetary alignment that occurs once every 175 years. This alignment allowed these spacecraft to use the gravitational force of one planet to propel them to the next, thus eliminating the need for large onboard propulsion systems.

6. These spacecraft converted gravitational potential energy into what other type of energy?
- A. Solar                      B. Kinetic  
C. Thermal                      D. Electrical

## Numerical Response

Use the following information to answer the next question.

A geostationary satellite of mass 805 kg in a circular orbit above the equator orbits Earth once in 24 hours. The mass of Earth is  $5.97 \times 10^{24}$  kg, while its equatorial radius is  $6.38 \times 10^6$  m. The value of the gravitational constant is  $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ .

7. The height of the geostationary satellite above the equator may be written as  $a.bc \times 10^d$  m. Give your answer in the form  $abcd$ . \_\_\_\_\_
8. A body is projected upward from the surface of Earth with a velocity that is equal to half the escape velocity. Given that  $R_E$  is the radius of Earth, the maximum height reached by the body is.
- A.  $\frac{R_E}{2}$                       B.  $\frac{R_E}{3}$   
C.  $\frac{R_E}{4}$                       D.  $\frac{R_E}{5}$

### Numerical Response

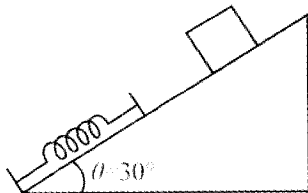
Use the following information to answer the next question.

The velocity required to put a satellite into orbit around a planet is called orbital velocity. A satellite orbiting close to Earth's surface has an orbital velocity of 7.9 km/s. Escape velocity is the minimum velocity with which a body is projected upward so that it escapes the gravitational pull of the planet. On Earth, escape velocity = 11.2 km/s,  $v_{\text{orbital}} = \sqrt{gR}$ , and  $v_{\text{escape}} = \sqrt{2gR}$ , where  $R$  is the radius of Earth.

9. A satellite with a mass of 400 kg is orbiting close to Earth's surface. The kinetic energy of the satellite, expressed in scientific notation, is \_\_\_\_ J.

Use the following information to answer the next question.

A massless spring can be compressed 2.00 m by a force of 200 N. This spring is placed at the bottom of a frictionless inclined plane, which makes an angle of  $30^\circ$  with the horizontal. A 20.0-kg mass is released from rest at the top of the inclined plane and is brought to rest momentarily after compressing the spring by 4 m.



10. What is the distance covered by the mass, before it comes to rest?
- A. 6.25 m                      B. 8.17 m  
C. 9.25 m                      D. 12.35 m

### Open Response

Use the following information to answer the next question.

A spring with a length of 8.0 cm is to be calibrated for use in a spring balance. To do this, the spring is loaded with different masses, and its new length is recorded in each case, resulting in the data below. The acceleration due to gravity is  $9.8 \text{ m/s}^2$ .

Load (kg)	Length of the spring (cm)
0.10	9.2
0.20	10.5
0.30	11.9
0.40	13.1
0.50	14.4
0.60	15.5

11. Using the experimental data above, find the spring constant of the spring in N/m, correct to two significant digits.

---



---



---



---

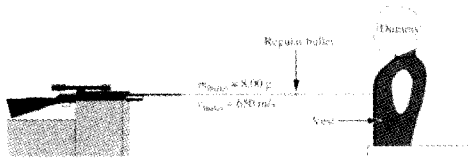


---

**Open Response**

Use the following information to answer the next question.

Several Canadian companies are redesigning and testing bulletproof vests. One company completed a test that involved firing a target rifle at a crash test dummy wearing a vest.

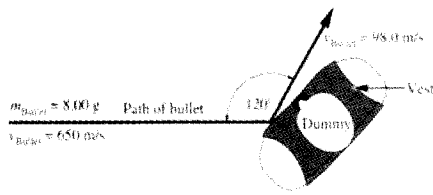


The company tested the vests with both regular bullets and armour-piercing bullets. The armour-piercing bullet travelled 1.20 times faster and had 1.20 times the mass of the regular bullet shown above.

A second test performed by the company had the regular bullet strike the vest at a glancing angle.

The mass of the vest and the dummy was 56.0 kg.

The bullet-vest collision was inelastic.



12. Determine the resultant speed of the vest and the dummy following the glancing collision shown above.

Clearly communicate your understanding of the physics principles that you are using to solve this question. You may communicate this understanding mathematically, graphically, with written statements, or a combination of all three methods.

---

---

---

---

---

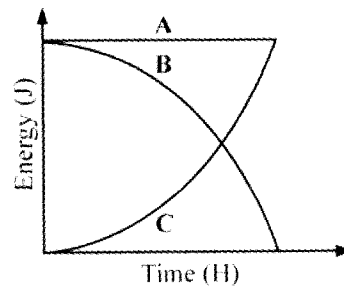
---

---

---

Use the following information to answer the next question.

The gravitational potential energy, kinetic energy, and mechanical energy of a bungee jumper during the free-fall portion of a jump are graphed below.



13. Lines A, B, and C represent, respectively,
- A. mechanical energy, gravitational potential energy, and kinetic energy
  - B. mechanical energy, kinetic energy, and gravitational potential energy
  - C. gravitational potential energy, mechanical energy, and kinetic energy
  - D. kinetic energy, gravitational potential energy, and mechanical energy

**Numerical Response**

*Use the following information to answer the next question.*

Mike uses a block sliding down an inclined plane to estimate the coefficient of kinetic friction,  $K$ , between the block and the plane. He adjusts the inclined plane to an angle of  $45^\circ$  above the horizontal, and allows the block to slide down the plane from rest. He uses a motion detector with a computer interface to get the velocity of the block when it leaves the plane.

The length that the block slides down the inclined plane is 1.15 m. The velocity of the block just before it leaves the plane is found to be 2.2 m/s.

- 14. Using conservation of energy, the estimated value of  $\mu_K$ , correct to two significant figures is \_\_\_\_.

**Open Response**

*Use the following information to answer the next question.*

Automatically inflating airbags are used as passive safety devices in vehicles to protect passengers.

- 15. Explain how an airbag reduces the chances of injury in a crash.

---

---

---

---

---

---

---



## SOLUTIONS

1. OR	5. B	9. 1.24	13. A
2. D	6. B	10. B	14. 0.69
3. D	7. 3 5 8 7	11. OR	15. OR
4. C	8. B	12. OR	

### 1. Open Response

The linear momentum of the three fragments, considered as part of the sphere, is conserved both during and after the explosion because no external forces were involved.

As the fragments do not exert forces on each other after the blast, this implies that they move with constant velocities.

Labelling the fragments  $A(0.5 \text{ kg})$ ,  $B(0.8 \text{ kg})$ , and  $C(1.2 \text{ kg})$ , each fragment's momentum can be broken down into its components.

Fragment  $A$ :

- $m_A = 0.5 \text{ kg}$
- $v_A = \frac{8.2 \text{ m}}{1.2 \text{ s}} = 6.8 \text{ m/s}$
- $p_A = 0.5 \text{ kg} \times 6.8 \text{ m/s} = 3.4 \text{ kg} \cdot \text{m/s}$
- $p_{Ax} = (3.4 \text{ kg} \cdot \text{m/s})\cos 33^\circ = 2.85 \text{ kg} \cdot \text{m/s}$
- $p_{Ay} = -(3.4 \text{ kg} \cdot \text{m/s})\sin 33^\circ = -1.85 \text{ kg} \cdot \text{m/s}$

Fragment  $B$ :

- $m_B = 0.8 \text{ kg}$
- $v_B = \frac{5.5 \text{ m}}{1.2 \text{ s}} = 4.6 \text{ m/s}$
- $p_B = 0.8 \text{ kg} \times 4.6 \text{ m/s} = 3.7 \text{ kg} \cdot \text{m/s}$
- $p_{Bx} = -(3.7 \text{ kg} \cdot \text{m/s})\cos 47^\circ = -2.52 \text{ kg} \cdot \text{m/s}$
- $p_{By} = -(3.7 \text{ kg} \cdot \text{m/s})\sin 47^\circ = -2.71 \text{ kg} \cdot \text{m/s}$

Fragment  $C$ :

- $m_C = 1.2 \text{ kg}$

The total momentum of the three fragments before the explosion was the null vector  $0$  because they were at rest as part of the sphere. So the total momentum after the explosion is also  $0$ . This results in the vector equation  $\vec{p}_A + \vec{p}_B + \vec{p}_C = 0$  or  $\vec{p}_C = -(\vec{p}_A + \vec{p}_B)$ .

This means that the components of  $\vec{p}_C$  can be obtained by adding up the corresponding components of  $\vec{p}_A$  and  $\vec{p}_B$  and then reversing the sign.

$p_{Cx}$  component:

- $p_{Cx} = -(p_{Ax} + p_{Bx})$
- $p_{Cx} = -(2.85 \text{ kg} \cdot \text{m/s} + -2.52 \text{ kg} \cdot \text{m/s})$
- $p_{Cx} = -0.33 \text{ kg} \cdot \text{m/s}$

$p_{Cy}$  component:

- $p_{Cy} = -(p_{Ay} + p_{By})$
- $p_{Cy} = -(-1.85 \text{ kg} \cdot \text{m/s} + -2.71 \text{ kg} \cdot \text{m/s})$
- $p_{Cy} = 4.56 \text{ kg} \cdot \text{m/s}$

The magnitude of the momentum of fragment  $C$  can be calculated using the Pythagorean theorem.

- $p_C = \sqrt{(p_{Cx})^2 + (p_{Cy})^2}$
- $p_C = \sqrt{(-0.33 \text{ kg} \cdot \text{m/s})^2 + (4.56 \text{ kg} \cdot \text{m/s})^2}$
- $p_C = 4.57 \text{ kg} \cdot \text{m/s}$

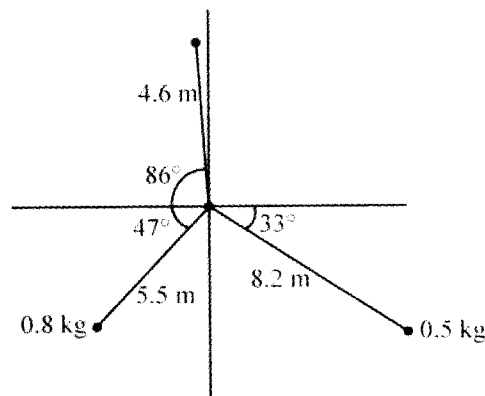
The direction of this momentum is given by the angle  $\tan^{-1} \frac{4.57}{0.33} = 86^\circ$  clockwise from the negative  $x$ -axis.

The speed of the 1.2-kg fragment is given by

$$v_C = \frac{p_C}{m_C} = \frac{4.57}{1.2} \text{ m/s} = 3.8 \text{ m/s}.$$

Finally, the distance of the 1.2-kg fragment from the origin after 1.2 s is

$3.8 \text{ m/s} \times 1.2 \text{ s} = 4.6 \text{ m}$  or as shown in the original graph.



### 2. D

A flywheel is a large, heavy wheel with a long, cylindrical axle passing through its centre. The centre of mass lies on its axis of rotation. It is constructed so that most of its mass is concentrated at the rim of the flywheel. This increases the moment of inertia about the axis of rotation.

The moment of inertia opposes any change in the uniform, rotatory motion; therefore, when a flywheel with a large moment of inertia is fitted to an engine, it assures smoother and steadier running of the engine.

### 3. D

The kinetic energy at the beginning and the kinetic energy at the end is zero; therefore, the frictional work done is equal to the change in the potential energy of the skier.



If the horizontal ice surface is the zero potential energy level, the change in potential energy is

$$E_p = mgh, \text{ where}$$

$m$  = Mass of the skier,

$g$  = Acceleration due to gravity,

$h$  = Height of the slope = 2.00 m.

Work done by friction over the inclined plane is:

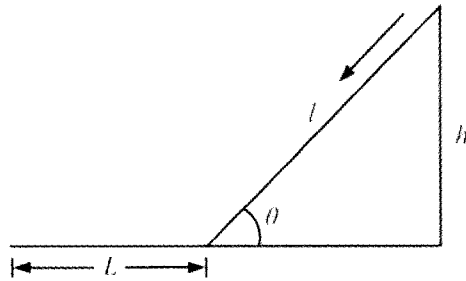
$$W_f = \mu_f F_N l = \mu_f mg l \cos \theta \\ = \mu_f gm \sqrt{l^2 - h^2}$$

$\theta$  = Angle between the inclined plane and the horizontal

Work done by the frictional force on the horizontal path is:

$$W_{f2} = \mu_f mgL$$

$L$  = Distance the skier cover before stopping



$$mgh = \mu_f mg \sqrt{l^2 - h^2} + \mu_f gml$$

$$\Rightarrow h = \mu_f \sqrt{l^2 - h^2} + \mu_f l$$

$$\Rightarrow 2 = 0.04 \sqrt{(3.00)^2 - (2.00)^2} + (0.04)L$$

$$\therefore L = 47.8 \text{ m}$$

Therefore, the distance the skier covers from the foot of the slope until he stops is 47.8 m.

#### 4. C

In an inelastic collision, the objects stick together after they collide. Some of their original kinetic energy is transformed into other types of energy—sound and heat.

#### 5. B

The work done is the product of the force and the displacement of the body in the direction of the force.

$$W = (150 \text{ N}) \times (12 \text{ m})$$

$$= 1800 \text{ N} \cdot \text{m}$$

$$= 1.8 \text{ kJ}$$

#### 6. B

It is given that the spacecraft used the gravitational potential energy of the planets to gain velocity.

$$E_k = \frac{1}{2} m \bar{v}^2$$

Therefore, by increasing their velocities ( $\bar{v}$ ), each spacecraft increased their kinetic energy  $E_k$  (motion).

The correct answer is alternative B.

#### 7. 3587

Using the fact that the gravitational force of Earth provides the centripetal force, you get the equation

$$T = \frac{2\pi r}{v} = 2\pi \sqrt{\frac{r^3}{GM}} \text{ or } T^2 = \frac{4\pi^2 r^3}{GM},$$

where  $M$  is the mass of Earth, and  $r$  is the distance of the satellite from the centre of Earth. This may be rewritten as a formula for the distance,

$$r = \left( \frac{GMT^2}{4\pi^2} \right)^{\frac{1}{3}}$$

$$\text{Here, } T = 24 \text{ h} = 24 \times 3600 \text{ s} = 8.64 \times 10^4 \text{ s}$$

Substituting the values of  $G$ ,  $M$ , and  $T$  in SI units into the equation gives:

$$r = \left( \frac{6.67 \times 5.97 \times 10^{13} \times (8.67 \times 10^4)^2}{4 \times \pi^2} \right)^{\frac{1}{3}} \text{ m}$$

$$r = 4.22 \times 10^7 \text{ m.}$$

The height of the satellite above the equator is this distance minus the equatorial radius of Earth.

$$h = 4.22 \times 10^7 \text{ m} - 6.38 \times 10^6 \text{ m}$$

$$h = 3.58 \times 10^7 \text{ m}$$

#### 8. B

The initial kinetic energy of the body:

$$= \frac{1}{2} m \left( \frac{v_c}{2} \right)^2$$

$$= \frac{1}{2} m \frac{2MG}{4R_E} \text{ since, } v_c = \sqrt{\frac{2MG}{R_E}}$$

$$= \frac{Gmm}{4R_E}$$

$$\text{The initial potential energy of the body} = -\frac{Gmm}{R_E}$$

The total initial mechanical energy

$$= \frac{Gmm}{4R_E} - \frac{Gmm}{R_E} = -\frac{3Gmm}{4R_E}$$

Let the maximum distance reached by the body from the centre of Earth be  $r$ .

Then, as the body comes to a rest at this point, the final

$$\text{potential energy} = -\frac{Gmm}{r}.$$

From the principle of conservation of mechanical energy: (The final potential total) = (initial kinetic energy)

$$-\frac{Gmm}{r} = -\frac{3Gmm}{4R_E}$$

$$\text{Therefore, } r = \frac{4}{3} R_E$$

$$\text{The maximum height} = r - R_E$$

$$= \frac{4}{3} R_E - R_E$$

$$= \frac{R_E}{3}$$



**9. 1.24**

$$\begin{aligned}
 E_k &= \frac{1}{2}mv_o^2 \\
 &= \frac{1}{2}(400 \text{ kg})(7.9 \times 10^3 \text{ m/s})^2 \\
 &= (200 \text{ kg})(62.4 \times 10^6 \text{ m}^2/\text{s}^2) \\
 &= 1.24 \times 10^{10} \text{ J}
 \end{aligned}$$

**10. B**

Since the spring is compressed by 2.00 m with the application of a force ( $F$ ) of 200 N, its force constant,  $k$ , is given by

$$k = \frac{F}{x} = \frac{200 \text{ N}}{2.00 \text{ m}} = 100 \text{ N/m.}$$

Let  $l$  be the distance along the inclined plane that the mass travels before it comes to rest.

Now, apply the conservation of energy,

$\frac{1}{2}kx_1^2 = mgh = mgl\sin\theta$	$[m = 20 \text{ kg}$ $g = \text{acceleration due to gravity}$ $x_1 = 4 \text{ m}]$
or	
$\frac{1}{2} \times 100 \times 4^2 = 20 \times 7.8 \times l \times \frac{1}{2}$	
or	
$l = \frac{800}{98} = 8.17 \text{ m}$	

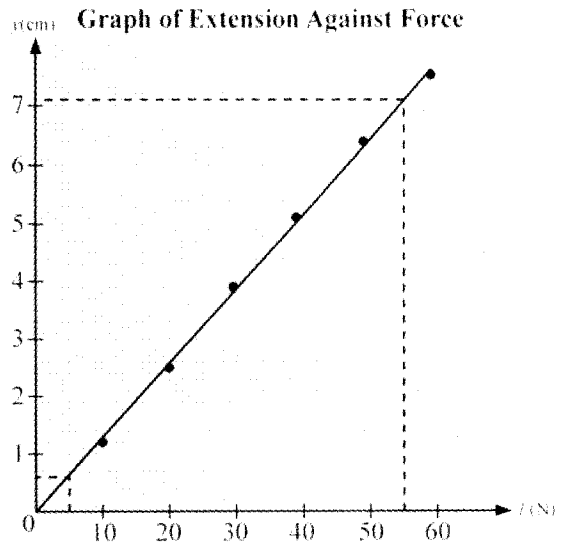
**11. Open Response**

The graph of either the length of the spring against the applied force, or the change of length against the applied force is expected to be a straight line. The spring constant can be obtained from the slope of the best fit line through the plotted points.

It is desirable to use the graph involving the change in length of the spring since it is known to pass through the origin. Here, the applied force comes from the weight of the variable mass,  $m$ , and can easily be calculated using the formula  $F = mg$ .

Force $F = mg$ (N)	Change of Length $y$ (cm)
9.8	1.2
19.6	2.5
29.4	3.9
39.2	5.1
49.0	6.4
58.8	7.5

Here,  $F$  is the independent variable and  $y$  is the dependent variable. Plotting the above values gives the following graph, with the line of best fit drawn through the plotted values.



The slope of the graph may be calculated using two widely separated points, as shown.

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{(7.1 - 0.6)\text{cm}}{(55.0 - 5.0)\text{N}} = 1.3 \text{ cm/N}$$

Since the change in length has been plotted on the vertical axis, the slope gives the reciprocal of the spring constant.

The spring constant is

$$k = \frac{1}{\text{Slope}} = \frac{1}{1.3} \text{ N/cm} = 77 \text{ N/m.}$$

**12. Open Response**

The resultant velocity of the vest and dummy:

Conservation of momentum in two dimensions requires two separate sets of calculations. Momentum is conserved in  $x$ -direction and momentum is conserved in  $y$ -direction. In the following calculations the subscripts a and b refer to the after collision and before collision cases.

$$\begin{aligned}
 (\vec{p}_B)_a &= m\vec{v} \\
 &= (8.00 \times 10^{-3} \text{ kg})(9.80 \text{ m/s})
 \end{aligned}$$

$$(\vec{p}_B)_a = 0.784 \text{ N} \cdot \text{s}$$

$$(\vec{p}_B)_{a_y} = 0.784 \sin 60^\circ$$

$$(\vec{p}_B)_{a_y} = 0.678 \text{ 96 kg} \cdot \text{m/s}$$

$$\begin{aligned}
 (\vec{p}_B)_{a_x} &= (\vec{p}_B)_a \cos 60^\circ \\
 &= 0.784 \cos 60^\circ
 \end{aligned}$$

$$(\vec{p}_B)_{a_x} = 0.392 \text{ kg} \cdot \text{m/s}$$



$$\begin{aligned}
 &(\vec{p}_B)_{b_x} + (\vec{p}_D)_{b_x} - (\vec{p}_B)_{a_x} + (\vec{p}_D)_{a_x} \\
 &(8.00 \times 10^{-3} \text{ kg})(650 \text{ m/s}) + 0 \\
 &= (0.329 \text{ kg} \cdot \text{m/s}) + (\vec{p}_D)_{a_x} \\
 &(5.20 \text{ kg} \cdot \text{m/s}) - (0.329 \text{ kg} \cdot \text{m/s}) + (\vec{p}_D)_{a_x} \\
 &4.808 \text{ kg} \cdot \text{m/s} = (\vec{p}_D)_{a_x} \\
 &(\vec{p}_B)_{a_y} + (\vec{p}_D)_{a_y} = (\vec{p}_B)_{a_y} + (\vec{p}_D)_{a_y} \\
 &0 + 0 = (\vec{p}_B)_{a_y} + (\vec{p}_D)_{a_y} \\
 &\text{Therefore: } (\vec{p}_D)_{a_y} + (\vec{p}_B)_{a_y} = 0 \\
 &(\vec{p}_D)_{a_y} = -0.678 \text{ 96 kg} \cdot \text{m/s}
 \end{aligned}$$

To find the resultant vector from  $x$ - and  $y$ -components, use the Pythagorean theorem:

$$\begin{aligned}
 (\vec{p}_D)_a &= \sqrt{((\vec{p}_D)_{a_x})^2 + ((\vec{p}_D)_{a_y})^2} \\
 &= \sqrt{(4.808)^2 + (0.678 \text{ 96})^2} \\
 (\vec{p}_D)_a &= m\vec{v}_a \\
 \vec{v}_a &= \frac{4.85 \text{ kg} \cdot \text{m/s}}{56.0 \text{ kg}} \\
 \vec{v}_a &= 0.0867 \text{ m/s} \\
 (\vec{p}_D)_a &= \sqrt{((\vec{p}_D)_{a_x})^2 + ((\vec{p}_D)_{a_y})^2}
 \end{aligned}$$

The vest and dummy had a speed of  $8.67 \times 10^{-2} \text{ m/s}$ .

### 13. A

Mechanical energy, which is the sum of kinetic and gravitational potential energies, does not change in a conservative system. Gravitational potential energy decreases as height decreases, while kinetic energy increases as speed increases.

### 14. 0.69

The principle of conservation of energy implies that the decrease in potential energy equals the increase in kinetic energy plus the work done against friction. This leads to the equation

$$\begin{aligned}
 mgh &= \frac{1}{2}mv^2 + \mu_K mgl \cos \theta, \\
 \text{or } gh &= \frac{1}{2}v^2 + \mu_K gL \cos \theta,
 \end{aligned}$$

where  $L$  is the length that the block slides down the inclined plane,  $h$  is the initial height of the block above the top of the plane, and  $\theta$  is the angle between the plane and the horizontal.

By basic trigonometry,

$$h = L \sin \theta = 1.15 \text{ m} \times \sin 45^\circ = 0.81 \text{ m.}$$

Rearranging the conservation equation gives

$$\begin{aligned}
 \mu_K &= \frac{gh - \frac{1}{2}v^2}{gL \cos \theta} \\
 \mu_K &= \frac{9.8 \times 0.81 - \frac{1}{2} \times 2.2^2}{9.8 \times 1.15 \times \cos 45^\circ} = 0.69
 \end{aligned}$$

## 15. Open Response

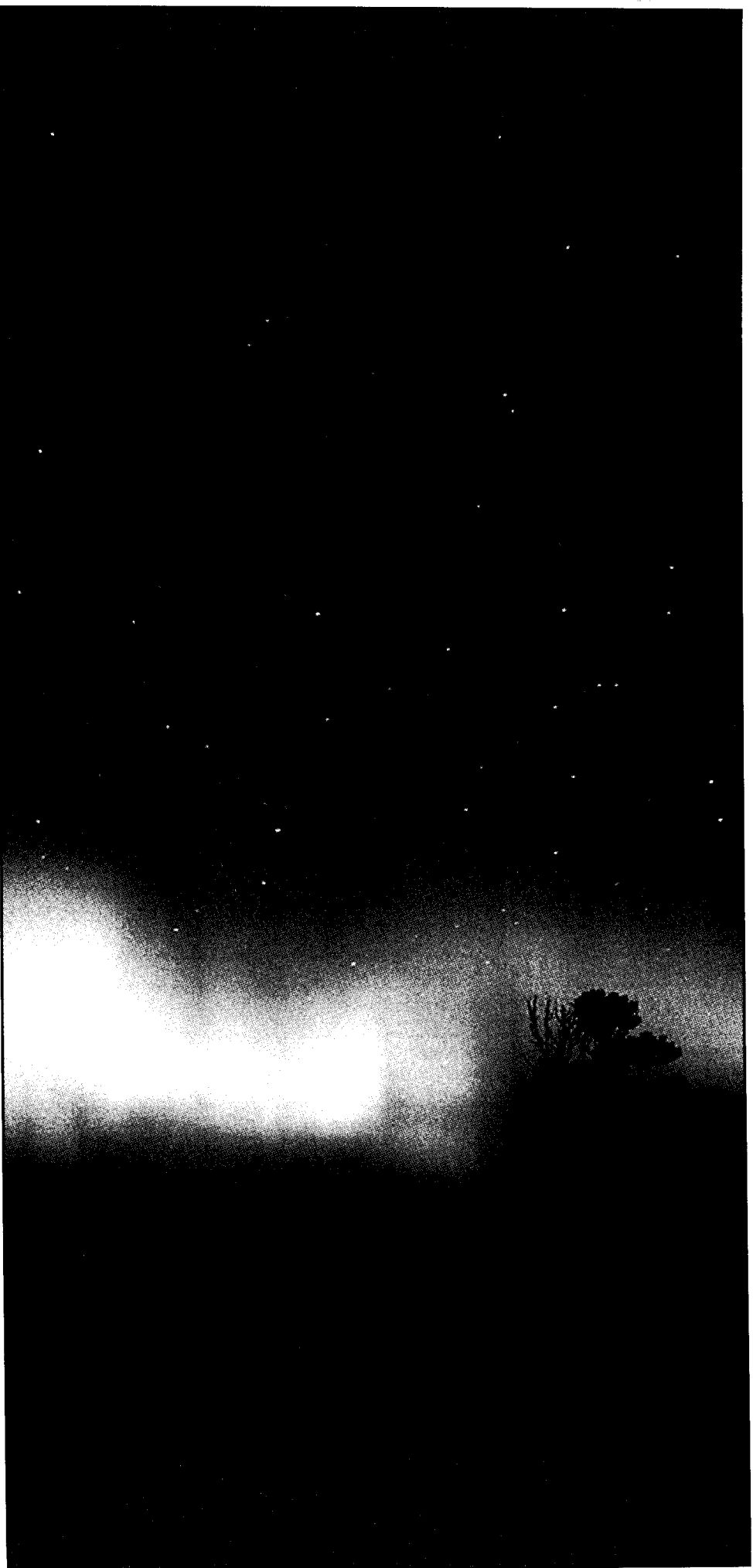
When a vehicle crashes, its velocity changes abruptly causing the driver and the passengers to be thrown violently forward or sideways. The injury to the driver or passengers depends on the large forces acting on those parts of the body that come into contact with other objects, like the car's dashboard or steering column, during the crash.

If a part of the body collides with a rigid object, there is a very rapid change of momentum during the collision, which amounts to a very large force having been involved. This follows from the equation

$$\text{Average Force} = \frac{\text{Change of Momentum}}{\text{Duration of Collision}}$$

However, when the collision is with an easily deformable object like an airbag, the effective duration of the collision is significantly increased, reducing the average force acting in the collision.

# Vector Fields





# Electric, Gravitational, and Magnetic Fields

## Table of Correlations

Specific Expectation	Practice Questions	Unit Test Questions
<b>12.3.1</b> Understanding Basic Concepts		
<b>12.3.1.1</b> <i>define and describe the concepts and units related to electric, gravitational, and magnetic fields</i>	1, 2	1
<b>12.3.1.3</b> <i>apply Coulomb's law and Newton's law of universal gravitation quantitatively in specific contexts</i>	4, 5, 6	3
<b>12.3.1.4</b> <i>compare the properties of electric, gravitational, and magnetic fields by describing and illustrating the source and direction of the field in each case</i>	7, 8, 9	4
<b>12.3.1.5</b> <i>apply quantitatively the concept of electric potential energy in a variety of contexts, and compare the characteristics of electric potential energy with those of gravitational potential energy</i>	10, 11, 12, 13	5, 6, 7
<b>12.3.1.6</b> <i>analyse in quantitative terms, and illustrate using field and vector diagrams, the electric field and the electric forces produced by a single point charge, two point charges, and two oppositely charged parallel plates</i>	14, 15, 16, 17	8, 9
<b>12.3.1.7</b> <i>describe and explain, in qualitative terms, the electric field that exists inside and on the surface of a charged conductor</i>	18	10
<b>12.3.1.8</b> <i>predict the forces acting on a moving charge and on a current-carrying conductor in a uniform magnetic field</i>	19	11, 12
<b>12.3.1.2</b> <i>state Coulomb's law and Newton's law of universal gravitation, and analyse and compare them in qualitative terms</i>	3	2
<b>12.3.2</b> Developing Skills of Inquiry and Communication		
<b>12.3.2.1</b> <i>determine the net force on, and resulting motion of, objects and charged particles by collecting, analysing, and interpreting quantitative data from experiments or computer simulations involving electric, gravitational, and magnetic fields</i>	20, 21, 22	13, 14
<b>12.3.2.2</b> <i>analyse and explain the properties of electric fields and demonstrate how an understanding of these properties can be applied to control or alter the electric field around a conductor</i>	23, 24	
<b>12.3.3</b> Relating Science to Technology, Society and the Environment		
<b>12.3.3.1</b> <i>explain how the concepts of a field developed into a general scientific model, and describe how it affected scientific thinking</i>	25	15
<b>12.3.3.3</b> <i>evaluate, using their own criteria, the social and economic impact of new technologies based on a scientific understanding of electric, gravitational, and magnetic fields</i>	28	
<b>12.3.3.2</b> <i>describe instances where developments in technology resulted in the advancement or revision of scientific theories, and analyse the principles involved in these discoveries and theories</i>	26, 27	



**12.3.1.1** *define and describe the concepts and units related to electric, gravitational, and magnetic fields*

## COMPARING FIELDS

The gravitational, electric, and magnetic fields are force fields and, like forces, are vector quantities. The direction of a force field is mapped by determining the force that would act on a test object (a mass, a positive charge, or the north pole of a magnetic dipole) placed at various locations within the field.

The units for a gravitational field are expressed in N/kg. This makes the value assigned to the gravitational field in a given location independent of the size of the test mass. For example, in a certain location, a 0.20 -kg mass might feel a gravitational pull of 1.5 N, while a 0.40 -kg mass would feel a gravitational pull of 3.0 N. In each case, the gravitational field strength is 75 N/kg.

Similarly, electric fields are expressed in N/C and magnetic fields are expressed in N/A · m. Fields and forces are related, but they are not identical quantities.

## MAGNETIC FIELD THEORY

Solid metal is made up of tiny regions, called domains, which are essentially tiny bar-magnets.

In most metals, these domains are arranged haphazardly and point in all directions; thus, all their magnetic fields cancel each other out.

In magnets, these domains are aligned, and their magnetic fields compound to make a single, large magnetic field that surrounds the magnet.

A magnet is attracted to ferromagnetic materials because its field temporarily aligns the domains of materials in proximity to it, turning them into magnets. Therefore, materials that do not have domains do not have magnetic properties.

## ELECTRIC FIELD THEORY

An electric field is the region around a charge in which a force is exerted upon other charges.

A force cannot exist within an electric field unless another charge is present for that force to act upon.

The direction of an electric field is defined as the direction in which a small positive test charge will move when released within the field.

The magnitude of the field emanating from a charged object can be determined by

$$|\vec{E}| = \frac{kq}{r^2},$$

where  $\vec{E}$  is the electric field in N/C,  $k$  is Coulomb's law constant ( $8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ ),  $r$  is the distance from the charged object's centre, and  $q$  is the magnitude of the charge on the object.

Electric fields can be added to and subtracted from one another according to their vector nature.

In this manner, two electric fields of equal strength and opposite direction may be used to cancel each other out.

## ELECTRIC FIELD THEORY

Uniform electric fields are typically created between two charged plates. The strength of such an electric field can be determined by the formula

$$|\vec{E}| = \frac{V}{d},$$

where  $\vec{E}$  is the strength of the electric field newtons/coulomb (N/C),  $V$  is the voltage potential between the plates, and  $d$  is the distance between the two plates.

Using this formula in combination with Newton's laws of motion and an understanding of accelerating systems, students will be able to determine the path a charged object will take in a uniform electric field.

### Practice

- The theoretical concept of a field was created to explain
  - action at a distance.
  - the net force on an object.
  - the Cavendish experiment.
  - collisions between objects.



2. James Maxwell first hypothesized that
- an electric current in a wire produces a magnetic field that circles the wire
  - a current is induced in a conductor that moves across a magnetic field
  - an electric field that changes with time generates a magnetic field
  - two parallel, current-carrying wires exert a force on each other

**12.3.1.2** *state Coulomb's law and Newton's law of universal gravitation, and analyse and compare them in qualitative terms*

## COMPARING COULOMB'S AND NEWTON'S LAWS

$$\text{Coulomb's law: } \vec{F}_c = \frac{kq_1q_2}{r^2}$$

$$\text{Newton's universal law: } \vec{F}_g = \frac{Gm_1m_2}{r^2}$$

While the causes of both forces differ, there is an obvious similarity in the relationships between the magnitudes of the forces and the distances separating the objects producing those forces. In both cases, the force diminishes as the distance between the objects increases. In other words, both forces are indirectly related to the distance between the objects:

$$\vec{F} \propto \frac{1}{r^2}$$

Compare the magnitudes of the two constants,  $k$  and  $G$ . The ratio of  $k/G$  approximately shows how much larger electrical forces are compared to gravitational forces.

$$\frac{k}{G} = \frac{8.99 \times 10^9}{6.67 \times 10^{-11}} = 1.35 \times 10^{20}$$

This is an enormous difference. You can see this in action by using a charged comb to pick up bits of paper. The electrical force is stronger than the gravitational force.

## Practice

3. Which of the following statements characterizes a relationship between the forces described by Coulomb's law and the forces described by Newton's law of universal gravitation?
- In atoms, gravitational forces are weaker than electrical forces.
  - Gravitational forces and electrical forces are attractive forces only.
  - Gravitational forces act at any distance, whereas electrical forces act at very short distances only.
  - As the distance between objects increases, electrical forces increase, whereas gravitational forces decrease.

**12.3.1.3** *apply Coulomb's law and Newton's law of universal gravitation quantitatively in specific contexts*

## COULOMB'S LAW

$$\text{Coulombs Law: } \vec{F}_e = \frac{kq_1q_2}{r^2}$$

$$\text{Newton's Universal Law: } \vec{F}_g = \frac{Gm_1m_2}{r^2}$$

While the causes of both forces differ, there is an obvious similarity in the relationships between the magnitudes of the forces and the distance separating the objects exerting the forces. In both cases, the force diminishes as the distance between the objects increases, such that

$$\vec{F} \propto \frac{1}{r^2}$$



## Practice

4. Two identical spheres of mass 20 g are placed at a distance of 1 m apart. Each sphere is given a charge,  $q$ . If the electrostatic force of repulsion just balances the gravitational force of attraction between the spheres, what is the value of  $q$ ?
- A.  $1.72 \times 10^{-12}$  C  
 B.  $5.4 \times 10^{-3}$  C  
 C.  $2.97 \times 10^{-24}$  C  
 D.  $2.9 \times 10^{-5}$  C
5. Assume a stationary electron and proton are separated by a distance of  $5.29 \times 10^{-11}$  m, the radius of the first Bohr orbit in the hydrogen atom. The ratio of the electrical force of attraction between the electron and the proton to the gravitational force of attraction between them is
- A.  $2.27 \times 10^{39}$   
 B.  $1.35 \times 10^{28}$   
 C.  $4.41 \times 10^{-40}$   
 D.  $3.47 \times 10^{34}$

## Numerical Response

6. Two point charges were placed  $6.75 \times 10^{-2}$  m apart. Point charge A had a mass of 125 mg and a charge  $200 \mu\text{C}$ , and point charge B had a mass of 450 mg and a charge of  $350 \mu\text{C}$ . Charge A was fixed to a non-conducting rod, while charge B was released and became free to move. The instantaneous acceleration of charge B as soon as it was released was  $a.bc \times 10^d \text{ m/s}^2$ . The values of  $a$ ,  $b$ ,  $c$ , and  $d$  are, \_\_\_\_, \_\_\_\_, \_\_\_\_, and \_\_\_\_\_. (Record your answer to three digits.)

**12.3.1.4** *compare the properties of electric, gravitational, and magnetic fields by describing and illustrating the source and direction of the field in each case*

## COMPARING GRAVITATIONAL, ELECTRICAL, AND MAGNETIC FIELDS

Gravitational fields are caused by the presence of matter. A large amount of matter is required to create a significant gravitational field. The direction of a gravitational field is defined as the direction that a test mass will move if placed within the field (usually toward the centre of gravity of the larger body generating the field).

Similarly, electric fields exist around charges. A uniform electric field can be created by a potential difference across two parallel metal plates attached to the opposite terminals of a battery. The direction of an electric field is defined as the direction that a positive test charge will move when placed within the field.

Magnetic fields exist around magnets (masses, usually of iron, in which the domains are aligned). They can also be created by moving electric charges. The direction of a magnetic field is defined as the direction that the north pole (south-seeking pole) of a magnet will point when placed within the magnetic field. This is the direction that a compass needle's north pole (south-seeking pole) will point since it is built to point toward Earth's magnetic south (geographic north) pole.



### Practice

7. An automobile's battery delivers a steady DC current to a headlight. The electric current in the wire produces a circular
- electric field around the wire
  - magnetic field around the wire
  - gravitational field around the wire
  - electromagnetic field around the wire
8. Which of the following statements about electric fields is **true**?
- Electric lines of force cannot intersect each other.
  - Electric lines of force begin and end on the same conductor.
  - The lines of force and an equipotential surface are parallel to each other.
  - The surface density of the of charge is equally distributed in a conductor irrespective of its shape.

### Open Response

9. Complete the following table:

Field	Source	Direction
Gravitational		
Electric		
Magnetic		

**12.3.1.5** *apply quantitatively the concept of electric potential energy in a variety of contexts, and compare the characteristics of electric potential energy with those of gravitational potential energy*

## COMPARING GRAVITATIONAL AND ELECTRICAL POTENTIAL ENERGIES

Assuming Earth's gravitational field is uniform over small distances, and a mass of 2.0 kg is raised a distance of 3.0 m at a constant velocity, the work done on the mass can be calculated. From the work completed, the increase in the gravitational potential energy of the mass can be determined (work-energy theorem).

$$W = \vec{F}d = m_2\vec{g}d = \frac{Gm_1m_2}{r} = E_p$$

When the mass was lifted, the work was done against the gravitational field. If the mass is dropped, the gravitational field does work on the mass, and the gravitational potential energy converts into kinetic energy.

Precisely the same transformations occur when a charge moves within a uniform electric field set up between two parallel charged plates. When a positive charge moves from the negatively charged plate to the positively charged plate, work is done against the electric field. The electric potential energy gained by the charge is equal to the work done to move the charge (assuming the charge moved at constant velocity).

$$W = \vec{F}d = q_2\vec{E}d = \frac{kq_1q_2}{r} = E_p$$

Again, if the charge is released, the electric field does work on the charge. The electric potential converts into kinetic energy.



**Practice**

10. Which of the following statements about electricity is false?
- A. The positive charge can move on an equipotential surface.
  - B. Electrons move from a lower potential to a higher potential.
  - C. A positive charge moves from a higher potential to a lower potential.
  - D. A neutral object always moves from a lower potential to a higher potential.

**Numerical Response**

11. A charge of 25 C moves through a potential difference of 2500 V in an electric field.
- What is the decrease in the potential energy? \_\_\_\_J

**Numerical Response**

*Use the following information to answer the next question.*

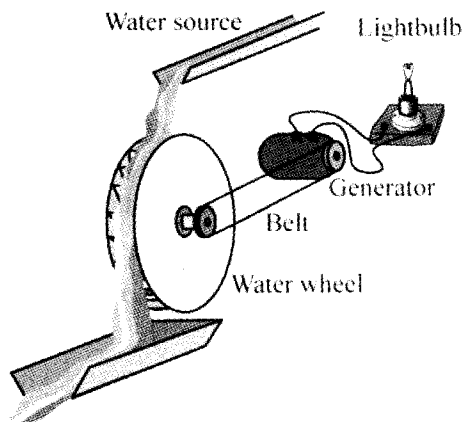
A negatively charged object between two parallel plates requires 9.00 J of work to be done to move it from the positive plate to the negative plate.

12. How much electric potential energy in joules is gained by the object? \_\_\_\_J

**Open Response**

*Use the following information to answer the next question.*

A student builds the small-scale hydroelectric system illustrated below. Water from a reservoir drains along a trough and spills onto a water wheel, causing the wheel to turn. Attached to the axle of the water wheel is a belt that turns a small generator, which is connected to an incandescent light bulb that emits light and feels warm when touched.



13. Using the principles of conservation of energy, electromagnetic induction, and electromagnetic radiation, analyse the small-scale hydroelectric generator described above. In your response,
- Identify one modification that could be made to this system to increase the electrical potential difference produced by the generator. Explain how your modification would accomplish this. Support your response with appropriate formulae.

---



---



---



---



**12.3.1.6** *analyse in quantitative terms, and illustrate using field and vector diagrams, the electric field and the electric forces produced by a single point charge, two point charges, and two oppositely charged parallel plates*

## CALCULATING ELECTRIC FIELD STRENGTH

Similar to the force exerted on a mass by the gravitational field, electric fields exert forces on charged objects. Therefore, the electric force on a charge can be calculated using

$$\vec{F}_e = \vec{E}q,$$

where  $\vec{F}_e$  is the force on the charged object (N),  $\vec{E}$  is the strength of the electric field (N/C), and  $q$  is the magnitude of the charge on the object in the electric field (C).

This can be rewritten in order to determine the strength of the electric field.

$$\vec{E} = \frac{\vec{F}_e}{q}$$

The direction of the field is the same as the direction of the electric force acting on a positive test charge. The electric field is oriented to repel positively charged objects and to attract negatively charged objects.

If the electric field is the result of a point charge, it will be a non-uniform field. If the electric field is the result of two charged parallel plates, the electric field will be uniform.

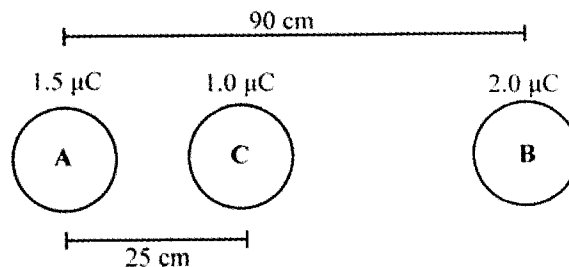
## ELECTRIC FORCES DUE TO TWO OR MORE CHARGES

Charge is a scalar quantity. As a result, the sign of the charge is not used in the formula when Coulomb's law is used. The numerical result of the formula is only the magnitude of the electric force.

Force, however, is a vector quantity, so the direction is important. This direction is determined by analysing the natures of the charges interacting (e.g., positive or negative) and the relative positions of the charges. This direction becomes essential if several point charges lie on a plane.

### Example

Two point charges,  $A = -1.5 \mu\text{C}$  and  $B = -2.0 \mu\text{C}$ , are 90.0 cm apart. Another charge,  $C = +1.0 \mu\text{C}$ , is placed between charges  $A$  and  $B$ , 25.0 cm from point charge  $A$ . What is the net electric force acting on point charge  $C$ ?



$$\vec{F}_{A \text{ on } C} = \frac{kq_1q_2}{r^2} \text{ of repulsion to the right} \\ = 0.216 \text{ N}$$

$$\vec{F}_{B \text{ on } C} = \frac{kq_1q_2}{r^2} \text{ of repulsion to the left} \\ = 0.0426 \text{ N}$$

$$\vec{F}_{\text{net}} = \vec{F}_{A \text{ on } C} + \vec{F}_{B \text{ on } C} \\ \vec{F}_{\text{net}} = 0.216 \text{ N} - 0.0426 \text{ N} \\ \vec{F}_{\text{net}} = 0.173 \text{ N}$$

The net force acting on charge  $C$  is 0.173 N acting to the right.

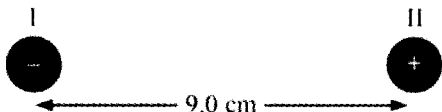


**Practice**

Use the following information to answer the next question.

**Electrostatics**

Two particles, I and II, of equal mass have opposite charges. The negative charge on particle I is three times greater than is the positive charge on particle II. The particles are placed 9.0 cm apart.



14. The electric field at a point halfway between the particles is
- A. zero
  - B. toward the top of the page
  - C. toward the left of the page
  - D. toward the right of the page

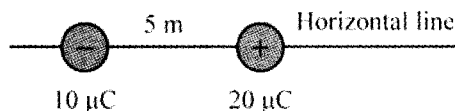
**Numerical Response**

15. Particles *A* and *B* have charges of  $8.0 \times 10^{-6} \text{ C}$  and  $-2.0 \times 10^{-6} \text{ C}$ , respectively, and are 20 cm apart. If a third charged particle is to be placed so that it does **not** experience a net electric force, how many cm away from charge *B* should it be placed? \_\_\_\_ cm

**Open Response**

Use the following information to answer the next question.

The given figure shows two charges placed 5 m apart.



16. Where should a unit positive test charge be placed on the horizontal line such that the net electric field is zero?

---



---



---



---

**Open Response**

Use the following information to answer the next question.

An oil drop is placed between two parallel plates such that the force of the electric field balances the force of gravity, causing the drop to be suspended with no measurable motion between the plates.

17. What would happen if the charges on the two parallel plates were reversed, causing the direction of the electric field to change direction?

---



---



---



---



**12.3.1.7** describe and explain, in qualitative terms, the electric field that exists inside and on the surface of a charged conductor

## DISTRIBUTION OF CHARGE ON CONDUCTORS AND INSULATORS

Insulators are materials that resist conducting electricity. Therefore, any charge placed on an insulator will remain where it is placed.

Conductors are materials that permit charges to flow through them. Most metals are good conductors, so their resistance to electrical current is low.

For example, in a solid metallic sphere, excess negative charges will mutually repel each other. The effect will distribute these negative charges uniformly across the surface of the solid metallic sphere. For a hollow metallic sphere, the result will be identical, with all excess charges residing on the surface of the hollow sphere and uniformly distributed across this surface.

If the conductor lacks spherical symmetry (it is oblate or it comes to a point), the excess charges still reside on the object's surface, but they will no longer be uniformly distributed on this surface.

The charges will concentrate where the radius of curvature is smallest. If the object comes to a point, there will be a huge concentration of charges at the point, since the radius of curvature of the surface here is extremely small.

## Practice

**18.** A conducting spherical shell has a surface charge density of  $\sigma$ . What are the electric fields inside ( $\vec{E}_{\text{in}}$ ) and outside ( $\vec{E}_{\text{out}}$ ) of the shell, respectively?

A.  $|\vec{E}_{\text{in}}| = \frac{k\sigma A}{r^2}$  and  $\vec{E}_{\text{out}} = 0$

B.  $\vec{E}_{\text{in}} = 0$  and  $|\vec{E}_{\text{out}}| = \frac{k\sigma A}{r^2}$

C.  $|\vec{E}_{\text{in}}| = |\vec{E}_{\text{out}}| = \frac{k\sigma A}{r^2}$

D.  $\vec{E}_{\text{in}} = \vec{E}_{\text{out}} = 0$

**12.3.1.8** predict the forces acting on a moving charge and on a current-carrying conductor in a uniform magnetic field

## MAGNETIC FORCE ON A MOVING CHARGE IN A UNIFORM MAGNETIC FIELD

The magnetic force exerted on a moving electric charge by a uniform magnetic field may be calculated using the formula

$$F_m = qvB_{\perp},$$

where,

$F_m$  = magnetic force,

$q$  = magnitude of charge,

$v$  = magnitude of velocity,

and,  $B_{\perp}$  = magnitude of magnetic field perpendicular to the motion of the charge.

Notice that the vector notation for the force, the velocity, and the magnetic field has been left out. This is because the direction of the magnetic force is most simply derived from the hand rules for charges moving in a uniform magnetic field.



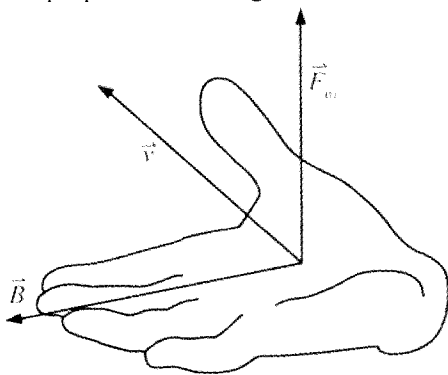
If you are given a magnetic field that is not perpendicular to the motion of the charge, it is a simple matter to break it down into components that are parallel and perpendicular to the motion of the charge. If you know the angle,  $\theta$ , that the magnetic field makes with the direction of motion of the charge, then

$$B_{\perp} = \vec{B} \sin \theta.$$

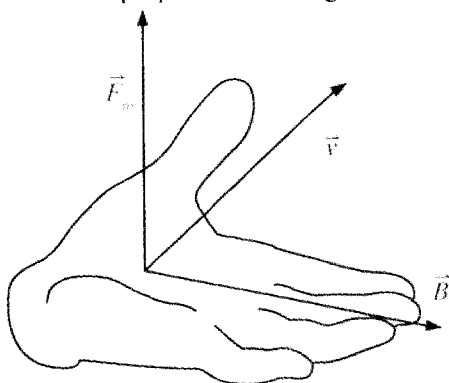
### HAND RULES FOR CHARGES MOVING IN UNIFORM MAGNETIC FIELDS

In order to determine the direction that the magnetic force acts upon a moving charge in a uniform magnetic field, we use the hand rules. The right hand is used to determine the force exerted upon a negatively charged particle, while the left hand is used for a positively charged particle. In both cases, use an open hand with the fingers pointing in the direction of the magnetic field. The thumb points in the direction of the motion of the charged particle, and the palm points in the direction of the magnetic force exerted on that particle. See the diagrams below to visualize how these hand rules work.

The right hand rule for a positive charge moving in a uniform perpendicular magnetic field:



The left hand rule for a negative charge moving in a uniform perpendicular magnetic field:



## THE MAGNETIC FORCE ON A CURRENT-CARRYING CONDUCTOR IN A UNIFORM MAGNETIC FIELD

In the same way that a single charged particle experiences a magnetic force when it moves through a uniform magnetic field, so does a current-carrying conductor. The force that a current-carrying conductor experiences can be calculated using the formula

$$F_m = B_{\perp} I l,$$

where,

$F_m$  = magnetic force,

$B_{\perp}$  = magnitude of magnetic field perpendicular to the current,

$I$  = electrical current,

$l$  = length of conductor within the perpendicular magnetic field.

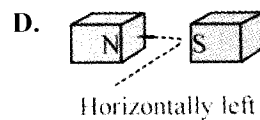
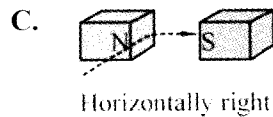
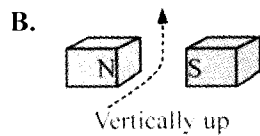
### HAND RULES FOR CURRENT CARRYING CONDUCTORS IN UNIFORM MAGNETIC FIELDS

The magnetic force exerted upon a current-carrying conductor within a perpendicular magnetic field can be determined in exactly the same way as the magnetic force exerted upon a charged particle. If you think of the current flowing within the conductor as being composed of moving charges, then it is a simple matter of using the same hand rules as shown above to determine the direction of the magnetic force upon the conductor. If you are considering the current to be conventional (positive) current, then we use the right hand with the fingers pointing in the direction of the magnetic field. The thumb points in the direction of the current, and the palm will point in the direction of the magnetic force. If the current is electron flow (negative), then the left hand is used.



### Practice

19. Which of the following diagrams correctly shows the path travelled by a moving proton in an external magnetic field?



**12.3.2.1** determine the net force on, and resulting motion of, objects and charged particles by collecting, analysing, and interpreting quantitative data from experiments or computer simulations involving electric, gravitational, and magnetic fields

## FORCES ON CHARGED OBJECTS

The force exerted by electric fields on charged objects is given by

$$\vec{F}_e = \vec{E}q,$$

where  $\vec{F}_e$  is the electric force on the charged object in newtons (N),  $\vec{E}$  is the vector electric field in newtons per coulomb (N/C), and  $q$  is the charge on the charged objects in coulombs (C).

When placed in the field, a positively charged particle will move in the direction of the field, while a negatively charged object will move opposite the direction of the electric field.

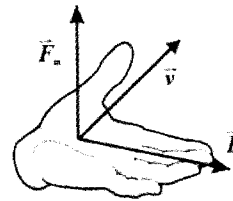
An electron placed in an electric field will move opposite the direction of the field, and the charge on the electron can be calculated if the strength of the electric field and the force experienced by the electric field are known.

The force a magnetic field exerts on a moving charge can be determined using the formula

$$\vec{F}_m = q\vec{v}B_{\perp},$$

where  $\vec{F}_m$  is the magnitude of the force,  $q$  is the magnitude of the charge on the object,  $v$  is the magnitude of the object's velocity, and  $B_{\perp}$  is the magnitude of the magnetic field perpendicular to the particle's motion. Ignore signs associated with these quantities and use the left hand and right hand rules to determine the direction of the force.

Flatten the hand and extend the thumb perpendicular to the fingers. If the thumb indicates the direction of velocity and the fingers indicate the direction of the magnetic field, then the force is said to come out of the palm. Use the left hand with negative charges and the right hand with positive charges.



### Practice

20. A charged oil drop is suspended in a uniform field of  $300 \text{ V cm}^{-1}$  so that the oil drop neither rises nor falls. The acceleration due to gravity =  $9.8 \text{ m/s}^2$ .

What is the charge on the oil drop, if its mass is  $9.75 \times 10^{-15} \text{ kg}$ ?

- A.  $2.33 \times 10^{-18} \text{ C}$
- B.  $2.46 \times 10^{-18} \text{ C}$
- C.  $2.59 \times 10^{-18} \text{ C}$
- D.  $3.18 \times 10^{-18} \text{ C}$



21. A proton and an alpha particle have identical circular orbits in a magnetic field. The proton has a speed of  $4.4 \times 10^5$  m/s. The speed of the alpha particle is
- A.  $1.1 \times 10^5$  m/s
  - B.  $2.2 \times 10^5$  m/s
  - C.  $4.4 \times 10^5$  m/s
  - D.  $8.8 \times 10^5$  m/s

### Open Response

*Use the following information to answer the next question.*

An experiment is conducted to determine the charge magnitude of the charge on a negatively charged particle using two parallel plates. The particle is initially at rest, and its speed as it reaches the positive plate is measured.

22. If the voltage between the parallel plates is mistakenly recorded to be higher than its actual voltage, how will the calculated magnitude of the charge on the particle be affected?

---



---



---



---



---

**12.3.2.2** *analyse and explain the properties of electric fields and demonstrate how an understanding of these properties can be applied to control or alter the electric field around a conductor*

## PROPERTIES OF ELECTRIC FIELDS AROUND CONDUCTORS

Electrons in conducting materials are free to move within the conductor and are at rest when they reach static equilibrium. This is when the charges are distributed evenly throughout the conductor so that no charge experiences a net force. If a conductor is a sphere, charges arrange themselves evenly on the surface of the sphere so that there is an equal distance between each charge, and no charge experiences a net force. Similarly, the charges on a hollow conductor will arrange themselves evenly on the outer surface of the material. There is no charge on the inner surface of a hollow conductor, and no electric field within the cavity. This is a result of the vector addition of the electric fields due to each charge cancelling out in the centre of the hollow conductor. A hollow tube of conducting material can be used to protect electrical equipment from external electric fields. This technique is commonly seen with things like coaxial cables used to transmit electrical signals. Surrounding the cable in either a hollow tube of conducting material or a very fine mesh of a substance such as copper creates a hollow tube with no electric field in the middle.



**Practice**

23. Which of the following statements correctly describes the charge distribution and resulting electric field of a hollow conducting object?
- A. Charge is distributed evenly on the outer surface, creating no electric field outside the conductor.
  - B. Charge is distributed evenly on the inner surface, creating no electric field inside the conductor.
  - C. Charge is distributed evenly on the outer surface, creating no electric field inside the conductor.
  - D. Charge is distributed evenly on the inner surface, creating no electric field outside the conductor.

**Open Response**

*Use the following information to answer the next question.*

You are concerned about the effect of the electric field created by your cellphone on nearby electrical equipment, so you create a hollow conducting tube to surround your cellphone.

24. Would this prevent the electric field of the phone from affecting other electrical devices?

---



---



---



---



---

**12.3.3.1** explain how the concepts of a field developed into a general scientific model, and describe how it affected scientific thinking

**FIELD THEORY AND ACTION AT A DISTANCE**

The observation that magnets attract and repel each other over a distance without direct contact contributed to the initial concept of fields. In the early 1800s Michael Faraday described a magnetic field to explain the effects of magnets, and an electric field to explain electrostatic effects. Newton had previously defined gravity as a force between objects capable of acting over great distances. As the concept of the field broadened, matter came to be thought of as having a three-dimensional gravitational field, that exerts its influence over objects entering the field.

The concept of gravitational fields explains the motions of celestial bodies in relation to each other, and allows predictions to be made about their positions. The movement of charged particles through an electric field can similarly be predicted based on the magnitude and direction of the electric field.

**Practice**

25. The theoretical concept of a field was created to explain
- A. collisions between objects
  - B. the Cavendish experiment
  - C. the net force on an object
  - D. action at a distance



**12.3.3.2** *describe instances where developments in technology resulted in the advancement or revision of scientific theories, and analyse the principles involved in these discoveries and theories*

## TECHNOLOGY ADVANCING SCIENTIFIC KNOWLEDGE

Advances in scientific understanding often lead to the production of new technologies; however, technological advances can also lead to greater scientific understanding. Particle accelerators consist of charged particles accelerated using electric and magnetic fields. When the particles collide, elementary particles are released. Because the forces present at the nuclear level are so strong, high energies are required to break them apart. This is why particle accelerators have become increasingly powerful. As more power is put into the collision, more hard to observe particles will be expected to be observed. Elementary particles are sometimes predicted to exist based on theoretical findings, but they cannot be observed because of technological constraints. In 1983, three particles mediating the weak nuclear force were found using powerful particle accelerators. These findings were key to understanding the structure of matter and would not have been observed without technological advances. The recently completed large hadron collider (LHC) at CERN in Geneva is the most powerful collider to date, and may provide new insights into the structure of matter.

### Practice

26. Sometimes technological advances lead scientific advances, and sometimes the reverse is true. Which of the following examples describes a technological advance that led to a scientific advance?
- The diffraction grating had to be invented before cathode rays could be further studied and explained.
  - Geissler, an expert glass blower, invented a new pump that produced extremely low pressures within glass tubes as they formed.
  - The ability to construct large solenoids, which form a uniform magnetic field, preceded the ability to study cathode rays in discharge tubes.
  - The intense interest in the Crooke's dark space, which formed between the cathode and anode, motivated the construction of discharge tubes in which this dark space occupied the entire tube.

### Open Response

27. Explain how scientific understanding of elementary particles was affected by knowledge of electric and magnetic fields.

---



---



---



---



---



*12.3.3.3 evaluate, using their own criteria, the social and economic impact of new technologies based on a scientific understanding of electric, gravitational, and magnetic fields*

## **SOCIAL APPLICATIONS OF FIELD THEORY**

New technologies based on a greater understanding of fields play an important role in our society.

Magnetic resonance imaging (MRI) is a widely used medical technique that uses properties of magnetic fields to visualize internal structures. It consists of a large solenoid, a coiled wire carrying a current that produces a strong uniform magnetic field. When a person is placed in the magnetic field, protons in the body respond and release energy, which is detected and interpreted to produce an image of internal structures. MRI is thought to be safer than older imaging techniques such as CAT scans, which use much higher energies.

The gravitational fields of objects can be used to an advantage in space exploration. A technique called gravity assist occurs when an object passes through a gravitational field and gains speed after passing through the field. In a kind of slingshot effect, gravitational forces cause the object to accelerate. By calculating the gravitational fields of planets and other celestial bodies, and the path of a space probe, travel time and fuel consumption can be reduced, making space exploration more feasible.

### **Practice**

#### **Open Response**

28. Why might medical researchers support the funding of research into fields and technologies based on field theory?

---

---

---

---

---



## SOLUTIONS—ELECTRIC, GRAVITATIONAL, AND MAGNETIC FIELDS

1. A	7. B	13. OR	19. A	25. D
2. C	8. A	14. C	20. D	26. B
3. A	9. OR	15. 20	21. B	27. OR
4. A	10. D	16. OR	22. OR	28. OR
5. A	11. 62500	17. OR	23. C	
6. 3, 0, 7, 8	12. 9.00	18. B	24. OR	

### 1. A

Before developing the concept of a field, scientists had difficulty explaining how two objects not in contact could affect each other, i.e., action at a distance. A field is a visual method to explain how masses attract other masses.

### 2. C

Maxwell suggested that electric fields that changed with time could produce a time-varying magnetic, creating an electromagnetic wave that could travel through space.

### 3. A

Gravitational forces are described by Newton's law of universal gravitation,  $\vec{F}_g = \frac{Gm_1m_2}{r^2}$ , while static

electrical forces are described by Coulomb's law,

$\vec{F}_e = \frac{kq_1q_2}{r^2}$ . In atoms, both charges and masses are very

small, but the constant in Coulomb's law,  $k$ , is approximately  $10^{20}$  times greater than the gravitational force constant,  $G$ .

### 4. A

Newton's law of gravitation states that is the force  $\vec{F}_g$  is  $\frac{Gm_1m_2}{r^2}$ , where  $m$  is mass,  $r$  is the distance separating the

objects, and  $G$  is the universal gravitational constant. The gravitational force is always an attractive force.

From Coulomb's law, the electric force,  $\vec{F}$ , between two point charges  $q_1$  and  $q_2$  separated by a distance,  $r$ , is  $\frac{kq_1q_2}{r^2}$ , where  $k$  is Coulomb's constant. The electric force

is attractive if the charges are different, and repulsive if the charges are the same.

$$F_e = F_g$$

$$\frac{kq_1q_2}{r^2} = \frac{Gm_1m_2}{r^2}$$

$$\frac{kq^2}{r^2} = \frac{Gm^2}{r^2}$$

$$kq^2 = Gm^2$$

$$q^2 = \frac{Gm^2}{k}$$

$$q^2 = \frac{(6.67 \times 10^{-11} \text{ N} \cdot (\text{m}/\text{kg})^2)(0.020 \text{ kg})^2}{8.99 \times 10^9 \text{ N} \cdot (\text{m}/\text{C})^2}$$

$$q^2 = 2.97 \times 10^{-24} \text{ C}^2$$

$$\therefore q = 1.72 \times 10^{-12} \text{ C}$$

### 5. A

$$\vec{F}_e = \frac{kq_1q_2}{r^2}$$

$$= \frac{\left(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}\right)(1.60 \times 10^{-19} \text{ C})^2}{(5.29 \times 10^{-11} \text{ m})^2}$$

$$= 8.22 \times 10^{-8} \text{ N}$$

$$\vec{F}_g = \frac{Gm_1m_2}{r^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}\right)(9.11 \times 10^{-31} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}{(5.29 \times 10^{-11} \text{ m})^2}$$

$$= 3.63 \times 10^{-47} \text{ N}$$

$$\left(\frac{\vec{F}_e}{\vec{F}_g}\right) = \frac{8.22 \times 10^{-8} \text{ N}}{3.63 \times 10^{-47} \text{ N}} = 2.27 \times 10^{39}$$



6. 3, 0, 7, 8

$$\vec{F}_e = \frac{kq_1q_2}{r^2}$$

$$= \frac{(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2})(200 \times 10^{-6} \text{ C})(350 \times 10^{-6} \text{ C})}{(6.75 \times 10^{-2} \text{ m})^2}$$

$$= 1.38 \times 10^5 \text{ N}$$

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{1.38 \times 10^5 \text{ N}}{450 \times 10^{-6} \text{ kg}} = 3.07 \times 10^8 \text{ m/s}^2$$

7. B

Oersted discovered that a current-carrying conductor is encircled by a magnetic field.

8. A

Two electric lines of force can never intersect each other. If they intersected each other, the electric field intensity at the point of intersection would act in two different directions. This is impossible. At any point, the electric field is oriented toward a particular direction; hence, only one line of force can pass through any point.

9. Open Response

Field	Source	Direction
Gravitational	All objects with mass	Towards source
Electric	Electrical charges	Away from positively charged source, towards negatively charged source
Magnetic	Moving electrical charge or current	Closed loop around source, exiting north pole and entering south pole

10. D

A positive charge moves from a higher potential to a lower potential. The electrons are negatively charged; hence, they move from a lower potential to a higher potential. A charge can move from one point to the other on an equipotential surface, and work is done on it.

11. 62500

$$E_p = qV = (25 \text{ C})(2500 \text{ V}) = 62\,500 \text{ J}$$

As the charge moves from higher potential to lower potential, its potential energy decreases.

12. 9.00

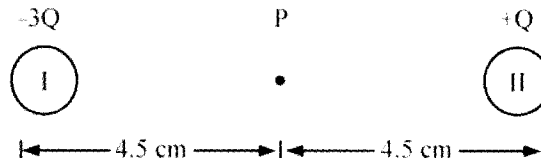
The change in electric potential energy is equal to the work done in moving it, 9.00 J.

13. Open Response

One way to increase the electrical potential difference generated is to increase the height of the reservoir, which would increase the potential energy. This increase in potential energy could make the wheel turn faster, thereby increasing  $v$ . Since  $V = B_{\perp}lv$ , increasing  $v$  increases  $V$ .

14. C

Suppose that the magnitude of the positive charge on particle II is  $Q$ . Then the charges will be in the conformation shown.



The magnitude of  $|\vec{E}| = \frac{kq}{r^2}$ .

The direction of  $|\vec{E}|$  is the direction of the electric force on an imaginary positive test charge placed at a particular location.

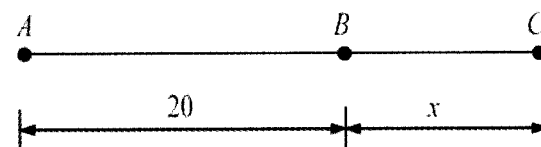
At point  $P$ , particle I will produce  $\vec{E} = \frac{k(3Q)}{(0.045)^2}$  to the left.

Particle II will produce  $\vec{E} = \frac{k(Q)}{(0.045)^2}$ , also to the left.

Thus, the resultant  $\vec{E}$  at point  $P$  will be to the left.

15. 20

In order for the net electric force on charge  $C$  to equal zero, the force due to charges  $A$  and  $B$  must be opposite in direction. Since charges  $A$  and  $B$  have opposite charges, charge  $C$  cannot lie between charges  $A$  and  $B$ . Hence, charge  $C$  should be placed closer to charge  $B$  than  $A$ . Let  $BC = x$  and the charge on  $C$  be  $Q$ .



From Coulomb's law, the electric force,  $\vec{F}$ , between two point charges  $q_1$  and  $q_2$  separated by a distance  $r$  is  $\frac{kq_1q_2}{r^2}$  where  $k$  is Coulomb's constant.

The force due to charge  $A$  is  $k \frac{8 \times 10^{-6} Q}{(20+x)^2}$ .

The force due to charge  $B$  is  $k \frac{2 \times 10^{-6} Q}{x^2}$ .



These two forces are opposite to each other. In order for the resultant to be zero, they should be equal in magnitude.

$$k \frac{8 \times 10^{-6}}{(20+x)^2} = k \frac{2 \times 10^{-6}}{x^2}$$

$$\frac{8}{(20+x)^2} = \frac{2}{x^2}$$

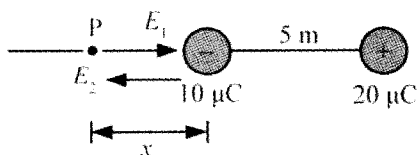
$$\frac{20+x}{x} = 2$$

$$x = 20 \text{ cm}$$

The third charge should be placed 20.0 cm past charge B along the line AB.

**16. Open Response**

The net electric field at a point P is zero, if the vector sum of all the fields at that point is zero. It can be seen in the given figure that the positive charge has greater magnitude than the negative charge. The given point must thus, lie behind the negative charge, so as to make the net field equal to zero. This can be shown as:



In the figure,  $E_1$  and  $E_2$  are the electric fields at point P caused by charges  $-10 \mu\text{C}$  and  $20 \mu\text{C}$ , respectively.

Net field at P,

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{k(-10 \times 10^{-6} \text{ C})}{x} + \frac{k(20 \times 10^{-6} \text{ C})}{x+5}$$

For  $\vec{E} = 0$ ,

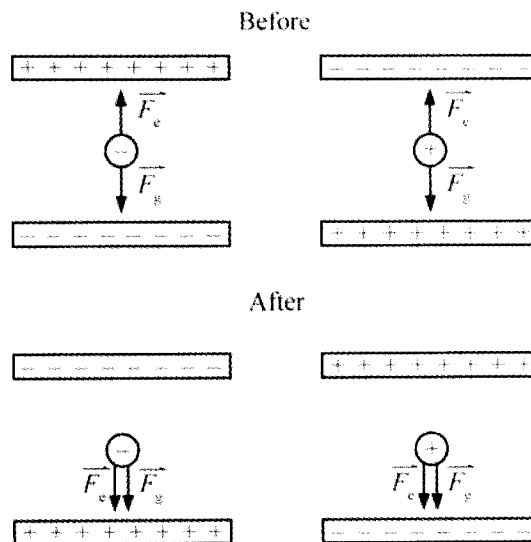
$$\frac{10}{x} = \frac{20}{5+x}$$

$$50 + 10x = 20x$$

$$x = \frac{50}{10} = 5 \text{ m}$$

**17. Open Response**

The oil drop would be accelerated downwards, as the force of gravity and the force due to the electric field would now be acting in the same direction.



**18. B**

The electric field inside a charged conductor is always zero.

The electric field outside a charged shell is equivalent to a point charge located at the shell's centre, such that all the charge is concentrated at the shell's centre.

The surface area of a sphere is given by  $A = 4\pi r^2$ .

$$\therefore q = \sigma A = \sigma(4\pi r^2)$$

The electric field outside of the conducting, spherical shell can be calculated using,

$$|\vec{E}| = \frac{kq}{r^2} = \frac{k\sigma A}{r^2}$$

Hence, the respective electric fields inside and outside the shell are:

$$\vec{E}_{\text{in}} = 0 \text{ and } |\vec{E}_{\text{out}}| = \frac{k\sigma A}{r^2}$$

**19. A**

Using the third right hand rule (positive particle), the thumb points in the direction of the particle movement, the fingers point in direction of the magnetic field, and the palm points in the direction of the force on the particle.

Since the proton moves into the page and the magnetic field points to the right, the palm faces downward.

This means that the proton will experience a force due to the magnetic field, directed vertically down.

**20. D**

Electric intensity,  $E$ , =  $300 \text{ V cm}^{-1} = 3 \times 10^4 \text{ V m}^{-1}$ .

Let the charge of the oil drop be  $q$ .

The force on the oil drop,  $\vec{F}_e$ , is given by  $\vec{F}_e = qE$ .

The weight of the drop,  $W$ , =  $mg$ .

The drop remains in equilibrium

$$\therefore \vec{F}_e = W$$

$$\therefore qE = mg$$

$$q = \frac{mg}{E} = \frac{9.75 \times 10^{-15} \times 9.8}{3 \times 10^4} = 3.18 \times 10^{-18} \text{ C}$$

**21. B**

Because the magnetic force is causing the charged objects to travel in a circle, make the magnetic force equal to the centripetal force requirements.

$$\vec{F}_c = \vec{F}_m$$

$$\frac{m\vec{v}^2}{r} = \vec{B}q\vec{v} \text{ (To compare, put what is changing on one}$$

side and what is constant on the other).

( $v/q$ ) changing between particles

$$= (\vec{B}r)\text{constant}$$

Since the magnetic field and the radius are the same, you can set the ratio below from the equation above.

$$\frac{m_{\alpha}\vec{v}_{\alpha}}{2q} = \vec{B}r = \frac{m_{\text{proton}}v_{\text{proton}}}{q}$$

$$6.65 \times 10^{-27} \text{ kg} \times \vec{v}_{\alpha}$$

$$= 2 \times 1.67 \times 10^{-27} \text{ kg} \times 4.4 \times 10^5 \text{ m/s}$$

$$v_{\alpha} = 2.2 \times 10^5 \text{ m/s}$$

**22. Open Response**

Since  $\Delta E = Vq$ , and you know the change in the energy of the particle since you were able to measure the velocity

with which it struck the positive plate,  $\Delta E = \frac{1}{2}m\vec{v}^2$ , you

can see that if the voltage increases, the charge must decrease. So if the voltage measured was accidentally too high, the calculated magnitude of the charge would have been too low.

**23. C**

The charge is distributed evenly over the outer surface of the conductor in order to put as much distance between all the individual charges as possible. This is because of the fact that all like charges repel, and when the charges are placed onto the conductor, they move to positions that are the farthest possible from one another. Because the charges are distributed evenly over the surface, any charge on one side is cancelled out by the same charge exactly opposite it. This is the farthest a charge could possibly be from another without leaving the surface of the conductor.

**24. Open Response**

A hollow conductor tube would create a zero electric field within the conductor, protecting the inside from external electric fields. It does not prevent fields generated inside the hollow conductor from exiting the conducting tube.

This is because the electric field inside the conductor would cause charges within the conducting material to move so that there would be an electrical charge distribution on the inside and outside surfaces of the conducting tube. This charge distribution would transfer the electric field from the inside of the tube to the outside.

**25. D**

Before developing the concept of a field, scientists had difficulty explaining how two objects, not in contact, could affect each other, i.e., action at a distance.

A field is a visual method for explaining how masses attract other masses, using field lines.

**26. B**

Geissler's vacuum pump enabled the construction of highly evacuated glass tubes and the discovery of cathode rays.

**27. Open Response**

Particle accelerators are used to determine properties of subatomic particles. These rely on electric and magnetic fields to accelerate the particles until they possess high amounts of energy, which would not be possible without knowledge of fields and their effects.

**28. Open Response**

Technology based on field theory has been applied to medical tools such as the PET and MRI scanners. Further insight and development of field-based technologies, may lead to more advances in medical technology, and promote quicker and easier diagnoses and treatments.

# Unit Test



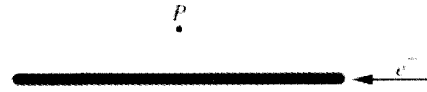
- In developing his law of universal gravitation, Isaac Newton used a mathematical analysis based on Kepler's first law to show that
  - $G$  is a constant.
  - $\vec{F}_g \propto \frac{1}{r^2}$ .
  - $T^2 \propto r^3$ .
  - $\vec{F}_g \propto a$ .
- If the distance between two bodies of masses  $m_1$  and  $m_2$  is doubled, then the gravitational field strength between them will
  - double
  - reduce to half
  - increase four times
  - reduce to one-fourth

### Numerical Response

- An electron and a proton are separated by the same distance as the first orbit radius of the Bohr model of the hydrogen atom. The electrical force of attraction is  $abc \times 10^d$  times as large as the gravitational attraction between the elementary particles. The values of  $abc$  and  $d$  are \_\_\_\_, \_\_\_\_, \_\_\_\_, and \_\_\_\_. (Record your answer to five digits.)

Use the following information to answer the next question.

Electrons move through a wire as shown below.



- What is the direction of the magnetic field at point  $P$ ?
  - Into the page.
  - Out of the page.
  - Toward the top of the page.
  - Toward the bottom of the page.
- Two points in an electric field have the same potential. What is the change in the potential energy of a charge flowing from one point to the other in the electric field?
 

A. 100 J	B. 80 J
C. 60 J	D. 0 J

### Numerical Response

- A charge of 18 C moving a certain distance in an electric field loses 1350 J of potential energy (PE). What is the potential difference between the two points in the electric field? \_\_\_\_ V

**Open Response**

Use the following information to answer the next question.

Two charged particles are placed in an electric field and are moved in the same direction. The electric potential energy of one increases, while the electric potential energy of the other decreases.

7. Explain why this happens.

---



---



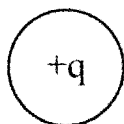
---



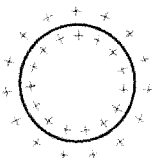
---

8. Which of the following charge distributions produces a region of non-zero, uniform, electric field?

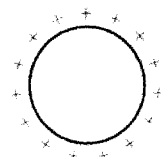
A.



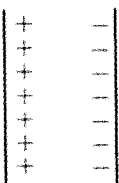
B.



C.



D.



**Open Response**

9. Compare the electric field produced by two oppositely charged point sources with the electric field produced by two oppositely charged parallel plates in terms of magnitude and direction.

---



---



---



---



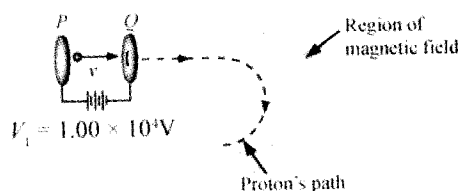
---

10. A copper ring of radius  $u$  is given a charge  $e$ . What is the field strength produced by the charge at the centre of the ring?

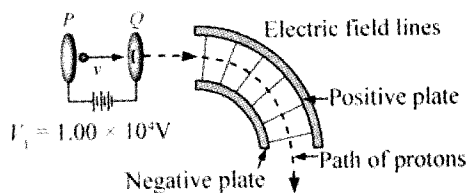
- A. 0
- B. 1
- C. 10
- D. Infinite

Use the following information to answer the next question.

A mass spectrometer uses either a magnetic field or an electric field to deflect charged particles.



Magnetic field mass spectrometer



Electric field mass spectrometer

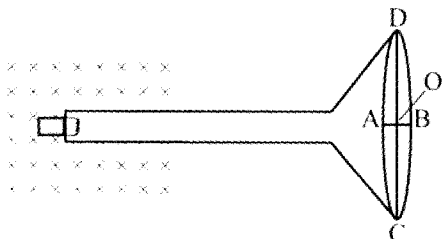
11. In the magnetic field mass spectrometer shown above, the direction of the magnetic field is

- A. into the page.
- B. out of the page.
- C. toward the left side of the page.
- D. toward the right side of the page.

**Open Response**

Use the following information to answer the next question.

The given figure shows the electron gun of a cathode ray tube placed in a region of magnetic field of strength 0.031 T. The acceleration of an electron in the magnetic field is approximately  $10^{16} \text{ m/s}^2$ .



12. Where would the electron strike the screen? If the direction of the magnetic field were reversed, where would the electron strike the screen?

---



---



---



---



---

**Numerical Response**

Use the following information to answer the next question.

The magnitude of charge on a conducting metal sphere is  $5.0 \times 10^{-6} \text{ C}$ . It is touched momentarily to an uncharged sphere, and then, the two spheres are separated by a distance of  $2.00 \times 10^{-2} \text{ m}$ .

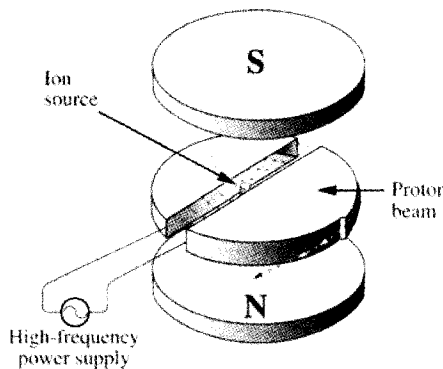
13. What is the electrostatic force between the two spheres? \_\_\_\_ N (State your answer to three significant digits).

**Open Response**

Use the following information to answer the next question.

**Cyclotron**

A cyclotron is a particle accelerator that is constructed of two hollow metal shells shaped like Ds in a perpendicular magnetic field created by magnets, as shown below. The entire apparatus is placed in a vacuum. An alternating voltage is maintained across the D separation. Positively charged particles, such as protons, are injected near the centre of the “D”s, and travel in circular paths, caused by the external perpendicular magnetic field. The frequency of the alternating voltage is adjusted to increase the speed of the particles each time they move across the D separation.



**Cyclotron Specifications**

Magnetic field intensity	0.863 T
Maximum voltage across D separation	20 000 V
D separation	5.00 cm

The speed of a particle moving with circular motion and the time it takes the particle to complete one circular orbit are given by the formulae

$$v = \frac{2\pi R}{T} \text{ and } T = \frac{2\pi m}{qB_{\perp}}$$

14. Beginning with force equations from the data sheets, derive the formula for the

$$\text{period } T = \frac{2\pi m}{qB_{\perp}}.$$

---

---

---

---

---

---

---

**Open Response**

15. Explain how scientists may predict the presence and size of a planet without actually observing the planet directly.

---

---

---

---

---



## SOLUTIONS

1. B	5. D	9. OR	13. 140
2. D	6. 75	10. A	14. OR
3. 2, 2, 7, 3, 9	7. OR	11. B	15. OR
4. B	8. D	12. OR	

## 1. B

Newton showed that elliptical orbits require that force follows an inverse square law.

In the formula  $\vec{F}_g = \frac{Gm_1m_2}{r^2}$ , the  $\vec{F}_g \propto \frac{1}{r^2}$ .

## 2. D

According to Newton's universal law of gravitation, the force between two bodies is given by the relation

$$\vec{F} = \frac{Gm_1m_2}{r^2}$$

This relation implies that gravitational force is inversely proportional to the square of the distance between two bodies. Hence, when the distance between the bodies is doubled, the gravitational field strength between them will reduce to one-fourth.

**Distractor Rationale**

A. The gravitational field strength between the two bodies will double if the distance between them is reduced by a factor of  $\sqrt{2}$ .

B. The gravitational field strength between the two bodies will reduce to half if the distance between them is increased by a factor of  $\sqrt{2}$ .

C. The gravitational field strength between the two bodies will increase four times if the distance between them is halved.

## 3. 2, 2, 7, 3, 9

$$\vec{F}_c = \frac{kq_1q_2}{r^2}$$

$$= \frac{\left(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}\right)(1.60 \times 10^{-19} \text{ C})^2}{(5.29 \times 10^{-11} \text{ m})^2}$$

$$= 8.224 \times 10^{-8} \text{ N}$$

$$\vec{F}_g = \frac{Gm_1m_2}{r^2}$$

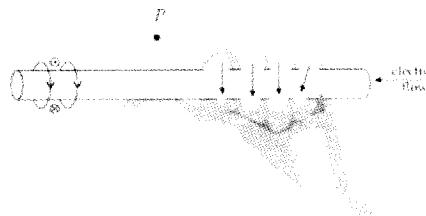
$$= \frac{\left(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}\right)(9.11 \times 10^{-31} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}{(5.29 \times 10^{-11} \text{ m})^2}$$

$$= 3.626 \times 10^{-47} \text{ N}$$

$$\frac{\vec{F}_c}{\vec{F}_g} = \frac{8.224 \times 10^{-8} \text{ N}}{3.626 \times 10^{-47} \text{ N}} = 2.27 \times 10^{39}$$

## 4. B

The direction of the magnetic field around a straight conductor carrying an electric current is given by the left hand rule.



## 5. D

Since the potential at the two points is equal, the potential difference between the two points is zero.

$$\begin{aligned} \text{Change in Potential Energy} &= \text{Charge} \times \text{Change in Potential} \\ &= \text{Charge} \times 0 = 0 \end{aligned}$$

## 6. 75

$$\begin{aligned} E &= qV \\ \Rightarrow V &= \frac{E}{q} = \frac{1350 \text{ J}}{18 \text{ C}} = 75 \text{ V} \end{aligned}$$

## 7. Open Response

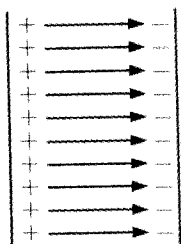
The particles are of opposite charge; one is positively charged and the other is negatively charged. If both are moved in the same direction and there is a change in their electric potential energies, then one is moving against the electric force of the field, meaning it gains electric potential energy, while the other is moving with the electric field, meaning it loses electric potential energy.

## 8. D

In the case of a single point charge, the electric field produced would be diverging and therefore non-uniform. In the cases of spheres with charges distributed over and under their surfaces, the electric field would also be diverging outside of the charged objects. However, inside these spheres there would be a region of zero electric field if the material used were conducting, and the charges were free to spread out. If the material used were non-conducting then the charges would remain where they were originally placed and would therefore have an associated electric field that was diverging in all locations around, and within, those spheres.



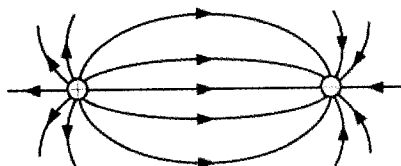
The electric field inside a parallel plate capacitor is shown in the following figure:



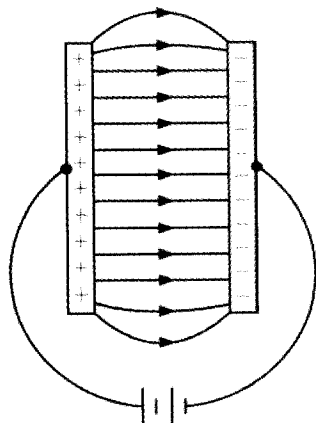
It can be observed in this figure that the electric field inside a parallel plate capacitor is uniform and non-zero. Alternative D depicts the charge distribution that produces a uniform region of electric field.

**9. Open Response**

The electric field produced by two oppositely charged point sources forms electric field lines that start on the positive point charge and end on the negative point charge. The strength of these electric field lines decreases with the distance away from each of the point charges according to Coulomb's law. Since the electric field is a vector field, the electric field due to one charge at any point in space is added with the electric field due to the other charge at that point.



The electric field between two charged parallel plates is unidirectional, perpendicular to the plates, and the strength of the field is uniform (within the space between the plates) starting from the positive plate and ending on the negative plate.



**10. A**

At every part of the ring, the effect of the charge,  $q$ , will be neutralized by a similar charge,  $q$ , diametrically opposite it. Therefore, the field strength at the centre is zero.

**11. B**

The moving particle is positive, so you must use your right hand. The thumb of your right hand will be in the direction of positive particle flow, the fingers will be in the direction of the magnetic field, and the palm will be in the direction of the force.

Because the particle's path is being bent in the direction shown, the palm of your right hand should face the centre of the half-circle in the grey area. Your thumb should point in the direction of particle motion. When you do this you will find your fingers will point out of the page.

**12. Open Response**

Since the electrons are travelling to the right in the given diagram, and the magnetic field is directed into the page, the **left hand rule**, for moving, negative charges in a perpendicular magnetic field, may be used. By letting your fingers represent the direction of the magnetic field and your thumb represent the direction that the electrons are travelling, the direction that the palm is pointing is left to show you which direction the magnetic force will push the electrons. In the initial case, this means that the electrons would be expected to strike the screen somewhere between O and C.

If the magnetic field were then reversed, you would use the same technique to determine the direction of the new magnetic force. In the second case, the magnetic force would cause the electrons to strike the scree between the O and the D.

**13. 140**

After touching the two conducting spheres together the charge on the first sphere is evenly distributed over both spheres leaving a charge of  $2.50 \times 10^{-6}$  C on each sphere. So the electrostatic force is given by

$$\vec{F}_e = \frac{kq_1q_2}{r^2}$$

$$\vec{F}_e = \frac{(8.99 \times 10^9)(2.50 \times 10^{-6})^2}{(2.00 \times 10^{-2})^2} \text{ N}$$

$$\vec{F}_e = 140 \text{ N}$$

**14. Open Response**

$$F_m = F_e$$

$$qvB_{\perp} = \frac{mv^2}{r}$$

$$q\left(\frac{2\pi R}{T}\right)B_{\perp} = \frac{m\left(\frac{2\pi R}{T}\right)^2}{R}$$

$$qB_{\perp} = \frac{m\left(\frac{2\pi R}{T}\right)}{R}$$

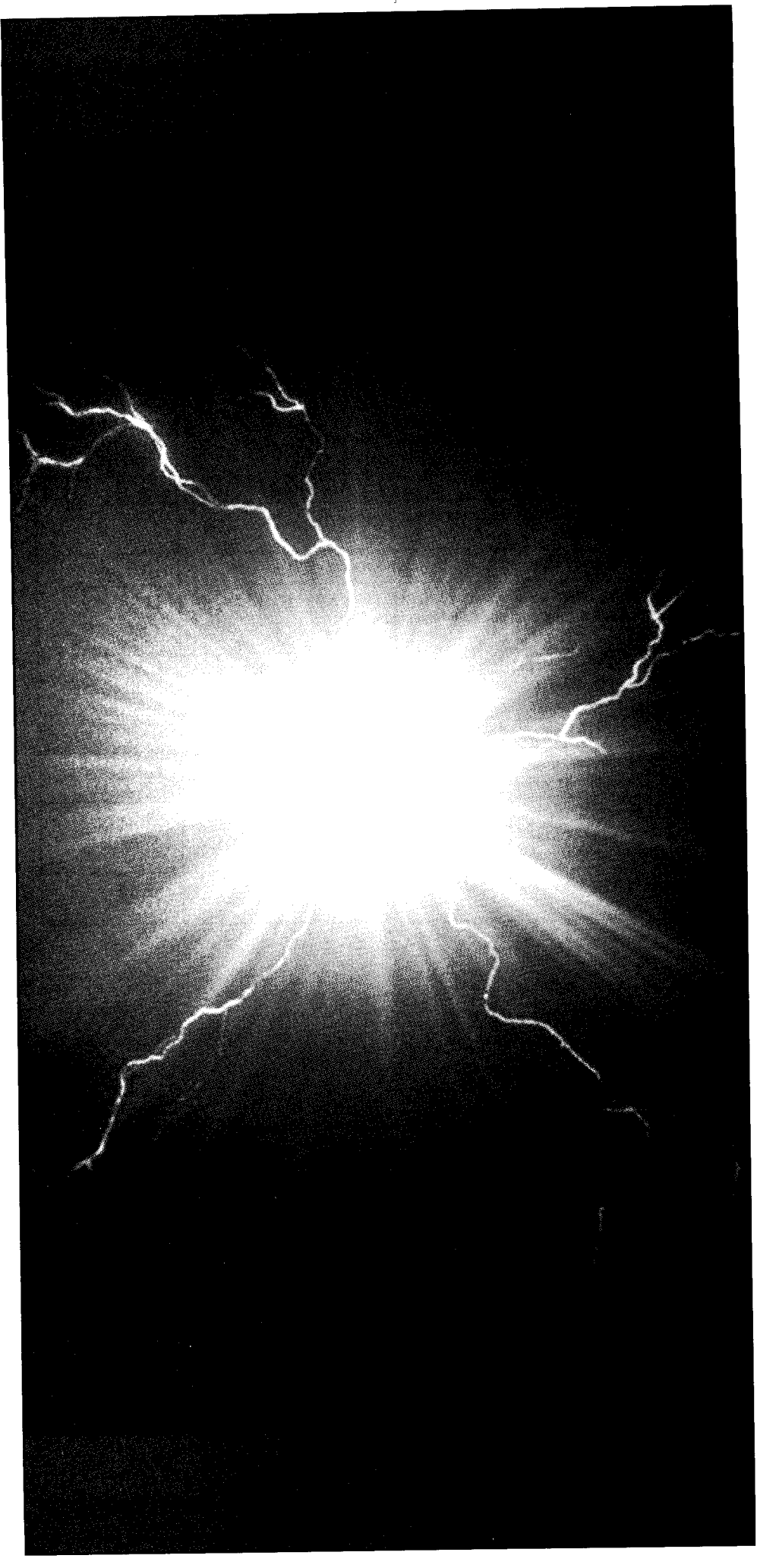
$$T = \frac{2\pi m}{qB_{\perp}}$$



**15. Open Response**

The movements of planets and other celestial bodies depends on the gravitational field produced by these bodies. Based on calculations of planetary movement, the presence of a planet may be predicted based on the gravitational forces necessary to produce the observed planetary motion.

# Wave Nature of Light





# The Wave Nature of Light

## Table of Correlations

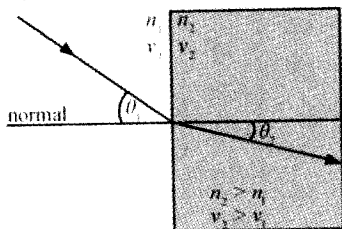
<b>Specific Expectation</b>	<b>Practice Questions</b>	<b>Unit Test Questions</b>
<b>12.4.1</b> Understanding Basic Concepts		
<b>12.4.1.1</b> <i>define and explain the concepts and units related to the wave nature of light</i>	1	1
<b>12.4.1.3</b> <i>describe the phenomenon of wave interference as it applies to light in qualitative and quantitative terms, using diagrams and sketches</i>	5, 6, 7	3, 5
<b>12.4.1.4</b> <i>describe and explain the phenomenon of wave diffraction as it applies to light in quantitative terms, using diagrams</i>	8, 9, 10a, 10b, 10c	4, 6
<b>12.4.1.2</b> <i>describe, citing examples, how electromagnetic radiation, as a form of energy, is produced and transmitted, and how it interacts with matter</i>	2, 3, 4	2
<b>12.4.2</b> Developing Skills of Inquiry and Communication		
<b>12.4.2.1</b> <i>identify the theoretical basis of an investigation, and develop a prediction that is consistent with that theoretical basis</i>	11	7
<b>12.4.2.3</b> <i>collect and interpret experimental data in support of a scientific theory</i>	15	10
<b>12.4.2.4</b> <i>analyse and interpret experimental evidence indicating that light has some characteristics and properties that are similar to those of mechanical waves and sound</i>	16, 17	11
<b>12.4.2.2</b> <i>identify the interference pattern produced by the diffraction of light through narrow slits (single and double slits) and diffraction gratings, and analyse it in qualitative and quantitative terms</i>	12, 13, 14	8, 9
<b>12.4.3</b> Relating Science to Technology, Society and the Environment		
<b>12.4.3.1</b> <i>describe instances where the development of new technologies resulted in the advancement or revision of scientific theories</i>	18, 19	12
<b>12.4.3.3</b> <i>analyse, using the concepts of refraction, diffraction, and wave interference, the separation of light into colours in various phenomena, which forms the basis for the design of technological devices</i>	21, 22	15
<b>12.4.3.2</b> <i>describe and explain the design and operation of technologies related to electromagnetic radiation</i>	20	13, 14



**12.4.1.1** define and explain the concepts and units related to the wave nature of light

### REFRACTION AND SNELL'S LAW

Refraction is defined as the bending of waves as they travel from one medium to another. In the case of light, it is a result of the different refractive indices present that the light is bent. As the index of refraction increases, more light will bend toward the normal and the velocity of that light will slow as it travels through the particular medium.



Snell's law describes this interface between different mediums using the following formula.

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2}$$

Another useful form of Snell's law is:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

It is also useful to note that the velocity of light in a medium can be described as:

$$v = \frac{c}{n}$$

Sometimes a situation can arise where the incident ray of light will be reflected off of the boundary as a result of the angle of incidence being too high. This situation only arises when the ray of light passes from a medium with a high index of refraction into a medium with a lower index of refraction. The minimum angle at which the incident ray of light will bounce off of the interface between the two mediums is called the critical angle.

$$\theta_{\text{critical}} = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$

### DISPERSION OF LIGHT

The refractive index of many materials varies with the wavelength of light used. This results in different colours of light being refracted at different angles. When white light is incident upon a glass prism, the many different wavelengths of light that compose white light are refracted at different angles resulting in a rainbow pattern.

### WAVE DIFFRACTION, INTERFERENCE, AND POLARIZATION

Diffraction refers to the spreading or bending of waves as the wave fronts move past the edge of a barrier or pass through a small opening.

This diffraction varies inversely with the size of the opening,  $d$ , and directly with the wavelength. Diffracted waves often interfere with one another.

Constructive interference occurs where a crest, or trough, from one source meets a crest, or trough, from another source. Points of constructive interference can be connected by a lines called antinodal lines. Destructive interference occurs where a crest from one source meets a trough from another. Corresponding lines connecting points of destructive interference are called nodal lines.

The relationship among the distance between the nodes or antinodes can be expressed by the formula:

$$n\lambda = \frac{dx}{l} = d \sin \theta,$$

where  $x$  is the distance between the nodes or antinodes,

$d$  is the distance between the two sources, and  $l$  is the distance from the slit sources to the screen.

The electric and magnetic fields of EMR vibrate along an infinite number of planes, which are always perpendicular to each other and to the direction of propagation of the radiation. Thus EMR travels as a transverse wave. Unlike other types of waves, transverse waves can be polarized.

Polarization is the process of filtering out the different planes of transverse waves. For polarizing EMR, the vibration of the electric field is filtered to a single plane. For example, the reason visible light is plane polarized when it reflects off a horizontal, non-metallic surface is that the electric field of the reflected light oscillates primarily along the horizontal plane.



Light can be polarized when it travels through certain natural crystals, such as tourmaline. A Polaroid lens is made of plastic that is strained in a manner that causes an alignment of its molecules. Light becomes plane polarized as it passes through the plastic.

Diffraction, interference, and polarization provide convincing empirical evidence to support the wave model of EMR.

## THE ELECTROMAGNETIC SPECTRUM

Different types of electromagnetic radiation differ from one another according to their frequencies, wavelengths, or energies. The frequency of EMR with a given energy never changes.

The corresponding wavelength is inversely related to the frequency.

$$\lambda \propto \frac{1}{f}$$

The proportionality constant is the speed of EMR,  $c$ .

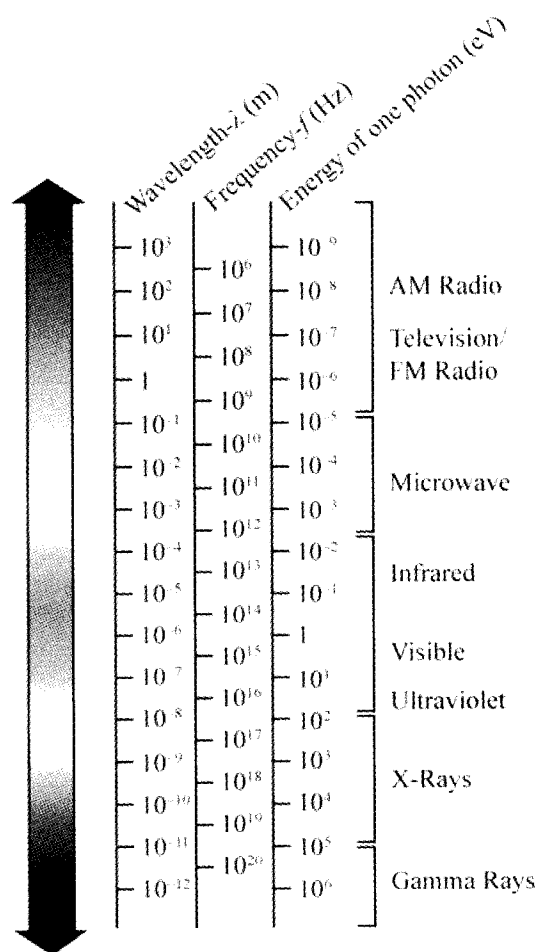
$$\lambda = \left( c \right) \frac{1}{f}$$

This results in the universal wave equation as it applies to EMR.

$$c = f\lambda$$

The value of  $c$  is  $3.00 \times 10^8$  m/s in vacuum (or air). This speed can be affected by the medium through which the EMR travels.

Placing the different forms of EMR in order of increasing frequencies or energies, or in order of decreasing wavelengths, the results in the electromagnetic spectrum.



*The electromagnetic spectrum*

### Practice

- The bending of light at a sharp edge or corner is referred to as \_\_\_\_\_.
  - reflection
  - refraction
  - diffraction
  - polarization



**12.4.1.2** describe, citing examples, how electromagnetic radiation, as a form of energy, is produced and transmitted, and how it interacts with matter

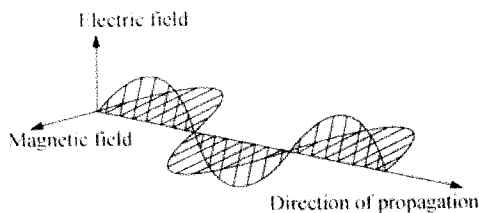
## PRODUCING ELECTROMAGNETIC RADIATION

Electromagnetic radiation (EMR) is a self-sustaining chain of interactions between electric charges and magnetic fields.

EMR always begins with an accelerating charge. This can be an electron changing its position within an atom and emitting visible light, the entire atom or molecule vibrating or rotating and emitting infrared or microwave radiation, or electrons oscillating within a conductor and emitting radio waves.

EMR can be considered as two waves.

One consisting of an electric field, and the other consisting of a magnetic field. Both waves have the same frequency travelling in phase together in perpendicular planes to form one EMR wave.



### Practice

- The electromagnetic radiations produced when high-speed electrons are decelerated by hitting a target are called
  - X-rays
  - Y-rays
  - beta rays
  - gamma rays
- Electromagnetic radiation is produced by charged particles that travel
  - at the speed of light
  - with zero acceleration
  - with a changing velocity
  - parallel to a fixed magnetic field

### Open Response

- Describe, in one line, how radio waves are produced. Give an example of an everyday object that uses this type of EMR.

---



---



---



---



---

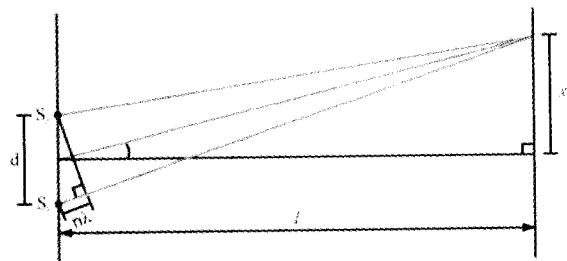


---

**12.4.1.3** describe the phenomenon of wave interference as it applies to light in qualitative and quantitative terms, using diagrams and sketches

## YOUNG'S DOUBLE-SLIT EXPERIMENT

Light from a coherent source was directed at an opaque screen with a single slit in it. The tiny ray emerging was used to illuminate two closely spaced slits. These two slits simulated two in-phase sources of light and the two rays were then projected on a screen.



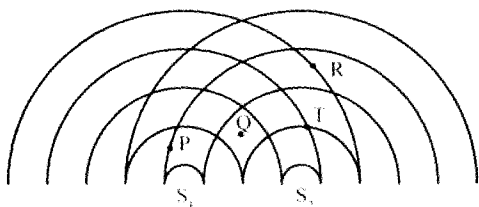
The interference pattern on the screen consisted of a bright central band with equally spaced dark bands (nodes) and bright bands (antinodes) on either side.

Thomas Young's experiment provided strong evidence in favour of the wave model of EMR.

**Practice**

Use the following information to answer the next question.

The given diagram illustrates the circular wavefronts generated by two point sources. Each line represents the crest of the wavefront, and the midpoint between each line is the trough of the wavefront.



5. Which of the following statements about the diagram is **true**?
- A. Both sources are in phase, and point Q is a nodal point.
  - B. Both sources are in phase, and points Q and T are antinodal lines.
  - C. Both sources are  $180^\circ$  out of phase, and point T is an antinodal point.
  - D. Both sources are  $90^\circ$  out of phase, and points P and R are nodal points.

**Numerical Response**

6. Monochromatic light is directed at a diffraction grating ruled with  $6.00 \times 10^4$  lines/m. The maxima on a screen 1.10 m away are  $9.89 \times 10^{-2}$  m apart. What is the wavelength of the light?  $\underline{\hspace{2cm}} \times 10^{-7}$  m

**Open Response**

7. How was Thomas Young able to obtain two sources that were in phase?

---



---



---



---



---



---

**12.4.1.4** describe and explain the phenomenon of wave diffraction as it applies to light in quantitative terms, using diagrams

**DIFFRACTION PROBLEMS**

Use the following formula to calculate problems involving double-slit or multi-slit apparatuses. A diffraction grating is a type of multi-slit apparatus.

$$n\lambda = \frac{dx}{l} = d \sin \theta$$

**Example**

When light with a wavelength of 650 nm shines on a diffraction grating producing a third-order maximum at an angle of  $22.0^\circ$ . How many lines/cm have been ruled on the diffraction grating?

$$\begin{aligned} n\lambda &= d \sin \theta \\ d &= \frac{n\lambda}{\sin \theta} \\ &= \frac{3(650 \times 10^{-9} \text{ m})}{\sin 22.0^\circ} \\ &= 5.206 \times 10^{-6} \text{ m} \\ &= \frac{1}{5.206 \times 10^{-6} \text{ m/line}} \\ &= 1.92 \times 10^5 \text{ lines/m} \\ &= 1.92 \times 10^3 \text{ lines/cm} \end{aligned}$$

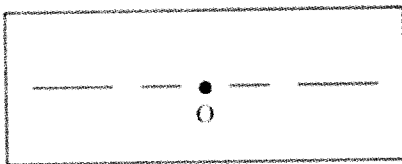


**Practice**

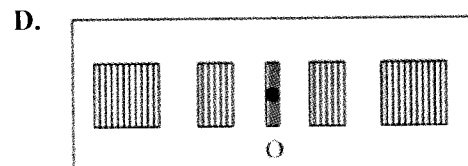
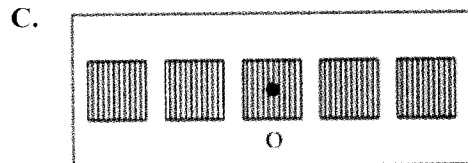
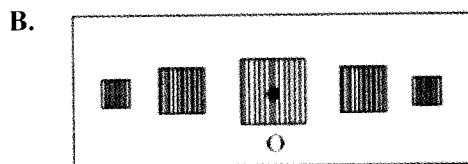
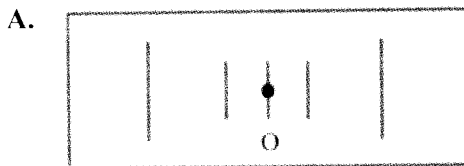
8. According to the wave model of light, the reason diffraction occurs is that a slit acts as a
- A. wave source.      B. mirror.  
 C. prism.              D. lens.

*Use the following information to answer the next question.*

The given figure shows the grating spectrum for light coming from a point source and passing through a diffraction grating.



9. Which of the following figures represents the grating spectrum if a vertical slit is placed between the point source and the diffraction grating?



*Use the following information to answer the next multipart question.*

10. Using Huygen's principle and appropriate diagrams, illustrate how a

Part A

**Open Response**

plane wave diffracts around an obstacle.

---

---

---

---

---

---

---

---

Part B

**Open Response**

plane wave diffracts through a small opening.

---

---

---

---

---

---

---

---

Part C

**Open Response**

curved wave front from a point source propagates over a short distance.

---

---

---

---

---

---

---

---



**12.4.2.1** *identify the theoretical basis of an investigation, and develop a prediction that is consistent with that theoretical basis*

## PREDICTING RESULTS OF EXPERIMENTS USING WAVE THEORY

### RIPPLE TANK EXPERIMENTS

Ripple tanks can be used to investigate how disturbances or ripples propagate on the surface of water. Such ripples are expected to behave as waves, and the results of qualitative experiments on them can be predicted using wave theory.

One significant experiment consists of placing an obstacle in the path of the ripples and observing the effect. Wave theory predicts that if the size of the obstacle is comparable to the wavelength of the ripples, diffraction effects can be noticed in the vicinity of the obstacle. On the other hand, as the size of the obstacle increases far beyond the wavelength, a “shadow” region behind the obstacle, where there are no ripples, becomes increasingly prominent.

Usually, the wavelengths of the ripples in a ripple tank are a few centimetres long. As the length of a ripple tank is limited to a few metres, it is expected that as the obstacle size changes from a few mm to many cm, diffraction effects will gradually give way to geometrical shadows within the region of ripple propagation.

### DIFFRACTION PATTERN OF A STRAND OF HAIR

An average strand of human hair has a width of  $50\ \mu\text{m}$ , though its actual thickness can vary by a factor of 3 or so above or below this number, depending on age and ethnicity. Visible light has a wavelength of the order of  $500\ \text{nm}$ . The central maximum of a diffraction pattern produced by an obstacle of width  $a$  in a monochromatic light source of wavelength  $\lambda$  is expected in theory to have an angular width of the order of  $\frac{\lambda}{a}$ .

This corresponds to a linear width of  $D\frac{\lambda}{a}$  on a screen at a distance of  $D$  behind the obstacle (i.e., about 1 cm on a screen 1 m away). This is big enough to be clearly visible. The pattern will consist of a few visible bright bands on either side of the brightest point, spaced about 1 cm apart.

As a strand of hair is very long compared to its width, the diffraction pattern will be restricted to only one dimension (the axis corresponding to the width of the strand). These predictions can be easily verified using a laser source for strong illumination and a dark screen a few metres away.

### CROSSED POLARIZING FILTERS

Since light is a transverse electromagnetic wave, it is expected to show the wave property of polarization. Polarizing filters have the property of transmitting light polarized in one direction, but blocking light polarized in the perpendicular direction.

Ordinary light comes as a large number of wave trains, each with a randomly oriented plane of polarization. This implies that on passing through a polarizing filter, the intensity should be half the original. Further, if two polarizing filters are used in succession, the intensity of the light transmitted by the combination depends on their relative orientation. If the two filters are aligned, both let the light of the same polarization pass through. In this case, the second filter has no effect, and the combination transmits the most light.

If the two filters are aligned at right angles, then they mutually block perpendicular planes of polarization. The component of the light transmitted by the first filter is then blocked by the second, and the combination of the two transmits the lowest possible amount of light.

### Practice

11. Which of the following waves would **not** be expected to experience polarization?
- A. A standing wave on a length of rope.
  - B. Sound waves in a gas or liquid.
  - C. Electromagnetic waves.
  - D. Waves in a ripple tank.



**12.4.2.2** *identify the interference pattern produced by the diffraction of light through narrow slits (single and double slits) and diffraction gratings, and analyse it in qualitative and quantitative terms*

## PHASE, COHERENT SOURCES, AND INTERFERENCE

Since light has wave properties, one important variable that enters any discussion of interference effects is the phase of a light wave at a point. The notion of phase is a familiar one from the study of mechanical waves and simple harmonic motion.

The mechanism of production of light plays an important role in determining how the phases of light from a light source vary. Ordinary sources like electric lamps produce light as a large number of wave trains, each with an independent phase. A pair of such sources will not produce stable interference effects.

Two light sources are said to be coherent if the phases of the light waves coming from them are related. Coherent light sources can produce stable interference patterns. For example, the two phases could be the same, or could have a constant difference. The easiest ways of producing coherent sources are to split light from a laser beam by reflection or refraction, or to illuminate two slits with the light from a single slit.

The intensity produced by one or more sources, of course, varies from point to point. When two coherent sources together illuminate a screen, their individual intensities do not combine. However, the amplitudes of the two waves at each point on the screen do combine according to the principle of wave superposition. This produces a stable interference pattern resulting from the redistribution of intensity into alternating bright and dark regions.

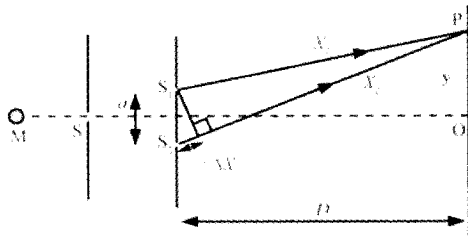
## CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE

- If the phase difference between the waves is an odd multiple of  $\pi$  (i.e.,  $n\pi$ , where  $n = \dots, -3, -1, 1, 3, \dots$ ), the resulting amplitude  $A = |A_1 - A_2|$  is the minimum possible. In this case, the waves are out of phase at that point. Which means that the trough of one arrives with the crest of the other, implying that the vibrations of the two waves cancel each other out. This condition is called destructive interference.
- If the phase difference between the waves is an even multiple of  $\pi$  (i.e.,  $n\pi$  where  $n = \dots, -4, -2, 0, 2, 4, \dots$ ), the resulting amplitude  $A = |A_1 + A_2|$  is the maximum possible. In this case, the waves arrive in phase at that point. which means that the trough of one arrives at the same time as the trough of the other, and the crest of one arrives with the crest of the other. This condition is called constructive interference.
- For other phase differences, the amplitudes will take values between the maximum and minimum values.
- If both sources have equal amplitudes ( $A_0$ ) and equal intensities ( $I_0$ ) then  $I_{\min} = 0$  and  $I_{\max} = 4I_0$ .
- In contrast, for incoherent sources a uniform intensity of  $2I_0$  is observed.



## DOUBLE SLIT INTERFERENCE

The most simple kind of interference pattern results from light passing through two slits  $S_1$  and  $S_2$ , illuminated by light passing through a single slit, which in turn is illuminated by a monochromatic light source  $M$ . The slits have widths that are negligible when compared to the distance between them. The resulting pattern consists of alternating dark and bright fringes of approximately constant width.



A quantitative description is illustrated using a general point  $P$  on the screen. Its vertical position with respect to the line of symmetry  $OS$  is  $y$ . The coherent light waves from  $S_1$  and  $S_2$  have to travel different distances  $X_1$  and  $X_2$  depending on the coordinate  $y$ .

### PATH DIFFERENCE

$$\Delta X = X_2 - X_1$$

$$\Delta X = \sqrt{D^2 + \left(y + \frac{a}{2}\right)^2} - \sqrt{D^2 + \left(y - \frac{a}{2}\right)^2}$$

If it is assumed that  $y$  and  $a$  are much smaller than  $D$ , then

$$\Delta X \approx \frac{ya}{D}$$

### FRINGE WIDTH

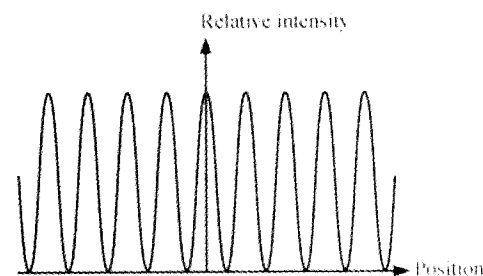
Constructive interference occurs at those points for which  $\Delta X = 2n\lambda$  where  $n$  is an integer. This may also be written as  $\Delta X = n\lambda$ , where  $n$  is an even integer. Such points locate the centres of bright fringes on the screen. The point  $O$  is one of these, because the path difference and phase difference are zero at this point. Using the condition for constructive interference and the expression for phase difference, the position of the  $n$ th bright fringe is given by  $y_n = \frac{nD\lambda}{a}$ . The reason the central point  $O$  ( $n = 0$ ,  $y = 0$ ) is always bright is that the path difference and phase difference are always zero there.

Destructive interference occurs at those points for which  $\Delta X = (2n + 1)\lambda$  where  $n$  is an integer.

Such points locate dark fringes on the screen. Using the condition for destructive interference and the expression for phase difference, the position of the  $n$ th dark fringe is given by  $y_{n+1} = (2n + 1)\frac{D\lambda}{2a}$ .

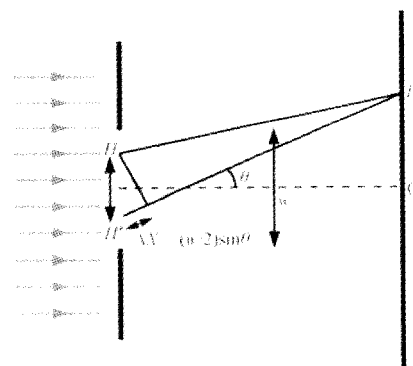
The fringe width is defined as the distance between the two successive maxima,  $y_{n+1} - y_n$ , which is a constant.

For a pair of slits of negligible width, the variation of intensity on the screen is given in the following illustration.



## SINGLE-SLIT DIFFRACTION

The quantitative results of two-slit interference may be used to explain some of the results of a single-slit diffraction pattern. This discussion applies to the case when all the wavefronts falling on the screen are approximately parallel. In practise, this is achieved by placing a lens between the slit and the screen, but the lens is omitted here.



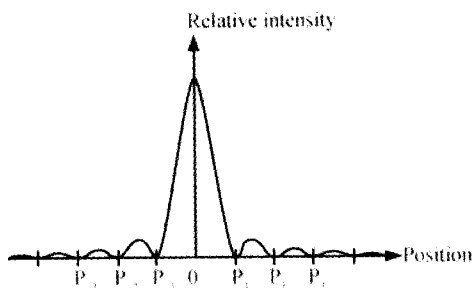
The slit can be imagined to be divided into a large number of points. It is then expected that the double-slit results can be applied to any pair of well separated points within the slit.



Consider the pair of points  $H$  and  $H'$ , separated by a distance  $\frac{w}{2}$  or exactly half the slit width  $w$ . If the point  $P$  is located such that  $\Delta X = \frac{\lambda}{2}$ , the double-slit results imply that the light waves from the pair of points  $H$  and  $H'$  produce destructive interference at  $P$ . The entire slit can be divided into such pairs of points, so  $P$  will be a dark point if the condition  $\Delta X = \frac{\lambda}{2}$  is met. Thus a minimum intensity point occurs when  $\Delta X = \frac{w}{2} \sin \theta = \frac{\lambda}{2}$ , (i.e.,  $\sin \theta = \frac{\lambda}{w}$ ).

There is a set of minimum intensity points at the positions  $\sin \theta = \frac{n\lambda}{w}$ , where  $n = \pm 1, \pm 2, \pm 3, \dots$

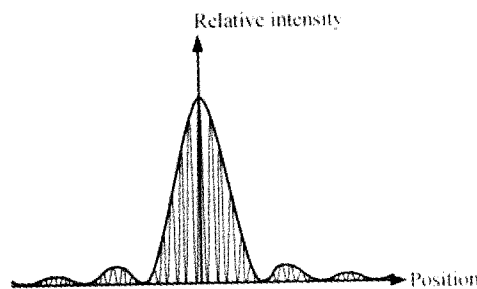
The intensity peaks between each pair of minima. In particular, the point  $O$  is a point of maximum intensity. If the distance of  $O$  from the slit is large, all the waves travel nearly equal distances to reach  $O$ , and their path differences are negligible. So, all the waves interfere constructively at  $O$ . The actual distribution of intensity is as shown on the given graph.



The intensity at  $O$  is much higher than that at the other peaks.

## DIFFRACTION FROM TWO OR MORE SLITS AND GRATINGS

If two or more slits of significant width are used, the resultant pattern shows the combined effects of the double-slit experiment with very thin slits, and the diffraction pattern for a single slit. The effect of the diffraction pattern appears as the light “envelope” curve of the actual distribution.



When the number of slits is increased beyond two, the pattern remains qualitatively similar, but the peaks become sharper. A set of negligible secondary peaks is also introduced. As the width of the slit is decreased, the fall off from the central maximum also becomes less pronounced. When a large number of equally spaced thin slits are used, the arrangement is called a diffraction grating. Gratings have proven to be extremely useful in experimental physics over the last century.

**Practice**

Use the following information to answer the next question.

The given table shows the angular spacing between two consecutive minima in a single-slit diffraction pattern. The wavelength of light used is  $5.0 \times 10^{-5}$  cm.

Minima	Angular spacing (radians)
1st – 2nd	$2.5 \times 10^{-3}$
2nd – 3rd	$2.5 \times 10^{-3}$
3rd – 4th	$2.5 \times 10^{-3}$

12. In the given experiment, if the width of the slit is increased, the angular separation between consecutive minima will
- A. increase      B. decrease  
C. remain same    D. become zero

**Numerical Response**

13. Monochromatic light illuminated two slits separated by  $1.50 \times 10^{-2}$  cm and formed a pattern on a screen 2.34 m away. The fifth maximum, not including the central maximum, was 6.22 cm from the centre of the screen. The wavelength of this light was  $a.bc \times 10^{-d}$  m. The values of  $a$ ,  $b$ ,  $c$ , and  $d$  are \_\_\_\_, \_\_\_\_, \_\_\_\_, and \_\_\_\_. (Give your answer to four digits.)

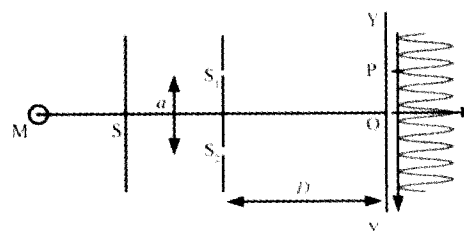
**Numerical Response**

14. A screen with two small slits, separated  $4.24 \times 10^{-4}$  m apart, is located 1.00 m from a solid screen. If the distance between the central fringe and the second fringe that appear on the solid screen is 2.35 mm, the wavelength of light creating this interference pattern must be  $a.bc \times 10^{-d}$  m. The values of  $a$ ,  $b$ ,  $c$ , and  $d$  are \_\_\_\_, \_\_\_\_, \_\_\_\_, and \_\_\_\_. (Give your answer to four digits.)

12.4.2.3 collect and interpret experimental data in support of a scientific theory

## IMPLICATIONS OF THE DOUBLE-SLIT EXPERIMENT FOR THE NATURE OF LIGHT

A key experiment that demonstrates the wave nature of light is the double-slit experiment. Two slits,  $S_1$  and  $S_2$ , are illuminated by a single slit, which in turn is illuminated by a monochromatic light source, M. This could be a laser, or just sunlight after passing through a prism. The slits have widths that are negligible compared to the separation  $a$  between them. The resulting pattern is observed on a dark screen YOY.



The overall effect is a regular pattern consisting of alternating dark and bright fringes of approximately constant width. The given diagram shows the graph of the intensity variation, rotated by a right angle, beside the screen. Points like O corresponding to the maximum intensity of the central fringe appear bright, while points like P corresponding to the minimum intensity appear dark.



Such a pattern is not possible to explain using a particle description of light. If light were thought to be a stream of particles, it would be reasonable to suppose that the brightness or intensity in a region would be proportional to the number of such particles hitting the region per unit time. Each of the two sources  $S_1$  and  $S_2$  would contribute to the brightness with a certain number of light particles (i.e., photons) per unit time. The total brightness would then just be proportional to the sum of the number of particles there, which would be the sum of the contributions from the two slits. This would give an intensity decreasing slowly on either side of the central maximum value at O. There would be no alternating dark and bright fringes in such a situation.

The pattern that results can only be explained as a wave phenomenon using the concept of interference of waves. The two slits act as sources of coherent waves that interfere on the screen in accordance with the principle of superposition. For any given point on the screen, the lengths of the paths from  $S_1$  and  $S_2$  are different in general. (The point O is an exception).

The waves from  $S_1$  and  $S_2$  produce a resultant combination that depends on the path difference. Points of maximum intensity occur where they interfere constructively, while points of minimum intensity occur where they interfere destructively. Qualitatively this implies periodically varying brightness forming the observed pattern of bright and dark fringes.

Quantitatively there is expected to be a nearly constant separation between two consecutive bright and dark regions. If the wavelength of light waves is assumed to be  $\lambda$ , wave theory predicts that this separation, or fringe width, is  $Y = \frac{D\lambda}{a}$ .

This can be directly tested by repeating the experiment with different values of  $a$  and  $D$ . A typical set of results using red light would resemble the data below. The fringe width can be found by dividing the total width of a number of fringes by the number of fringes.

Further, if the colour of light sent through the double slits is varied, the fringe width is seen to vary as well. This can be explained in terms of a relation between the colour of light and the wavelength. If light with many components like sunlight is used, it is seen that fringes of different colours overlap, producing an indistinct pattern.

The explanation of the experimental observations using wave concepts and their quantitative predictive power, support the wave theory of light.

### Practice

#### Open Response

*Use the following information to answer the next question.*

An experiment is set up to determine the wavelength of a light source. Monochromatic light is shone through a double slit, and a pattern is observed on a screen behind the setup.

15. What measurements must be taken to determine the wavelength of the monochromatic light?

---



---



---



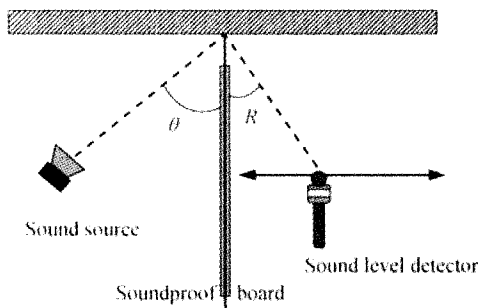
---



**12.4.2.4** *analyse and interpret experimental evidence indicating that light has some characteristics and properties that are similar to those of mechanical waves and sound*

## REFLECTION OF SOUND

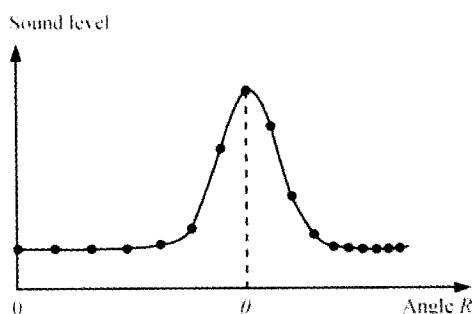
An experiment to demonstrate the reflection of sound may be set up as shown below.



A long soundproof board a few metres long is placed perpendicular to a wall, with a gap of approximately 30 cm between the soundproof board and wall.

A source of sound emitting a high pitched, pure tone about four octaves above middle C is activated on one side of the board. On the other side of the board, a sound level detector records the loudness of the sound at different positions along a straight line when the source is activated. The source and the detector are maintained at the same height above the floor.

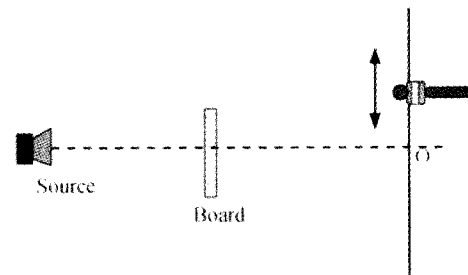
The detector is placed at different positions and the level of sound is recorded. The angle  $R$  is calculated from the position and noted at each position. When the data are plotted, the results are illustrated by the given graph.



The peak sound level is observed to occur when  $R = \theta$ . This peak position may be interpreted as the position in the direction of the reflected sound. This demonstrates that the reflection of sound, which is a mechanical wave, obeys the same law as that for the reflection of light.

## DIFFRACTION OF SOUND

The reason diffraction of sound is a very noticeable effect is that the wavelength of sound in air is large enough to be comparable to the sizes of doors and windows. An experiment to demonstrate the diffraction of sound may be set up as shown in the given diagram.



A source of sound emitting a pure, high-pitched tone is placed at the focus of a concave reflector which, by reflection, makes sound travel in a beam.

A soundproof board with a length of approximately 50 cm is placed in the path of the sound. A detector measures the sound level at varying heights along a straight line. The height can be converted to an angle using a simple trigonometric calculation.

The data show that the highest sound level occurs at the point O, which lies in the geometrical shadow of the board. This shows that sound waves bend around the board (i.e., show diffraction effects).

Higher pitched sounds, which correspond to higher frequencies, bend the least, while lower pitched sounds, which correspond to lower frequencies, bend the most. The analogous phenomenon for light may be observed by placing a strand of hair in front of a laser beam and observing the resulting pattern of illumination on a screen.

Such experiments show that mechanical waves like sound share properties like reflection, refraction, and diffraction with light. Similarly, a ripple tank can be used to demonstrate interference effects of mechanical waves on the surface of water.



**Practice**

16. An astronomical instrument that analyzes light in order to determine the chemical composition of stars is a
- A. spectrometer    B. satellite dish  
C. telescope        D. prism

**Open Response**

17. Mechanical waves can be described as “a disturbance moving through a medium.” How does this statement apply to light waves?

---

---

---

---

---

---

---

---

**12.4.3.1** describe instances where the development of new technologies resulted in the advancement or revision of scientific theories

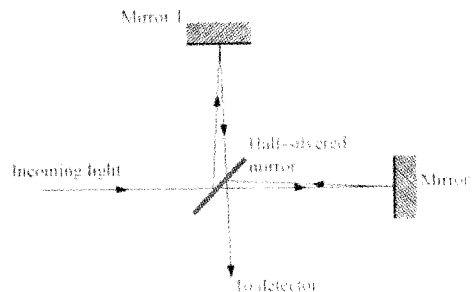
**INTERFEROMETERS AND THE MICHELSON–MORLEY EXPERIMENT**

The interferometer is one of the devices that have had a direct bearing on the history of scientific theory. Maxwell’s discovery in 1862 that light was a kind of electromagnetic wave raised questions about the nature of space itself. The speed of light deduced from Maxwell’s equations depends only on the properties of free space. In contrast, the speed of familiar mechanical waves depends on the properties of the medium in which they propagate. Further, the speed makes physical sense because it is defined relative to the particles in the medium. As light can travel through a vacuum, one of the questions that naturally arose was which medium the speed of light was relative to.

A common answer at that time was that the speed was relative to the “ether”, an invisible medium that filled space. If this were true, then it would, in principle, be possible to measure variations in the speed of light with respect to different frames of reference. This was a result of the fact that if light moved within the ether, and the ether had some velocity relative to some arbitrary zero velocity, then the speed of light would be expected to show variations dependent upon whether the light waves were moving with or against the motion of the ether.

The Michelson–Morely experiment was conducted to try to observe such a variation using the varying directions of Earth’s movement. Since the speed of light compared to the speed of Earth around the sun, is significantly faster, such an experiment would need to be extremely accurate.

The high accuracy required was made possible by the use of Michelson’s interferometer. This is a device in which light from a source is split using a half silvered mirror that reflects some of the light incident on it and transmits the rest. The two components are then arranged to combine using the same half silvered mirror as shown in the given diagram. Since the components travel along different paths, they combine with different phases and produce interference fringes in the detector.



The time intervals taken by light after being split along the two paths depend on the speed of light relative to the apparatus along each path. The motion of Earth was expected to make these speeds vary as a result of it’s rotation and orbital velocity, and hence shift the interference fringes produced in the detector.



Despite repeated experiments that had increasing accuracy and sophistication, no such shift was observed within the limits of accuracy of the experiments. The consistent failure of this experiment to detect the motion of Earth with respect to the ether had no satisfactory theoretical explanation at the time. Over a period of time, it came to be realized that the idea of the ether as a medium for light propagation had no physical relevance. In the meantime, Einstein formulated the special theory of relativity, of which one of the basic postulates stated that the speed of light is the same regardless of the motion of the observer. This conclusion hinted at but did not initially explain the Michelson–Morley experiments. The theory of relativity drastically changed the scientific view of nature.

More than a hundred years after these developments, laser interferometers are now being employed in many different fields. Interferometers are expected to detect possible gravitational waves. Gravitational waves are predicted as a consequence of Einstein’s general theory of relativity, and if they are discovered will be invaluable experimental sources in the study of the universe.

## ELECTRON MICROSCOPY

As ordinary microscopes use visible light to produce an image of the object under study, they are fundamentally limited by the wavelength of visible light. In particular, they cannot resolve details much smaller than the wavelength of the light being used. Following the discovery of the wave nature of the electron, the possibility of using electron waves instead of light in microscopy became apparent. The de Broglie wavelength of a beam of electrons can be made much smaller than the wavelength of light, allowing electron microscopes to probe atomic size details.

Since their invention in the 1930s, electron microscopes have played a major role in the investigation of small scale systems in biology, chemistry, material physics, and condensed matter physics. Electron microscopy made it possible to map the structure of biological cells in much greater detail than that provided by optical microscopy. It also led to the first direct observation of the composition of viruses. It is now the standard technique for studying biomolecules like proteins and nucleic acids, which are too small to examine with optical microscopes.

Electron microscopes are indispensable in material science and the semiconductor industry, where they are routinely used to characterize materials and structures and to investigate faults.

### Practice

18. A well-known radio interferometer is the
- A. Very Big Array
  - B. Very Large Array
  - C. Large Radio Telescope
  - D. American Radio Telescope

### Open Response

19. Explain why an electron microscope has greater resolution and magnification than a light microscope.

---



---



---



---



---



---

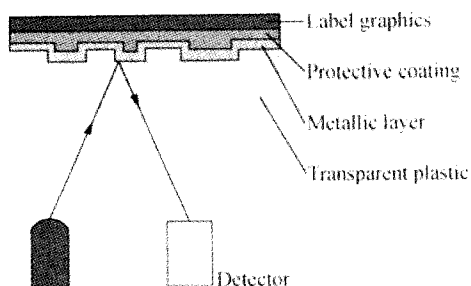


### 12.4.3.2 describe and explain the design and operation of technologies related to electromagnetic radiation

## COMPACT DISCS

Large, fast, and portable auxiliary storage is often required when using computers. One of the most important devices meeting this requirement is the compact disc, a device based on the use of laser light to retrieve stored data.

The operating principle of a compact disc or CD is the dependence of the intensity of light reflected from a surface on the local properties of the surface. The actual design depends on whether the disc is readable only or writable as well. A compact disc is typically 12 cm in diameter and made of a suitable transparent plastic. In a read only disc, the data to be recorded is physically encoded in a long, thin spiral in the form of a series of bumps within the plastic. The height and width of a bump are comparable to the wavelength of laser light used to read the disc. The undulating side of the plastic is covered with a metallic layer to reflect the laser light, with a protective coating above it.



A laser casts light onto the disc through the transparent plastic on the playable side of the disc. The intensity of light reflected off the track is detected in the CD drive. The variation of intensity due to the series of bumps is converted into digital data. During the process, a precise tracking mechanism keeps the laser beam on the spiral track.

This design is modified to produce a blank CD-R disc, on which data can be recorded by a CD-R drive only once per disc. An entire spiral groove corresponding to a track in an ordinary CD is created on the plastic. The groove is coated with a special dye and a metallic layer. To record data, the drive laser sends intense pulses of light that heat the dye on contact, and permanently alter its properties. This changes the reflection of light to an extent similar to the presence of bumps on a prewritten CD. The laser uses a lower power setting during read operations, so there is no change in the dye when data is read.

Another modification is used in CD-RW discs, which can be written and erased many times. They employ an alloy layer that can exist in two different states, each with different reflection properties. During recording, the laser sends pulses that change the state of the alloy and again stores information.

## OPTICAL FIBRES

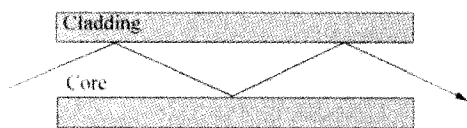
Long-distance communications have been revolutionized by the use of flexible optical fibres, which can carry light signals and at the same time can be bent to fit most paths.

Optical fibres can be used to transmit actual optical data from small probes, such as the image of an organ in a surgical procedure. As a result of their flexibility and low space requirements, they are suitable for minimally invasive medical operations. They are often used to carry light to illuminate the region where surgery is being performed.

Optical fibres can also carry electrical signals that have been converted to light. They have become especially important in this context because of the rapidly increasing need for data transfer between computers. They carry signals with much smaller energy losses and higher data transfer rates than electrical wires.



An optical fibre has a flexible solid inner tube called the core, made of a transparent, high refractive index material. An outer layer of lower refractive index material, called the cladding, encloses the core. Since the light is sent into the higher refractive index core, the lower refractive index cladding results in complete internal reflection because the angle of incidence of the light is greater than the critical angle between the two materials as shown in the given diagram. More accurately, the fibre acts as a waveguide for the light waves.



## PHOTOGRAPHIC FILTERS

A photographic filter is a transparent object through which light is made to pass before it reaches lens of the camera. They are used to produce different effects and enhancements by filtering out certain components of light.

One class of filters is based on blocking certain wavelengths while letting other wavelengths through. For example, an ultraviolet filter blocks ultraviolet light, and can reduce haziness. Filters of other colours may be used to alter the colour balance of the image.

Another class of filters produces special effects by using embedded optical elements. For example, a diffraction grating with suitable geometry can produce star patterns. A grid of thin wires can be used to soften the image to produce a diffuse effect.

Polarizing filters are used to block light with a given plane of polarization while allowing light with the perpendicular plane of polarization to pass through freely. Many polarizing filters use a set of closely spaced parallel metal wires. Such filters allow electromagnetic waves with their electric field vectors perpendicular to the wires to pass through, while blocking the parallel component. Since light reflected from transparent media like water and glass is partially polarized, these filters can eliminate reflections from water or glass. They can also make the sky appear less bright.

## Practice

### Open Response

20. X-ray astronomy is the study of celestial sources of X-rays, such as neutron stars and black holes. Explain why all astronomical X-ray detectors are located aboard satellites in orbit.

---



---



---



---



---

**12.4.3.3** *analyse, using the concepts of refraction, diffraction, and wave interference, the separation of light into colours in various phenomena, which forms the basis for the design of technological devices*

## SEPARATION OF WAVELENGTHS BY DIFFRACTION GRATINGS

The dispersion of light by an optical element like a prism, based on refraction, is well known. The effect of splitting light into its different colours can also be achieved using diffraction elements, like gratings.

The theory of diffraction applied to single slits and gratings yields results that depend on the wavelength of light used.

In particular, the angular positions of the primary maxima in the diffraction pattern caused by a grating, vary with the wavelength  $\lambda$  according to the relation

$$\sin \theta = n \frac{\lambda}{d},$$

where  $n$  numbers the order of the maxima and  $d$  is the separation between successive slits in the grating. Thus if light with a combination of different wavelengths is incident on the grating, the emerging light is split into its components in each order of diffraction depending on the wavelength.

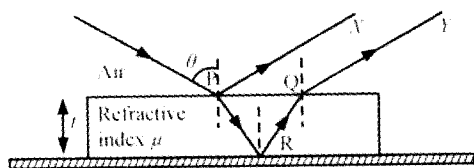


This is the principle used in the grating spectroscopy, which essentially comprises a diffraction grating and a spectrometer. The grating spectroscopy has played an important role in several major experiments in the last century. For example, it allowed the observation of the emission spectrum of hydrogen. Quantum theory implies that wavelengths or frequencies of light correspond to energy level differences in atomic systems. States with energy differences corresponding to the near visible region invariably occur at some level of organization. Thus the grating spectroscopy is a commonly used device to investigate atoms and molecules.

## SEPARATION OF WAVELENGTHS BY THIN FILMS

Thin films of optically transparent media also show the property of dispersion. This can be analyzed using the concepts of refraction and interference.

The given diagram shows a thin film of a transparent medium upon which light of a single wavelength is incident. The refractive index of the medium is  $\mu$ .



The thin film is placed on a medium of higher refractive index. When a wavefront of light is incident at a point like P, it splits because a part of the intensity is refracted while the rest is reflected along P $X$ . The refracted light travelling in the direction PR, in turn, is reflected when it strikes the base of the film. This light eventually emerges after a second refraction in the direction Q $Y$  which is parallel to P $X$ .

In this situation, it is evident that the light from P follows two different paths with different optical lengths. If light emerging from P and Q is brought to focus at a point with a lens, interference effects can be observed. The actual difference in the distance travelled is just the length of the path PRQ.

Using trigonometry, this distance is  $\frac{2t}{\sin \theta}$  if the

index of refraction of the thin film were the same as that of air. However, the optical path difference is greater than this by a factor of  $\mu$ .

$$\Delta X = \frac{2\mu t}{\sin \theta}$$

When light reflects off of a denser medium, an additional phase difference of  $\pi$  is added as well.

So the phase difference is given as

$$\pi + \Delta X \left( \frac{2\pi}{\lambda} \right) = \frac{4\mu t}{\lambda \sin \theta} + \pi$$

Constructive interference occurs if the phase difference is an even multiple of  $\pi$ .

$$\frac{4\mu t}{\lambda \sin \theta} + \pi = 2n\pi; n = 1, 2, 3, \dots$$

On the other hand, destructive interference occurs when the phase difference is an odd multiple of  $\pi$ . The conditions for constructive and destructive interference depend on the wavelength of light being passed through the thin film.

If light with many colour components is used, each wavelength or colour has a different set of angles at which it appears bright. This gives rise to an appearance of varying colours as the viewpoint is changed if the incoming light is spread along a range of angles.

### Practice

21. Light passing through a grating spectroscopy is separated into its various colours as a result of
- A. transmission      B. refraction  
C. reflection          D. diffraction



**Open Response**

22. A thin film is placed upon a material of higher refractive index. The phase difference for a beam of light incident on the thin film is  $\frac{4\mu t}{\lambda \sin \theta} + \pi$ . If the phase difference is equal to  $2n\pi$  when  $n = 1, 2, 3, \dots$ , what is the resulting interference?

---

---

---

---

---



## SOLUTIONS—THE WAVE NATURE OF LIGHT

1. C	6. 1.50	Part B- OR	14. 4,9,8,7	19. OR
2. A	7. OR	Part C- OR	15. OR	20. OR
3. C	8. A	11. B	16. A	21. D
4. OR	9. D	12. B	17. OR	22. OR
5. B	10. Part A- OR	13. 7,9,7,7	18. B	

### 1. C

Diffraction is defined as the spreading of a wave front around and past an obstacle.

### 2. A

When an electron is made to enter the target material, it loses its energy and is brought to rest. The electrons collide with the atoms of the target material before they stop. The energy that is lost may create a photon, and a stream of photons radiate as X-rays.

### 3. C

Maxwell proposed that electromagnetic radiation is produced by accelerating charged particles.

### 4. Open Response

Radio waves (low frequency EMR) are produced by oscillations of electric current. Since radio waves are used mainly in communication, many everyday devices rely on this type of EMR. Some examples are: televisions, cellphones, and radios.

### 5. B

The reason the sources are in phase is that the crests were produced simultaneously. When two lines meet, two crests are intersecting, so point T lies at a point of constructive interference. In contrast, point Q lies at a point where the midpoints between two different pairs of lines meet. This is a point of constructive interference if two troughs. Thus both points are antinodal points.

### 6. 1.50

$$\begin{aligned}
 d &= \frac{1}{6.00 \times 10^4 \text{ lines/m}} \\
 &= 1.67 \times 10^{-5} \text{ m} \\
 \lambda &= \frac{dx}{nl} \\
 &= \frac{(1.67 \times 10^{-6} \text{ m})(9.89 \times 10^{-2} \text{ m})}{(1)(1.10 \text{ m})} \\
 &= 1.50 \times 10^{-7} \text{ m}
 \end{aligned}$$

### 7. Open Response

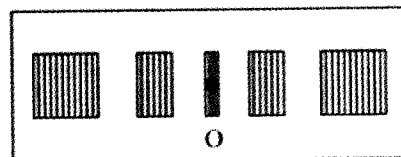
Thomas Young did his experiment in the early 1800s, long before incandescent light bulbs were invented and certainly before lasers were invented. He probably used an oil lamp and the emitted light was skewed towards the red part of the visible spectrum. By passing light through a single slit, he was ensuring the wave fronts of the dominant colour from his source would strike the slits simultaneously.  $S_1$  and  $S_2$ , according to Huygen's principle, would behave as in-phase sources of this dominant colour.

### 8. A

Each slit in a diffraction grating acts like a source of circular waves generating an interference pattern.

### 9. D

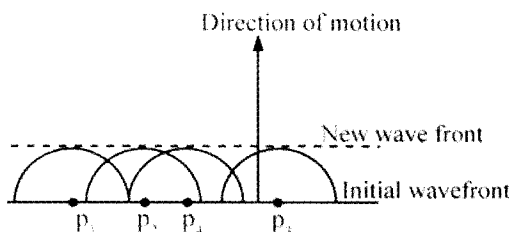
A diffraction grating consists of a number of slits with very small widths. Light coming from a point source gets diffracted by each slit of the grating in a horizontal direction. Thus, a grating spectrum is produced as shown in the given figure. If a vertical slit of a finite width is placed between the point source and the grating, then the grating diffracts each light ray coming through the slit in both the vertical and horizontal directions. Therefore, the following grating spectrum is obtained.



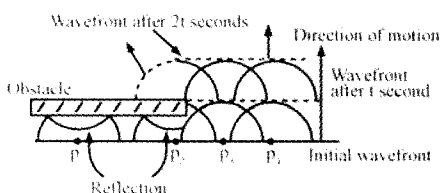


### 10. Part A – Open Response

Huygen's principle states that every point on a wave front behaves like a point source of circular wave fronts that are in phase and propagate out in front of the wave to produce a new wave front.



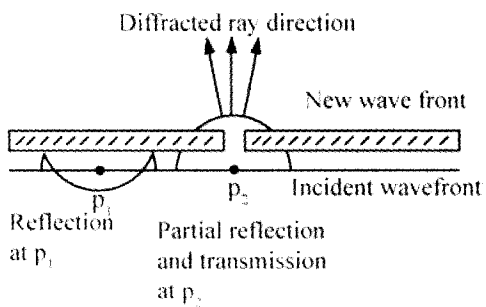
Huygen's used this idea to explain diffraction, the bending of a wave front around an obstacle.



Wave fronts from point sources at times  $t$  and  $2t$  show that energy in the wave front is transmitted behind the obstacle. This is diffraction.

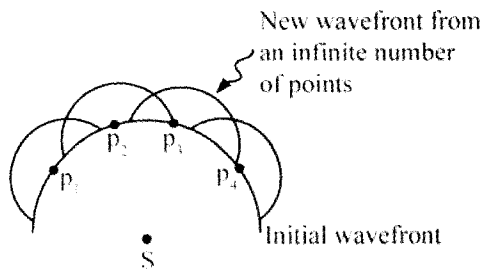
### Part B – Open Response

Diffraction by a small slit is similar to the situation when waves are diffracting around objects. Every point on the wavefront acts as an individual circular wavefront propagating out from the slit.



### Part C – Open Response

Huygen's principles easily explains the propagation of a curved wave front from a point source.



### 11. B

While transverse wave are easily polarizable, longitudinal waves are not since the motion of the wave is parallel with the direction that the wave is travelling.

Electromagnetic waves, a standing wave on a length of rope, and waves in a ripple tank are all transverse waves (i.e., the wave motion is perpendicular to the direction of the wave propagation). Only sound waves in a gas or liquid are longitudinal and therefore do not exhibit polarization.

### 12. B

In the given experiment, if the width of the slit is increased, the angular separation between consecutive minima will decrease. The condition for the minima in a single slit diffraction pattern is  $d \sin \theta = n\lambda$

where  $d$  = the width of the slit

$\theta$  = the angular spacing of the  $n$ th dark fringe

$\lambda$  = the wavelength of the light used.

For very small values of  $\theta$ ,  $\sin \theta = \theta$

$$\therefore \theta = \frac{n\lambda}{d}$$

The angular separation between  $n$ th and  $(n + 1)$ th

$$\theta_{n+1} - \theta_n = \frac{(n+1)\lambda}{d} - \frac{(n)\lambda}{d}$$

$$= \frac{n\lambda + \lambda - n\lambda}{d}$$

$$= \frac{\lambda}{d}$$

$$\therefore (\theta_{n+1} - \theta_n) \propto \frac{1}{d}$$

Therefore, the angular separation between two consecutive minima is inversely proportional to the width of the slit.

Hence, when the slit width increases in a single slit diffraction experiment, the angular separation between the consecutive minima will decrease.

### 13. 7,9,7,7

$$n\lambda = \frac{dx}{l}$$

$$\Rightarrow \lambda = \frac{dx}{nl}$$

$$= \frac{(1.50 \times 10^{-4} \text{ m})(6.22 \times 10^{-2} \text{ m})}{5(2.34 \text{ m})}$$

$$= 7.97 \times 10^{-7} \text{ m}$$

### 14. 4,9,8,7

$$n\lambda = \frac{dx}{l}$$

$$\lambda = \frac{dx}{nl} = \frac{(4.24 \times 10^{-4} \text{ m})(2.35 \times 10^{-3} \text{ m})}{2(1.00 \text{ m})}$$

$$= 4.98 \times 10^{-7} \text{ m}$$

**15. Open Response**

You must measure the distance between the two slits,  $a$ . You must measure the distance between the grating and the screen,  $d$ . You must also measure the distance between the maxima and minima observed on the screen,  $y$ .

**16. A**

While a telescope and a satellite dish both capture electromagnetic radiation from distant sources neither are directly used to determine the chemical composition of stars. A spectroscope is the device that separates the wavelengths of electromagnetic radiation that come from a star by using either prisms or diffraction gratings. By itself a prism is not used specifically for determining chemical composition of a star. It is just a component of some spectroscopes.

**17. Open Response**

Light waves can propagate through a medium, but they do not need a medium in which to propagate. Light waves can travel through a vacuum, and mechanical waves cannot. This is because mechanical waves are dependant upon the motion of the medium for propagation, while light waves are composed of perpendicular magnetic and electric fields, oscillating in phase with one another and as such, require no medium to propagate.

**18. B**

A well-known radio interferometer is the Very Large Array.

**19. Open Response**

Both electron and light microscopes have resolution and magnification limitations imposed by their wavelengths. An electron microscope has much greater resolution and magnification than a light microscope because its limiting wavelength, an electron, is much smaller than that of a photon.

**20. Open Response**

X-rays are absorbed by Earth's atmosphere. In order to capture the clearest images of these celestial bodies all X-ray detection instruments must be located above the atmosphere. This is true of both visible light telescopes, such as Hubble, and X-ray telescopes, such as the Chandra X-ray Observatory.

**21. D**

In a grating spectroscopy, a diffraction grating is used to separate the different wavelengths of light. Diffraction gratings bend light according to the wavelength of electromagnetic radiation that is passing through. Different wavelengths of light bend to a different degree resulting in a complete spectrum being observed.

**22. Open Response**

The resulting interference is constructive since the phase difference is always an even multiple of  $\pi$ . This results in the light reflected from the surface of the thin film being in phase with the light that reflected off the boundary between the thin film material and the higher index of refraction substrate.



# Unit Test



- In the electromagnetic spectrum, visible light lies between
  - ultraviolet rays and infrared rays
  - infrared rays and radio waves
  - microwaves and radio waves
  - gamma rays and X-rays
- Which of the following types of electromagnetic radiation do not involve accelerating electrons within atoms?
  - visible light
  - radio waves
  - infrared radiation
  - ultraviolet radiation

### Numerical Response

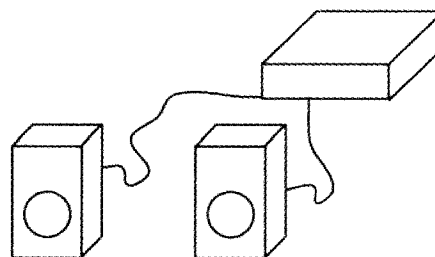
- Calculate the angle of deviation of the second-order maximum produced by directing monochromatic light ( $\lambda = 3.90 \times 10^2 \text{ nm}$ ) through a diffraction grating in which the slits are  $6.50 \times 10^{-6} \text{ m}$  apart? \_\_\_\_°
- Which of the following statements is **true**?
  - The amount that a wave diffracts is directly proportional to the size of the opening through which the wave travels.
  - The least sharp image produced by a pinhole camera will occur when the pinhole is extremely small.
  - The amount of that a wave diffracts is directly proportional to the magnitude of the wavelength.
  - Diffraction only applies to EMR with wavelengths longer than those of ultraviolet light.

### Open Response

Use the following information to answer the next question.

A group of physics students are experimenting with sound waves.

They attach an electronic sound generator to two speakers, as shown to produce two in phase sources of sound.



They are looking to find places in the room where constructive and destructive interference of the sound waves occurs.

- Draw a diagram to show where a location of constructive interference might be. Explain the reason constructive interference occurs.

---



---



---



---



---



---

### Open Response

- A diffraction grating has  $6.73 \times 10^5 \text{ lines/m}$ . How many orders of maxima can be observed if the grating is illuminated with monochromatic light of wavelength  $4.35 \times 10^{-7} \text{ m}$ ?

---



---



---



---



---



---

**Open Response**

7. Describe Poisson's spot and why it occurs.

---



---



---



---

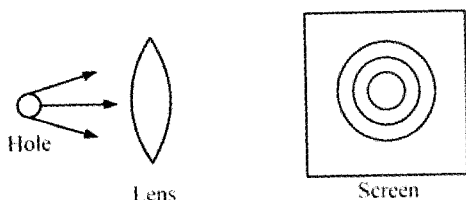


---



---

Use the following information to answer the next question.



Joseph performs an experiment to observe the diffraction of light by a circular hole with a radius of 0.02 cm, as shown in the given figure. The screen is placed 20 cm from the aperture. He observes that the radius of the first dark ring obtained on the screen  $3.6 \times 10^{-2}$  cm.

8. What is the wavelength of the light that Joseph uses in the given experiment?
- A.  $2.9 \times 10^{-3}$  cm
  - B.  $5.9 \times 10^{-5}$  cm
  - C.  $6.9 \times 10^{-4}$  cm
  - D.  $7.9 \times 10^{-9}$  cm

**Numerical Response**

9. Light with a wavelength of 565 nm passes through two narrow slits. Adjacent maxima near the centre of the interference pattern are separated by  $1.50^\circ$ . The distance between the slits is  $a.bc \times 10^{-d}$  m. Give your answer in the form *abcd*.

Use the following information to answer the next question.

A set of results is given for a classic double-slit experiment:

<i>l</i> (m)	<i>d</i> (mm)	<i>x</i> (mm)
2.0	0.4	2.4

10. The colour of light used in this experiment was
- A. blue.
  - B. green.
  - C. violet.
  - D. yellow.

**Open Response**

11. Describe the main difference between light waves and mechanical waves.

---



---



---



---

12. What is the maximum magnification of electron microscopes?
- A. 2000 times
  - B. 100 000 times
  - C. 1 million times
  - D. 2 million times

13. An endoscope is a long tube used by surgeons to look inside the human body with minimal invasiveness. Endoscopes need a light source in order to be functional, but the source is always located outside of the body. This is possible due to the principle of

- A. refraction.
- B. diffraction.
- C. illumination.
- D. total internal reflection.

**Open Response**

14. Explain how the Polarizing material in sunglasses greatly reduces the glare caused by sunlight reflecting off surrounding surfaces.

---

---

---

---

---

---

---

15. The laser used to read a compact disc in a CD drive has a wavelength of

- A. 250 nm            B. 560 nm
- C. 780 nm            D. 1000 nm



## SOLUTIONS

1. A	5. OR	9. 2165	13. D
2. B	6. OR	10. A	14. OR
3. 6.89	7. OR	11. OR	15. C
4. C	8. B	12. D	

### 1. A

The electromagnetic spectrum is the frequency range of all possible electromagnetic radiation. It is arranged in increasing order of wavelength and includes gamma rays, X-rays, ultraviolet rays, visible light, infrared rays, microwaves, radio waves, and long waves.

Frequency (Hz)		Wavelength
$10^{22}$	Gamma rays	0.1 Å
$10^{18}$		1 Å
$10^{17}$	X-rays	0.1 nm
$10^{16}$		1 nm
$10^{15}$	Ultraviolet	10 nm
$10^{14}$	Visible	100 nm
$10^{13}$		1 000 nm
$10^{12}$	Infrared	1 μm
$10^{11}$		10 μm
$10^{10}$		100 μm
$10^9$		1 000 μm
$10^{11}$	Microwaves	1 mm
$10^{10}$		1 cm
$10^9$		10 cm
$10^8$	Radio, TV	1 m
$10^7$		10 m
$10^6$		100 m
$10^5$	Longwaves	1 000 m

In the electromagnetic spectrum, visible light lies between ultraviolet and infrared rays.

### 2. B

Radio waves are produced when electrons accelerate in conductors... antennae for example.

### 3. 6.89

$$\lambda = \frac{d \sin \theta}{n}$$

$$\sin \theta = \frac{n \lambda}{d}$$

$$\sin \theta = \frac{(2)(3.90 \times 10^2 \text{ nm})}{6.50 \times 10^{-6} \text{ m}}$$

$$= 0.12$$

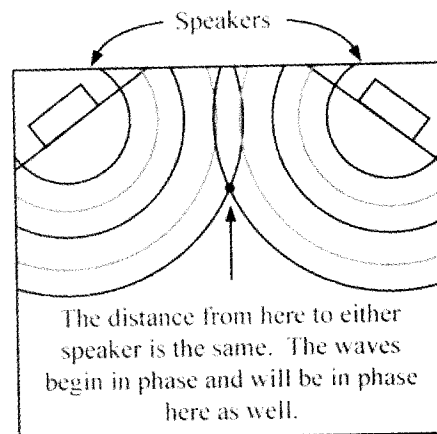
$$\theta = 6.89^\circ$$

### 4. C

$$\text{Since, } \lambda = \frac{x d}{n l} \Rightarrow x = \frac{\lambda n l}{d}$$

So,  $x \propto \frac{\lambda}{d}$ , so as  $\lambda$  increases, the diffraction distance increases.

### 5. Open Response



Any location where the distance from each of the speakers is equal to a whole number of the wavelengths of the waves is a location where constructive interference occurs.

### 6. Open Response

$$d = \frac{1}{6.73 \times 10^5 \text{ lines/m}} = 1.49 \times 10^{-6} \text{ m}$$

$$\lambda = \frac{d \sin \theta}{n}$$

$$n = \frac{d \sin \theta}{\lambda}$$

$$n = \frac{(1.49 \times 10^{-6} \text{ m}) \sin \theta}{4.35 \times 10^{-7} \text{ m}}$$

$$n = 3.43 \sin \theta$$

The maximum value of  $\sin \theta$  is 1, so the maximum value of  $n$  is 3.43. However,  $n$  can only be a whole number, so therefore the numbers of order of maxima that can be observed is 3.

### 7. Open Response

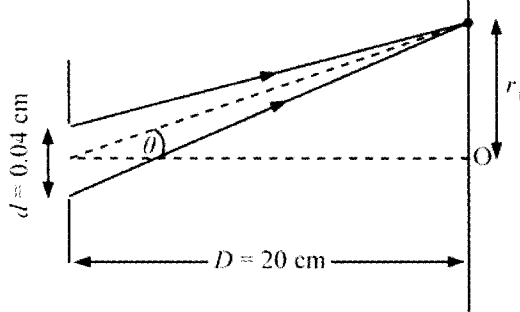
Poisson's Spot occurs when a sphere or circular disc is illuminated by monochromatic light. The centre of the shadow of the sphere/circular disc is equally distant from all points on the circumference of the sphere/circular disc, therefore the monochromatic light reaches the centre of the shadow with a path difference of zero. This is constructive interference, and is shown by a bright spot in the centre of the object's shadow.

**8. B**

In the diffraction pattern obtained from a circular aperture diffraction experiment, the angular distance ( $\theta$ ) of the first dark ring from the central axis of the circular aperture is related to the wavelength ( $\lambda$ ) of the light and the diameter ( $d$ ) of the hole as

$$\sin \theta = \frac{1.22\lambda}{d}$$

In the given situation,  $d = 2 \times 0.02 \text{ cm} = 0.04 \text{ cm}$



For a small value of  $\theta$ ,  $r_1 = D\theta$ ,  $D =$  where the distance between the aperture and the screen, and  $r_1 =$  the radius of the first dark ring.

In the given situation,  $D = 20 \text{ cm}$  and

$$r_1 = 3.6 \times 10^{-2} \text{ cm}$$

$$\therefore \theta = \frac{3.6 \times 10^{-2}}{20}$$

Again, for a small value of  $\theta$ ,  $\sin \theta \approx \theta$

$$\therefore \frac{1.22\lambda}{d} = \frac{3.6 \times 10^{-2}}{20}$$

$$\therefore \frac{1.22\lambda}{0.04} = \frac{3.6 \times 10^{-2}}{20} \left[ d = 0.04 \text{ cm} \right]$$

$$\therefore \lambda = \frac{(3.6 \times 10^{-2}) \times (0.04)}{(20) \times (1.22)}$$

$$= \frac{1.44 \times 10^{-3}}{24.4}$$

$$= 5.9 \times 10^{-5}$$

Hence, the wavelength of light used in the given experiment is approximately  $5.9 \times 10^{-5} \text{ cm}$ .

**9. 2165**

$$n\lambda = d \sin \theta$$

$$d = \frac{n\lambda}{\sin \theta}$$

$$= \frac{(1)(565 \times 10^{-9} \text{ m})}{\sin 1.50^\circ}$$

$$= 2.16 \times 10^{-5} \text{ m}$$

**10. A**

Given the formula  $\lambda = \frac{xd}{nl}$ , it is simple matter to plug in the data given and calculate the wavelength of light used. Assume  $n = 1$ .

$$\lambda = \frac{(2.4 \times 10^{-3} \text{ m})(0.4 \times 10^{-3} \text{ m})}{(1)(2.0 \text{ m})}$$

$$\lambda = 4.80 \times 10^{-7} \text{ m} = 480 \text{ nm}$$

Light with a wavelength of 480 nm is known to be blue light.

**11. Open Response**

Mechanical waves require some sort of medium in which to propagate, light waves do not. Light waves can propagate in a vacuum.

**12. D**

The maximum magnification of electron microscopes is 2 million times.

**13. D**

The light that an endoscope uses to be able to see inside of a patient's body, gets there in the same way that light signals are sent down fibre optic cables, total internal reflection. While this is a consequence of refraction, refraction alone does not explain how the light can travel long distances down fibre optic cable without losing intensity. It is the principle of total internal reflection that bounces the beam of light into the patient's body.

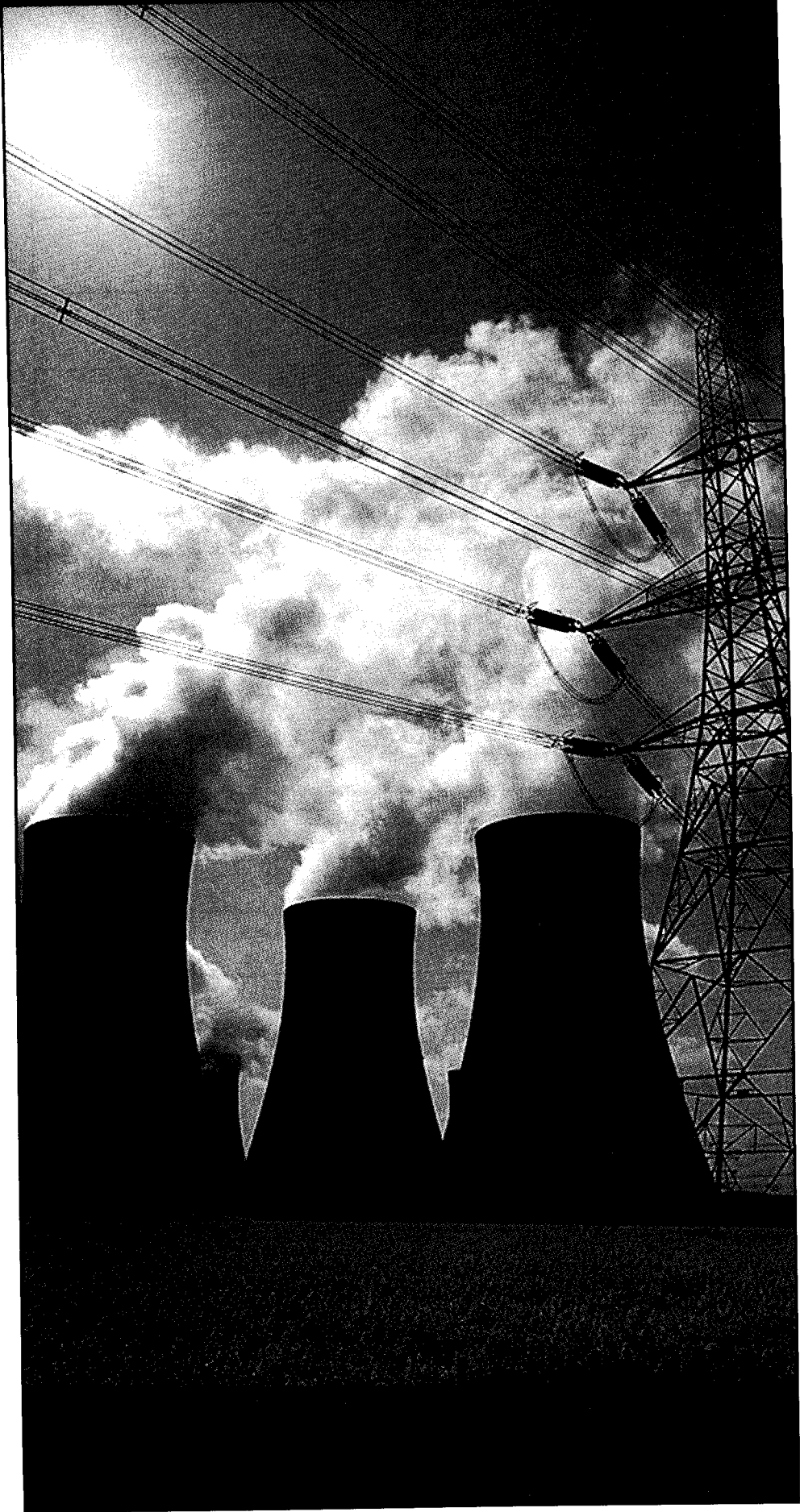
**14. Open Response**

Light bouncing off of hard surfaces has been partially polarized. The polarizing material in sunglasses only allows vertically polarized light through, while stopping partially polarized light. This allows direct light through the glasses, while blocking out light that has already been reflected from other surfaces, greatly reducing glare.

**15. C**

CD disc drives use a read laser with a wavelength of 780 nm, which is at the end of visible red light and entering the infrared region of the electromagnetic spectrum.

# Matter-Energy Interface





# Matter-Energy Interface

## Table of Correlations

<b>Specific Expectation</b>	<b>Practice Questions</b>	<b>Unit Test Questions</b>
<b>12.5.1</b> Understanding Basic Concepts		
<b>12.5.1.1</b> <i>define and describe the concepts and units related to the present-day understanding of the nature of the atom and elementary particles</i>	1	1
<b>12.5.1.2</b> <i>describe the principle forms of nuclear decay and compare the properties of alpha particles, beta particles, and gamma rays in terms of mass, charge, speed, penetrating power, and ionizing ability</i>	2, 3, 4	
<b>12.5.1.3</b> <i>describe the photoelectric effect in terms of the quantum energy concept, and outline the experimental evidence that supports a particle model of light</i>	5, 6	2, 3
<b>12.5.1.4</b> <i>describe and explain in qualitative terms the Bohr model of the (hydrogen) atom as a synthesis of classical and early quantum mechanics</i>	7, 8	4
<b>12.5.1.6</b> <i>apply quantitatively the laws of conservation of mass and energy, using Einstein's mass-energy equivalence</i>	12, 13, 14, 15	7, 8
<b>12.5.1.7</b> <i>describe the Standard Model of elementary particles in terms of the characteristic properties of quarks, leptons, and bosons, and identify the quarks that form familiar particles such as the proton and neutron</i>	16, 17	9
<b>12.5.1.5</b> <i>state Einstein's two postulates for the special theory of relativity and describe related thought experiments</i>	9, 10	5, 6
<b>12.5.3</b> Relating Science to Technology, Society and the Environment		
<b>12.5.3.3</b> <i>describe examples of Canadian contributions to modern physics</i>	27	
<b>12.5.3.2</b> <i>describe how the development of the quantum theory has led to scientific and technological advances that have benefited society</i>	26	16
<b>12.5.3.1</b> <i>outline the historical development of scientific views and models of matter and energy, from Bohr's model of the hydrogen atom to present-day theories of atomic structure</i>	24, 25	15
<b>12.5.2</b> Developing Skills of Inquiry and Communication		
<b>12.5.2.1</b> <i>collect and interpret experimental data in support of a scientific theory</i>	18, 19	10, 11
<b>12.5.2.3</b> <i>analyse images of the trajectories of elementary particles to determine the mass-versus-charge ratio</i>	20, 21	13
<b>12.5.2.4</b> <i>compile, organize, and display data related to the nature of the atom and elementary particles, using appropriate formats and treatments</i>	22, 23a, 23b, 23c	14a, 14b, 14c, 14d
<b>12.5.2.2</b> <i>conduct thought experiments as a way of developing an abstract understanding of the physical world</i>	11	12



**12.5.1.1** *define and describe the concepts and units related to the present-day understanding of the nature of the atom and elementary particles*

### DE BROGLIE'S HYPOTHESIS

A.H. Compton showed that photons had momentum, a particle property. This is represented by his formula

$$\vec{p} = \frac{h}{\lambda} = \frac{hf}{c}$$

Louis de Broglie reasoned that if a photon could have both particle and wave properties, then perhaps this dual nature extended to other particles as well. He suggested that electrons, neutrons, and even baseballs have a wave nature. By combining the formulae for photon momentum and the momentum of matter, de Broglie derived a formula that suggested particles of matter have wave properties.

De Broglie's hypothesis was verified experimentally in 1927 by C. J. Davisson and L. H. Germer. A few years earlier, W.H. Bragg predicted and empirically verified that X-rays could be diffracted by thin crystals. He showed that the regular spacing of the atoms in the crystal behaved like a multi-slit diffraction grating. Davisson and Germer showed that a beam of electrons reflect off a thin crystal in a very similar manner, producing a diffraction pattern essentially the same as the pattern observed by Bragg.

This observation was convincing evidence for de Broglie's matter waves. It also supported the wave-particle duality theory, because it showed that a particle could have wave properties, just as a wave could have particle properties.

### PHOTOELECTRIC EFFECT AND THE PHOTON MODEL

Prior to Max Planck's work on blackbody radiation, theories used to predict the behaviour of radiation could not be applied to EMR with wavelengths shorter than ultraviolet radiation (i.e., higher frequency). Planck introduced the idea that energy came in quanta, or packets, in order to explain blackbody radiation. In other words, Planck assumed that the energy of an oscillator, or a vibrating particle, could only accept or release fixed quantities of energy. He derived the following formula to explain the blackbody radiation curves obtained through experimentation.

$$E = nhf$$

where  $E$  is the energy of the oscillator,  
 $n$  is a whole number,

$h$  is Planck's constant ( $6.63 \times 10^{-34} \text{ J}\cdot\text{s}$ ),  
and  $f$  is the frequency of the emitted radiation.

Planck's value of  $n$  was later explained to represent the number of quanta, or energy packets, of light being emitted. Albert Einstein called these packets of light energy **photons**.

### EINSTEIN'S MASS-ENERGY EQUIVALENCE

Einstein derived the equation  $E = mc^2$ . This equation indicates that mass can be converted into energy. It also indicates that the conversion of very small amounts of matter will create vast amounts of energy.

In radioactive decay, nuclear fission, and nuclear fusion, the energy released is the result of the mass defect. The mass defect is the difference in the masses of the reactants and products. Since the mass defect in these subatomic reactions is always very small, the masses must be known to very precise figures (five or six decimal places) to determine the mass defect.



## NUCLEAR REACTIONS

The half-life of an element is the time it takes for a sample of the element to decay to one-half its original mass. The activity of a sample (number of decays per second) decreases at the same rate as the mass of the sample because the two are proportional to one another. The activity of a sample after a given amount of time can be determined using the

$$\text{formula } N = N_0 \left( \frac{1}{2} \right)^n$$

where  $N_0$  = activity of the original sample

$n$  = number of half-lives.

$$n = \frac{t}{\frac{t_1}{2}}$$

where  $t$  = time elapsed

$\frac{t_1}{2}$  = element's half-life

The mass of a sample can be determined by substituting the mass in to the equation for the activity of the sample ( $m$  for  $N$  and  $m_0$  for  $N_0$ ).

Half-life can be given for any unit of time, from seconds to years. However, the elapsed time must be converted to the same unit of time as the half-life.

### Practice

Use the following information to answer the next question.

Louis de Broglie first proposed that both microscopic and macroscopic materials may have characteristics of both particle and waves. A de Broglie wave is the wave movement of a particle.

1. The equation de Broglie used to describe the wavelength of a particle is

A.  $\lambda = \frac{h}{m\bar{v}}$

B.  $\lambda = hm\bar{v}$

C.  $\lambda = h + m\bar{v}$

D.  $\lambda = h + \frac{m}{\bar{v}}$

**12.5.1.2** describe the principle forms of nuclear decay and compare the properties of alpha particles, beta particles, and gamma rays in terms of mass, charge, speed, penetrating power, and ionizing ability

## NUCLEAR REACTIONS

Physicists have identified that a nucleus undergoing a transformation emits one of three types of radiation. The nucleus will emit either alpha rays, beta rays, or gamma rays.

- Alpha rays—beams of alpha particles (helium nuclei, consisting of two protons and two neutrons). These particles are represented by the symbol  ${}^4_2\alpha$ . Alpha particles can travel only very short ranges in matter. For example, a 5 MeV alpha particle (a high-energy particle) will travel only 4 cm in air, and they cannot penetrate a sheet of paper. However, an alpha particle will undergo collisions with many atoms in the short distance it travels. Although alpha particles can ionize the atoms they collide with, their low range makes them low biological hazards.
- Beta rays—beams of beta particles (high-speed electrons emitted from the nucleus). These particles are represented by the symbol  ${}^0_{-1}\beta$ . The positron is the antiparticle of the beta particle. A positron is represented by the symbol  ${}^0_{+1}\beta$ . Beta particles have ranges typically one hundred times greater than the ranges of alpha particles. Beta particles only moderately ionize the atoms they collide with, so they are low biological hazards.
- Gamma rays—beams of high-energy photons. These photons can travel great distances in air and can completely penetrate the human body. Thus gamma rays are considered high biological hazards because they can cause molecular damage. Their energies are sufficient to disrupt the chemical bonds holding atoms together.

The amount of radiation absorbed by an object is proportional to the intensity of the radiation and the length of time of exposure. The intensity of radiation is the amount of radiation emitted per unit of area. Since radiation is emitted in all directions, the area in question is the surface area of a sphere.



If the distance of the absorbing object from the source increases, the radius of the sphere also increases. The surface area also increases. Since a larger surface area would be exposed to the same amount of radiation, the intensity of the radiation is then less. Therefore, the intensity of radiation is inversely proportional to the distance from the source of radiation,  $I \propto \frac{1}{r^2}$ .

Alpha, beta, and gamma radiation are all ionizing forms of radiation. They will turn atoms in the human body into ions, making normally unreactive substances in the body reactive. Even atoms in DNA can become reactive, which can cause genetic mutations.

Symptoms caused by the exposure to radiation are collectively called radiation sickness. In the short term these symptoms may include loss of appetite, nausea, and vomiting. Long term symptoms may include a weakening of the immune system (more severe with greater exposure), cataracts, and serious gastric disorders. Severe radiation sickness usually results in death within three weeks.

### Practice

2. Which of the following types of radiation is a form of electromagnetic radiation produced during nuclear decay?
- A. X-rays                      B. Beta rays  
C. Alpha rays                D. Gamma rays

### Numerical Response

Use the following information to answer the next question.

Three forms of radiation are produced during nuclear decay:

1. Alpha rays
  2. Beta rays
  3. Gamma rays
3. Place the given forms of radiation in order of the greatest biological risk to least biological risk. \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

### Open Response

4. Describe the difference between alpha rays,  $\alpha$ , and gamma rays,  $\gamma$ . Be sure to include any difference in mass and the methods needed to stop both types of radiation.

---



---



---



---



---



---

**12.5.1.3** describe the photoelectric effect in terms of the quantum energy concept, and outline the experimental evidence that supports a particle model of light

## THE PHOTOELECTRIC EFFECT

The photoelectric effect occurs when a metal surface in a photoelectric cell is exposed to light. When light illuminates the surface, a current begins to flow in the circuit connected to the photoelectric cell. Observations made with the photoelectric effect created a problem for classical physicists, requiring new explanations to understand what happens.

The energy of the photoelectron increases with an increase in the frequency (or decrease in wavelength) of the incident light. An increase in the intensity of light (the number of photons) causes an increase in the photoelectric current. The material used in the photoelectric cell determines how tightly bound the electrons are to the atoms. This characteristic determines how much energy must be absorbed by the surface to eject these electrons out of the surface atoms. Therefore, the threshold frequency,  $f_0$ , necessary to eject photoelectrons will be different for different photoelectric surfaces.

**Practice**

5. A photocell is an electronic device with an electrical output that varies according to the radiation that shines upon the cell. Which of the following statements about photocells is **true**?
- They are based on the photoelectric effect.
  - They are based on the electrochemical effect.
  - They convert mechanical energy to solar energy.
  - They convert solar energy to mechanical energy.

**Open Response**

6. The model of the atom has been modified and experimentally tested for well over 100 years. Many scientists have contributed to the ever-changing understanding of the atomic model. In a brief paragraph, identify and explain Max Planck's and Albert Einstein's major contributions to the development of the quantum model of the atom, using the given equation and terms in your explanation.

- $E = h \times \nu$
- photoelectric effect
- quantum or quanta
- photon

---



---



---



---



---

**12.5.1.4** describe and explain in qualitative terms the Bohr model of the (hydrogen) atom as a synthesis of classical and early quantum mechanics

**ELECTRON ENERGY STATES**

The phenomena of line emission and absorption spectra suggest that electrons can only orbit nuclei at certain energy levels. The Balmer equation predicts the wavelength of a photon emitted when an electron moves to a lower energy level from a higher energy level. The equation can also predict the wavelength of a photon needed to cause an electron to jump up to a higher energy level from a lower energy level. The following equation is the Balmer equation for a hydrogen atom.

$$\frac{1}{\lambda} = R_{\text{H}} \left( \frac{1}{n_{\text{l}}^2} - \frac{1}{n_{\text{u}}^2} \right)$$

where  $\lambda$  = wavelength of the photon emitted or absorbed

$R_{\text{H}}$  = Rydberg constant for

hydrogen =  $1.10 \times 10^7 \text{ m}$  (only for the hydrogen spectra)

$n_{\text{l}}$  = lower energy level of the electron

$n_{\text{u}}$  = upper energy level of the electron.

The values of  $n_{\text{l}}$  and  $n_{\text{u}}$  can only have whole number values.

Neils Bohr fixed the planetary model of the atom proposed by Ernest Rutherford by stating that the electrons exist in fixed energy states and only emit photons when they jump down energy states. The difference in energy between states will become the energy of the emitted photon.

**Practice**

7. Which of the following scientists first explained the structure of the atom using quantum theory?
- Neils Bohr
  - Albert Einstein
  - Amedeo Avogadro
  - Werner Heisenberg



**Open Response**

*Use the following information to answer the next question.*

The given list shows the first five energy levels for the electron in the hydrogen atom.

- $E_1 = -13.60 \text{ eV}$
- $E_2 = -3.40 \text{ eV}$
- $E_3 = -1.51 \text{ eV}$
- $E_4 = -0.85 \text{ eV}$
- $E_5 = -0.54 \text{ eV}$

8. If a hydrogen atom emitted a photon with a wavelength of 435 nm, what was the change in the energy levels of the electron that released this photon?

---



---



---



---



---



---

**12.5.1.5** *state Einstein’s two postulates for the special theory of relativity and describe related thought experiments*

**12.5.2.2** *conduct thought experiments as a way of developing an abstract understanding of the physical world*

**SPECIAL THEORY OF RELATIVITY**

Albert Einstein initially proposed the special theory of relativity in his 1905 paper “On the Electrodynamics of Moving Bodies.” In this paper, Einstein attempted to explain the failure of various classical experiments and theories to explain the propagation of light, such as the Michelson–Morley experiment in 1887.

The Michelson–Morley experiment attempted to measure the velocity of light relative to the motion of Earth through the “luminiferous ether.” This ether was a theoretical substance between Earth and the sun. The null result of this experiment could not be explained until Einstein proposed special relativity. The key to Einstein’s explanation was that light did not move with a relative velocity. Instead, he suggested that its speed in a vacuum was always  $c$ , regardless of the inertial reference frame from which it was measured.

**EINSTEIN’S POSTULATES FOR THE SPECIAL THEORY OF RELATIVITY**

1. The laws of physics are identical in all inertial reference frames. (The principle of relativity)
2. The speed of light in a complete vacuum is identical in all inertial reference frames. (Invariability of  $c$ )

**THOUGHT EXPERIMENTS**

A thought experiment is a hypothetical situation that can be considered but cannot be quantitatively measured do to the immeasurable variables involved. Einstein’s special theory of relativity is explained using thought experiments. The time dilation thought experiment is one of these experiments. It considers the variability of time in relation to the speed of light.

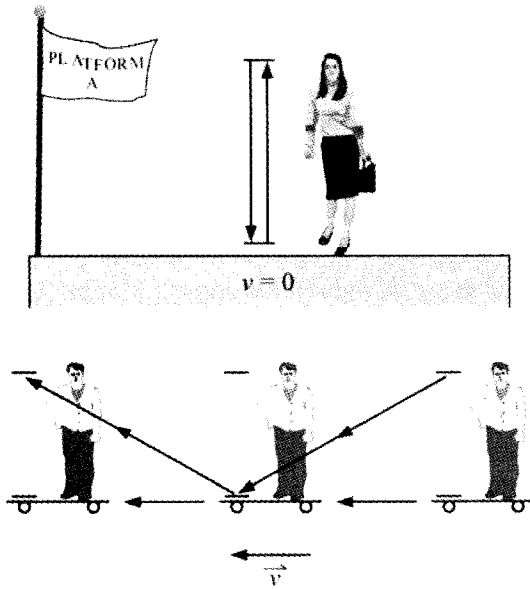
**THE TIME DILATION THOUGHT EXPERIMENT**

Stacey stands on railway platform A. She has both a standard wristwatch and a light clock with her on the platform. Consider the light clock to be a timing device that measures each tick of the clock by ticking each time a beam of light travels from an upper mirror to a bottom mirror and back again. Both the wristwatch and the light clock are perfectly synchronized.

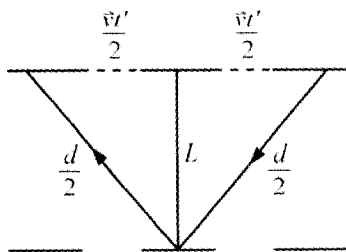
At the same time, Brian stands on a moving train car. Brian also has both a wristwatch and a light clock on the train with him, and the watch is synchronized with the light clock. If the train travels to the left with a velocity of  $\vec{v}$ , what can be noticed about the passage of time as measured by Stacey and by Brian?



**Note:** The purpose of the wristwatches that Stacey and Brian are wearing is to show that in their own frames of reference time is passing at its normal rate. They would not experience any difference in the passage of time. It is only when the passage of time in one reference frame is compared to the passage of time in another, that any useful conclusions can be made.



- Let  $L$  = distance from the top mirror to the bottom mirror in the light clock
- $\bar{v}$  = Brian's velocity while riding on the train
- $t$  = time between ticks on Stacey's watch
- $t'$  = time between ticks on Brian's watch, as observed by Stacey
- $d$  = total distance that the light in the light clock has travelled
- $c$  = speed of light



There are two methods that can be used to determine the distance the light travelled in Brian's light clock. First, calculate the distance travelled with the distance formula  
 $d = ct'$

Second, calculate the distance that the light travelled using the Pythagorean theorem.

$$d = 2\sqrt{L^2 + \left(\frac{\bar{v}t'}{2}\right)^2} = 2\sqrt{L^2 + \frac{\bar{v}^2(t')^2}{4}}$$

$$d = 2\sqrt{\frac{4L^2}{4} + \frac{\bar{v}^2(t')^2}{4}} = 2\frac{\sqrt{4L^2 + \bar{v}^2(t')^2}}{\sqrt{4}}$$

$$\therefore d = \sqrt{4L^2 + \bar{v}^2(t')^2}$$

Since both formulae express the value  $d$ , they can be set as being equal.

$$ct' = \sqrt{4L^2 + \bar{v}^2(t')^2}$$

$$\Rightarrow c^2 t'^2 = 4L^2 + \bar{v}^2 t'^2$$

$$c^2 t'^2 - \bar{v}^2 t'^2 = 4L^2$$

$$t'^2(c^2 - \bar{v}^2) = 4L^2$$

$$t'^2 = \frac{4L^2}{c^2 - \bar{v}^2}$$

$$t'^2 = \frac{4L^2}{c^2 \left(1 - \frac{\bar{v}^2}{c^2}\right)}$$

$$\therefore t' = \frac{2L}{c\sqrt{1 - \frac{\bar{v}^2}{c^2}}}$$

However, the time duration for the light travelling in either light clock as measured by the observer that is at rest with that clock, is given by

$$t = \frac{2L}{c}$$

Therefore,

$$t' = \frac{t}{\sqrt{1 - \frac{\bar{v}^2}{c^2}}}$$

This is the formula for time dilation. It can also be written by defining

$$\gamma = \frac{1}{\sqrt{1 - \frac{\bar{v}^2}{c^2}}}$$

Therefore, the time dilation formula becomes

$$t' = \gamma t$$



As measured by Brian, time would not slow down or speed up. Time will pass exactly as he would measure it in any inertial frame of reference. However, the time that Brian measured on the train would appear slower when observed by Stacey. This is because the train is moving relative to Stacey's inertial frame of reference. This is known as time dilation because the time that Stacey measured from the platform is dilated, or is longer than the time that Brian measured. Time slowed down for Brian with respect to Stacey.

**Practice**

Use the following information to answer the next question.

An observer within a frame of reference measures the time interval between two events in the same frame to be  $\Delta t$ . A second observer in a frame of reference moving with a velocity  $\vec{v}$  with respect to the first observer's frame of reference, observes the time interval to be  $\Delta t'$ .

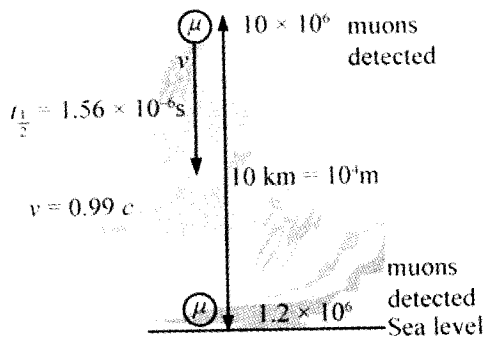
9. Which of the following expressions describes the relationship between  $\Delta t$  and  $\Delta t'$ ?

- A.  $\Delta t = \Delta t'$
- B.  $\Delta t < \Delta t'$
- C.  $\Delta t > \Delta t'$
- D.  $\Delta t = 2\Delta t'$

**Open Response**

Use the following information to answer the next question.

The muon experiment was performed to verify Einstein's special theory of relativity.



In this experiment, scientists measured that there were 10 million muons present in the atmosphere at a height of  $10^4$  m above sea level. Then they measured that there were 1.2 million muons at sea level. To predict the number of muons that should have been measured at sea level, the scientists used the following formula.

$$\frac{N}{N_0} = (2)^{-\frac{t}{t_{1/2}}}$$

where

$N$  = Number of muon at sea level

$N_0$  = Number of muons at  $10^4$  m above sea level =  $10^7$  muons

$t_{1/2}$  = half-life of muon =  $1.56 \times 10^{-6}$  s

$t$  = time duration of muons travelling from  $10^4$  m above sea level to sea level

The velocity of the muons was measured to be  $0.99c$ .



10. Prove the validity of Einstein's special theory of relativity using the concept of length contraction and the given information.

---



---



---



---



---



---

11. The Milky Way galaxy has a radius of about  $3 \times 10^{20}$  m. In order for an object to travel to the centre of the galaxy from the perimeter in twenty years, the object must travel with a speed of
- A.  $0.58c$                       B.  $0.7949c$   
 C.  $0.875\ 247c$                 D.  $0.999\ 9996c$

**12.5.1.6** *apply quantitatively the laws of conservation of mass and energy, using Einstein's mass-energy equivalence*

## EINSTEIN'S MASS-ENERGY EQUIVALENCE

In 1905, Albert Einstein also proposed his now famous mass-energy equivalence equation,  $E = mc^2$ . This equation indicates that mass and energy are interchangeable quantities and that a very small amount of mass can convert into a large amount of energy. For example, if a single gram of matter, about the mass of a nickel, were converted entirely into energy, it would produce the same amount of energy as approximately 21 kilotons of exploding TNT.

## THE MASS DEFECT

The conservation of mass and energy is a universal theory to which even nuclear reactions, such as fission, fusion, and radioactive decay, must conform. However, in each type of nuclear reaction there is a very small difference between the mass of the reactants involved and the products produced. This difference in mass is called the mass defect.

The mass defect is a result of some of the mass involved in the reaction converting into some form of energy. Since the mass defect is extremely small, it is necessary to know the masses of the reactants and the products very precisely. For example, to calculate the energy released by a reaction using Einstein's mass-energy equivalence equation  $E = mc^2$ , the masses of the reactants and products need to be measured to approximately five or six decimal places.

## Practice

12. In a fission reaction, the mass of the reactants is 236.05 atomic mass units and the mass of the products is 235.86 atomic mass units. Which of the following statements **best** describes the discrepancy in the masses that occurred in this nuclear fission reaction?
- A. Mass has been converted into energy in this reaction.  
 B. Energy has been converted into mass in this reaction.  
 C. The discrepancy in the mass was caused by inaccurate measuring equipment.  
 D. The mass discrepancy is the result of the emission of undetectable neutrino particles produced during the fission reaction.



### Numerical Response

Use the following information to answer the next question.

The given measurements indicate that the uranium-235 nucleus has a mass smaller than the mass of the number of free protons and neutrons that compose the nucleus. This difference in mass is called the mass defect.

Einstein's mass-energy equivalence equation can be used to predict the energy that binds a nucleus together from the mass defect.

$$m_{\text{U-235 nucleus}} = 3.9021 \times 10^{-25} \text{ kg}$$

$$m_{\text{proton}} = 1.6726 \times 10^{-27} \text{ kg}$$

$$m_{\text{neutron}} = 1.6749 \times 10^{-27} \text{ kg}$$

13. The mass defect of uranium-235, expressed in scientific notation, is  $b \times 10^{-x}$  kg.

What is the value of  $b$ ? \_\_\_\_\_. (Record your answer to **three** digits.)

### Numerical Response

14. Given that the mass of a proton is 1.007 28 u, the mass of a neutron is 1.008 67 u, and the mass of the carbon-13 nucleus is 13.003 35 u, the binding energy of a carbon-13 nucleus is  $a.bc \times 10^{-de}$  J. The values for  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  are \_\_\_\_\_ (Give your answer to **five** digits.)

### Numerical Response

15. Two deuterium nuclei (mass of each nucleus is 2.012 10 u) collide to form a tritium nucleus (3.016 03 u) and a neutron. Assuming the tritium nucleus has essentially zero kinetic energy after the fusion of the deuterium nuclei, the speed of the ejected neutron is  $a.bc \times 10^d$  m/s. The values of  $a$ ,  $b$ ,  $c$ , and  $d$  are \_\_\_\_, \_\_\_\_, \_\_\_\_, and \_\_\_\_\_. (Give your answer to four digits.)

**12.5.1.7** describe the Standard Model of elementary particles in terms of the characteristic properties of quarks, leptons, and bosons, and identify the quarks that form familiar particles such as the proton and neutron

## THE STANDARD MODEL OF ELEMENTARY PARTICLES

The standard model of particle physics is the commonly accepted theory of all of the known physical forces, except gravitation, and the elementary particles involved in these interactions. These elementary particles compose all of the known matter in the universe and can be separated into either fermions or bosons. The standard model is a quantum theory that includes the theory of strong interactions (quantum chromodynamics) and the unified theory of weak and electromagnetic interactions (electroweak force).

Fermions can be broken down to include leptons and quarks. Leptons include electrons, muons, and tau leptons, as well as their related neutrinos (electron neutrino, muon neutrino, and tau neutrino).

## QUARKS

Scientists have found that protons and neutrons belong to a family of subatomic particles known as hadrons. Hadrons are particles that interact with one another with the strong nuclear force. All hadrons are composed of some combination of the six different types of quarks. The six types of quarks are: up, down, strange, charm, bottom, and top.



The neutron and proton are composed of three quarks each. The difference between the two particles is that the neutron consists of two down quarks and one up quark, while the proton is composed of two up quarks and one down quark.

## BOSONS

Bosons are considered the force carriers of the Standard Model, obey Bose-Einstein statistics, and have an integer spin. Bosons come in two forms, either elementary or composite. The following particles are examples of elementary bosons:

- Photons, which act as force carriers for the electromagnetic field
- Gluons, which act as the force carriers for the strong nuclear force
- Higgs boson, which give mass to matter through the Higgs mechanism
- W and Z bosons, which act as the force carriers for the weak nuclear force
- Gravitons, which are theoretical bosons proposed to act as the force carriers for gravity

Composite bosons are particles that contain an even number of fermions. The following particles are examples of composite bosons:

- Mesons
- Helium-4 atom
- Nucleus of a carbon-12 atom

## SUBATOMIC PARTICLES AND ANTIPARTICLES

Both the up and down quarks have fractional charges. An up quark has a fractional charge of  $+\frac{2}{3}$ , and a down quark has a fractional charge of  $-\frac{1}{3}$ .

Therefore, the neutron (down-down-up, or ddu) has a charge of  $2\left(-\frac{1}{3}\right) + \left(+\frac{2}{3}\right) = 0$  and a mass of  $938.23 \text{ MeV}/c^2$ . In contrast, the proton (uud) has a charge of  $2\left(+\frac{2}{3}\right) + \left(-\frac{1}{3}\right) = +1$  and a mass of  $939.52 \text{ MeV}/c^2$ .

## LEPTONS

Leptons are the family of subatomic particles that do not bond with the strong nuclear force. Similar to the hadron particles, there are six types of leptons. The types of leptons are: electrons, electron neutrinos, muons, muon neutrinos, taus, and tau neutrinos. Each lepton particle has an antiparticle.

Recall that the electron has a mass of  $0.511 \text{ MeV}/c^2$  and a charge of  $-1$ . The antiparticle of the electron is the positron.

Neutrinos travel close to the speed of light, have no electric charge, and are able to pass through ordinary matter almost undisturbed. They are extremely difficult to detect, and the consensus is that neutrinos have an extremely tiny mass. The antiparticle of the neutrino is the antineutrino.

### Practice

16. The subatomic particles that make up protons and neutrons are called
- |              |                   |
|--------------|-------------------|
| A. quarks    | B. gluons         |
| C. positrons | D. beta particles |

### Open Response

17. What are the types of quarks that form a proton, and what is the value of the charge of each?

---



---



---



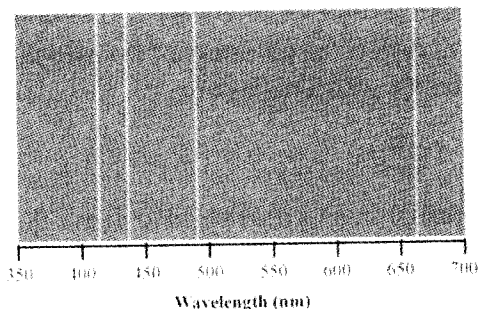
---



### 12.5.2.1 collect and interpret experimental data in support of a scientific theory

## THE HYDROGEN SPECTRUM

A hydrogen discharge tube produces an emission spectrum of light. A number of separate lines in the visible region of the EMR spectrum appear when the hydrogen emission spectrum is observed with a spectroscope. This spectrum of only a few lines of light contrasts the continuous spectrum produced by diffracted white light.



The hydrogen emission spectrum

## THE BOHR MODEL EXPLANATION

Bohr's model attempted to explain how the separate lines are produced. He proposed that the electron in a hydrogen atom could only have certain specified energy states. Then radiation would be emitted from the atom if the electron fell from a higher energy state to a lower one. The emission spectrum would then show separate lines corresponding to the energy difference between pairs of energy states.

The theoretically predicted energy levels are given by the equation

$$E_n = \frac{-E_0}{n^2}, \text{ when } n = 1, 2, 3, \dots$$

and  $E_0$  = base energy level of

$$\text{hydrogen} = -2.18 \times 10^{-18} \text{ J} = -13.6 \text{ eV}$$

The allowed values of the wavelengths of the photons emitted or absorbed by the hydrogen atom can be found using the Rhydberg formula:

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_u^2} - \frac{1}{n_l^2} \right)$$

where,

$\lambda$  = wavelength of the emitted or absorbed photon

$R_H$  = Rydberg constant for hydrogen

$$= 1.10 \times 10^7 \text{ m}^{-1}$$

$n_l$  = lower energy level of the electron in hydrogen

$n_u$  = upper energy level of the electron in hydrogen

If  $n_l$  is set to 2, the predicted wavelengths fall in the visible region, with the first four values being 414 nm, 438 nm, 490 nm, and 662 nm. This agrees with the visible spectrum obtained experimentally. Similar results also occur in the infrared and ultraviolet regions of hydrogens emission spectrum.

### Practice

Use the following information to answer the next question.

An atom is composed of a central nucleus with a varying number of electrons orbiting it. The electrons are free to jump between energy levels by gaining or losing energy. In the simplest neutral hydrogen atom, the energy of an electron occupying the first energy level is  $-13.6 \text{ eV}$ , and the energy of an electron occupying the second energy level is  $-3.4 \text{ eV}$ .

18. In a hydrogen atom, the energy required for an electron to jump from the first energy level to the second energy level is
- $-17.6 \text{ eV}$
  - $-3.4 \text{ eV}$
  - $10.2 \text{ eV}$
  - $13.6 \text{ eV}$

**Numerical Response**

Use the following information to answer the next question.

A physicist carried out an experiment in which electrons with increasing levels of energy bombarded a gaseous sample of a metallic element. The physicist then recorded the two given observations.

1. Electrons with less than 1.55 eV of kinetic energy collided elastically with the atoms of the gas.
2. Electrons with a kinetic energy of exactly 1.92 eV scattered with a kinetic energy of 0.36 eV.

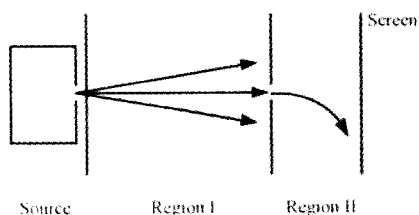
19. What was the energy of the stationary state for the atoms in the gaseous sample?  
 \_\_\_ eV. (Give your answer to **three** digits)

**12.5.2.3** analyse images of the trajectories of elementary particles to determine the mass-versus-charge ratio

## DETERMINING THE MASS-TO-CHARGE RATIO

The mass-to-charge ratios of charged elementary particles can be determined by observing their motion in electric and magnetic fields.

In a typical mass-to-charge experiment, the velocity of the particles is determined, and then the curvature of the particles' paths in the magnetic field are measured.



In region I of the given diagram, the incoming particles are subject to both an electric field and a magnetic field. The fields are perpendicular to the path between the two slits and to each other. For those particles that pass undeflected, the electric force cancels the magnetic force.

$$\vec{E}q = q\vec{v}(B_{\perp})_I \Rightarrow \vec{v} = \frac{\vec{E}}{(B_{\perp})_I}$$

The particles that manage to enter into region II are known to have this speed.

In region II, only a perpendicular magnetic field of strength  $(B_{\perp})_{II}$  acts on the particles. The magnetic field will deflect the trajectory of the particles into a circular arc with a radius of  $r$ .

$$\frac{m\vec{v}^2}{r} = q\vec{v}(B_{\perp})_{II}$$

This equation can be rearranged to derive the mass-to-charge ratio.

$$\frac{m}{q} = \frac{r(B_{\perp})_{II}}{\vec{v}} = \frac{r(B_{\perp})_{II}}{\frac{\vec{E}}{(B_{\perp})_I}} = \frac{r(B_{\perp})_{II}(B_{\perp})_I}{\vec{E}}$$

Although the circular trajectory can be difficult to see, it can be easily determined from the point on the screen that the stream of particles strikes.

## CLOUD CHAMBER

A cloud chamber is a device used to make the trajectory of a charged particle visible.

A supersaturated gas is saturated in a chamber, through which the particles pass. Tracks in the cloud chamber can then be photographed and analysed using the given calculations.



## Practice

Use the following information to answer the next question.

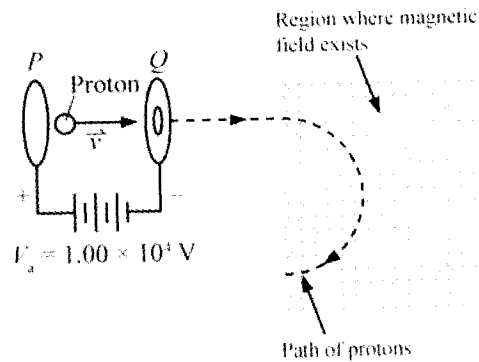
A proton with 894 eV of energy, travelling perpendicularly to a magnetic field, moves in a circular path with a radius of  $3.60 \times 10^{-4}$  m.

20. If an alpha particle were to enter into the magnetic field with the same speed as the proton, what will be the radius of the alpha particle?
- A.  $1.80 \times 10^{-4}$  m
- B.  $3.60 \times 10^{-4}$  m
- C.  $5.40 \times 10^{-4}$  m
- D.  $7.20 \times 10^{-4}$  m

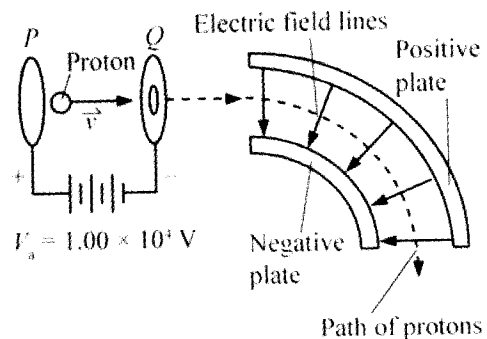
## Numerical Response

Use the following information to answer the next question.

A mass spectrometer uses either a magnetic field or an electric field to deflect charged particles.



*Magnetic field mass spectrometer*



*Electric field mass spectrometer*

21. In the magnetic field mass spectrometer shown, If the radius of curvature of a proton's path in the given magnetic field mass spectrometer is 3.00 m, the magnetic field intensity, expressed in scientific notation, is  $a.bc \times 10^{-d}$  T. What are the values of  $a$ ,  $b$ ,  $c$ , and  $d$  in the form  $abcd$ ?

(Record your answer to **four** digits.)



**12.5.2.4** *compile, organize, and display data related to the nature of the atom and elementary particles, using appropriate formats and treatments*

## DETERMINING HALF-LIVES

Radioactive decay is the emission of either alpha or beta particles and gamma radiation.

These emissions can be detected, and the detected occurrences can be counted using a device, such as a scintillation counter.

Over a short interval of time the count is proportional to the activity of the sample and the number of atoms of the active species. By observing how the count varies over a period of time comparable to the species half-life,  $t_{\frac{1}{2}}$ , and by using

the radioactive decay equation, the half-life of the radioactive species can be measured.

The radioactive decay equation for a situation with a single kind of activity is

$$N = N_0 \left( \frac{1}{2} \right)^n$$

$$\text{where } n = \frac{t}{t_{\frac{1}{2}}}$$

This expression can be rearranged using natural logarithms:

$$\ln N = \ln N_0 - \left( \frac{\ln 2}{t_{\frac{1}{2}}} \right) t$$

If the logarithm of the activity is plotted against time, the result will be a straight line with a slope of

$$-\left( \frac{\ln 2}{t_{\frac{1}{2}}} \right). \text{ This method is suitable for half-lives of}$$

the order of several minutes to several days.

The following information is the half-life data of various radioisotopes used in medicine and their uses.

### STRONTIUM-89

- half-life = 50.5 days
- typical observation time = 20 days
- fraction of original activity after typical observation time = 0.760
- used in the treatment of bone pain due to prostate cancer

### IRIDIUM-192

- half-life = 73.8 days
- typical observation time = 25 days
- fraction of original activity after typical observation time = 0.791
- used as the gamma radiation source in brachytherapy for cancer treatment

### TECHNETIUM-99M (METASTABLE FORM OF TECHNETIUM-99)

- half-life = 6.01 hours
- typical observation time = 5.0 hours
- fraction of original activity after typical observation time = 0.562
- used as a radioactive tracer in medical tests

### IODINE-123

- half-life = 13.2 hours
- typical observation time = 10.0 hours
- fraction of original activity after typical observation time = 0.591
- used as a radioactive tracer in imaging

### IODINE-131

- half-life = 8.02 days
- typical observation time = 5.0 days
- fraction of original activity after typical observation time = 0.649
- used as radiation therapy for cancer and thyroid diseases

### Practice

22. A given radioactive isotope has a half-life of 10 years and an active mass of 12 g. How much mass of this isotope will remain after 20 years?
- A. 0.6 g                      B. 1.2 g  
C. 3.0 g                      D. 6.0 g



Use the following information to answer the next multipart question.

23. Iodine-131 is a radioactive element used in the medical diagnosis and treatment of thyroid problems. Iodine-131 undergoes simultaneous beta and gamma decay and has a half-life of 8.00 days.

Part A

**Open Response**

Write the complete decay equation for iodine-131.

---

---

---

---

---

---

---

Part B

**Open Response**

Complete the table below by entering the amount of iodine-131 remaining over 40 days. Explain how you obtained the data for the table.

<b>Time (days)</b>	0	8.00	16.0	24.0	32.0	40.0
<b>Mass (g)</b>	2.00					

---

---

---

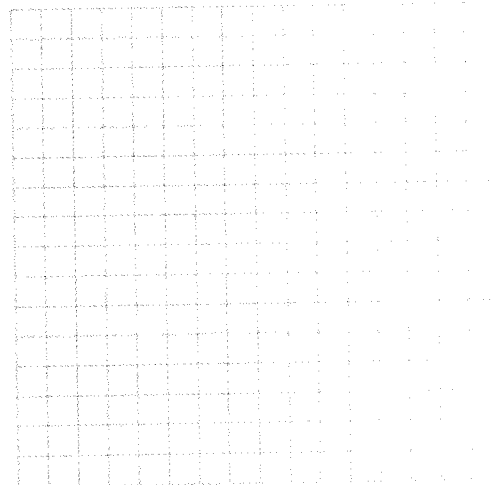
---

---

Part C

**Open Response**

Provide a graph to show the data for the theoretical decay of 2.0 g of iodine-131.



*12.5.3.1 outline the historical development of scientific views and models of matter and energy, from Bohr's model of the hydrogen atom to present-day theories of atomic structure*

**THE DEVELOPMENT OF THE STANDARD MODEL**

Bohr's model of the hydrogen atoms marked an important step in the understanding of atomic particles. Building on the photon concept introduced by Einstein to explain the photoelectric effect, the Bohr model explained the emission spectrum of hydrogen in terms of the atom's internal energy states. While the Bohr model contained radically new ideas, the scope of its application was rather limited, and it fell short of being a general physical theory.



## DE BROGLIE'S HYPOTHESIS

The Bohr model was expanded by Louis de Broglie in 1924 when de Broglie suggested that material particles have a dual nature and can behave like waves. De Broglie's hypothesis was inspired by the photoelectric effect, in which light, which was otherwise known to behave as a wave, seemed to possess particle qualities. He proposed that the quantitative relation between the wave and the particle aspects of light could be applied to ordinary matter as well. Thus any particle, including an electron within an atom, would be associated with its own matter wave.

Experimental confirmation of de Broglie's hypothesis followed only three years later. Davisson, Germer, and Thomson independently observed that electrons could be diffracted by crystals. As diffraction is a characteristic of waves, the experiments established the wave-particle duality of electrons. These experiments also confirmed de Broglie's equations.

## QUANTUM MECHANICS

In 1924, Heisenberg, Jordan, and Born used the mathematical concept of matrices to express the transitions of electrons from one orbit to another. This may be considered the origin of quantum mechanics, the physical theory that generalizes the mechanics of atomic scale and smaller objects. Heisenberg originally focused on the physically observable quantities to describe the hydrogen atom. He realized that the orbits of electrons could no longer be thought of as being sharply defined.

Schrödinger independently pursued de Broglie's ideas to devise a wave equation for material particles. In particular, he successfully found an equation that predicted the energy levels of the hydrogen atom. This equation explained the experimentally observed emission spectrum of hydrogen. Schrödinger's work was initially favoured while Heisenberg's matrix mechanics met with some opposition. However, Dirac soon showed that both theories represented the same mathematical scheme.

Schrödinger's equation did not agree with the theory of relativity. However, Dirac was able to express it as a relativistic equation for the electron.

Remarkably, when interpreted correctly, Dirac's equation predicted the existence of a particle with the same mass as an electron but the opposite charge. This particle was called the positron, and it was discovered in 1932 by Carl Anderson. Subsequently, it became obvious that each elementary particle has a corresponding antiparticle. The antiparticles of the proton and the neutron were discovered twenty years later.

Another related idea that developed in parallel was that the electron had an intrinsic spin. This could explain the results of an experiment by Stern and Gerlach that aimed to determine whether particles had intrinsic angular momentum. Theoretical studies of spin in quantum mechanics were initiated by Uhlenbeck and Goudsmit. The Dirac equation included spin in a natural way, and showed it to be of relativistic origin.

## QUANTUM ELECTRODYNAMICS

Though Schrödinger's equation predicts the emission spectrum of hydrogen, it does not explain the reason electrons change states by emitting radiation. This requires a quantum theory of the electromagnetic field, or quantum electrodynamics (QED).

QED was developed by Feynman, Dyson, Schwinger, and Tomonaga, giving what is perhaps the most accurately verified physical theory to date. QED is based on quantum field theory, which extends quantum mechanical ideas to objects like electromagnetic fields. One key idea in QED is that particles are excited states of such fields. Another essential idea is that the forces of interaction between particles are themselves carried by other particles. In QED, the interaction between electrons and positrons are due to the exchange of photons.

## QUANTUM CHROMODYNAMICS

The neutron, which had long been assumed to be present in the nucleus of atoms, was experimentally identified by Chadwick in 1932. Because the masses of the neutron and the proton were nearly the same, it was proposed that these two particles were just different states of the same particle, distinguished by a property called isospin as well as by charge.



More and more elementary particles were discovered in the 1950s. It was eventually accepted that the proton and neutron particles were composed of smaller particles. The classification of these particles by charge, isospin, and other properties showed certain regularities. Gell-Mann suggested in 1965 that the class of particles called hadrons was composed of a set of smaller particles called quarks, which are distinguished by a property called colour charge. The strong forces between protons and neutrons in a nucleus, and between other hadrons, arise because of their colour charge. These interactions are described by a quantum field theory called quantum chromodynamics.

### **ELECTROWEAK THEORY**

In the 1930s, Pauli proposed a hypothetical particle called the neutrino to explain the process of beta decay. The neutrino was experimentally observed in 1956. The neutrino does not possess either charge or colour charge, and its mass is negligible. The interactions among neutrinos are fundamentally from strong nuclear, electromagnetic, and gravitational forces. These interactions are known as weak nuclear forces.

In the 1960s, Glashow, Salam, and Weinberg showed that the theory of weak interactions and electromagnetic interactions could be described as different aspects of a single quantum field theory. This quantum field theory is called the electroweak theory.

### **THE STANDARD MODEL**

The Standard Model of elementary particle physics combines quantum electroweak theory and quantum chromodynamics into a single theory. The Standard Model largely accounts for the interactions among experimentally confirmed elementary particles. The Standard Model groups elementary particles as leptons and quarks, and includes force mediating particles, or bosons. The Standard Model predicts the existence of the Higgs boson, for which experimental evidence is currently being sought using the large hadron collider at CERN in Geneva, Switzerland.

### **Practice**

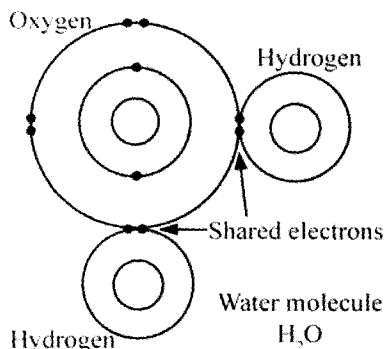
24. Which of the following physicists wrote the first relativistic quantum theoretical equation for the electron?
- A. Werner Heisenberg
  - B. Erwin Schrödinger
  - C. Wolfgang Pauli
  - D. Paul Dirac



**Open Response**

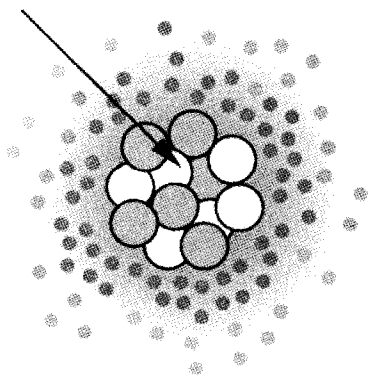
Use the following information to answer the next question.

Niels Bohr visualized the atom as a nucleus of protons and neutrons surrounded by orbiting electrons.



The use of more sophisticated technology has led to research that proposes a better model of the atom. Scientists now realize that electrons have both wave and particle characteristics. Electrons do not move in fixed orbits around the nucleus, but in high energy electron clouds around the nucleus.

Nucleus of protons and neutrons



25. What type of technology is used to see the atom? Explain the reason the electron cloud proposal is a more realistic model of an atom than the Bohr model.

---

---

---

---

---

12.5.3.2 describe how the development of the quantum theory has led to scientific and technological advances that have benefited society

**TECHNOLOGICAL ADVANCES MADE THROUGH QUANTUM THEORY**

Although quantum mechanics is most useful in the study of atomic or subatomic systems and elementary particles, it has led to a number of useful and practical applications beyond experimental physics.

**SEMICONDUCTOR DEVICES**

The rapid growth of electronics and the subsequent computer revolution were made possible by the development of semiconductor devices, such as transistors and diodes. The development of semiconductor physics was based on the quantum mechanical understanding of solids.

The electron energy levels in a periodic array of atoms are grouped into a number of distinct bands, or energy ranges. The electrons are distributed two per energy level to completely fill the lowest energy bands, starting from the ground state. Each energy level contains two electrons because Pauli's exclusion principle states that no two electrons may occupy the same quantum state simultaneously (i.e., each energy level can contain only one electron with an up spin and one electron with a down spin). The highest energy band that is completely filled is called the valence band. The next higher energy band is called the conduction band.

The electrons in the valence band and lower bands may be thought to be bound to the individual atom. The electrons in the conduction band are then free electrons that can move through the bulk of the solid. These electrons are responsible for electrical conduction. A solid is either an insulator or conductor, depending on whether the valence and conduction bands in its atoms overlap or not. If these bands overlap, the solid is a conductor. If the gap is large, the solid is an insulator.



Semiconductors are a class of solids with narrow band gaps, which prevents the solid from conducting at ordinary temperatures. However, these solids can be doped with impurity atoms that allows them to conduct in one of two different manners. Junctions, or contact boundaries, between two semiconductor blocks with different kinds of doping have remarkable current-voltage characteristics. In particular, a junction of layers of the two types does not obey Ohm's law  $\left(I = \frac{V}{R}\right)$ .

Such a device, called a junction diode, allows current to flow in only one direction, a property that finds many applications in electronics. One variant of the junction diode, a light emitting diode or LED, produces light when a current is sent through it. LEDs are now being explored as an alternative to large scale conventional light sources. Another version of the diode, the photovoltaic cell, can be used to convert light energy into electrical energy.

More complicated devices can be formed using three or more layers of semiconductors together. The most important of these is the transistor, which can be used as a current amplifier or as an electrically controlled switch. These electronic components can be miniaturized, which led to the invention of integrated circuits in which a very large numbers of components can be packed into a small semiconductor wafer. These integrated circuits form the heart of modern computers.

Quantum mechanical effects are also used in a number of semiconductor devices. One of these effects is tunnelling, which allows a particle to pass through a barrier. A semiconductor component based on tunnelling is now routinely used in flash memory components for computers.

## LASERS

A central idea in quantum mechanics is that electrons in atoms have a set of discrete energy states. Transitions between these states can be made by the emission or absorption of electromagnetic radiation of the correct energy (i.e., frequency or wavelength). An electron in an excited state can fall to a lower energy state on its own, emitting a photon that carries the energy lost by the electron. The phase of the emitted photon is random; this process is known as spontaneous emission.

However, it is also possible to cause the transition of the electron and emission of a photon by the interaction of another photon of the correct frequency. In this case, the emitted photon has the same phase as the photon that induced the emission. This process is called stimulated emission. If the process is somehow made to repeat itself, a large number of photons with related phases can be created. In other words, the system will act as a source of coherent light. Of course, it is also necessary to ensure that these photons are created faster than they are absorbed, since the same atoms can absorb the photons by raising the energy states of their electrons. This is achieved by pumping.

To make emitted photons pass through the medium repeatedly before it is emitted, the medium is enclosed between mirrors in a cavity of suitable length. A small opening in the cavity allows for some of the light to be emitted. The resulting setup is a laser (light amplification through stimulated emission of radiation).

There are many uses for lasers that produce an intense beam of coherent light. Lasers are used in optical storage devices, such as compact disc drives in computers, and they are used as light sources in fibre optic systems. The high intensity of a laser can be used to burn through materials, which makes lasers useful for diverse purposes, such as eye surgery, modifying metal structures, and military applications. Laser lighting and optical fibres are now used for decorative lighting as well.

## ELECTRON MICROSCOPES

Following the discovery of the wave nature of the electron, the possibility of using electron particle waves instead of light in microscopy became apparent. The de Broglie wavelength of a beam of electrons can be made much smaller than the wavelength of light, allowing electron microscopes to probe atomic size details. The lenses in optical microscopes are mimicked by electromagnetic lenses, which use spatial variation of electric potential to converge or diverge beams of electrons.

Since their invention in 1931, electron microscopes have played a major role in the investigation of small scale systems in biology, biochemistry, chemistry, material science, and condensed matter physics.



## Practice

26. Which of the following forms of energy was decisively advanced by the use of semiconductor materials?
- A. wind energy    B. solar energy  
C. atomic energy    D. thermal energy

**12.5.3.3** *describe examples of Canadian contributions to modern physics*

## CANADIAN CONTRIBUTIONS TO MODERN PHYSICS

Several Canadians have been involved in the development of modern physics. The following contributors are only a sample of the many Canadians that have impacted the increasing understanding of the physical world.

### BERTRAM BROCKHOUSE

Bert Brockhouse won the Nobel Prize for physics in 1994, along with Clifford Shull, for designing the triple-axis neutron spectroscope. This device aided in the development of neutron scattering techniques used to study condensed matter. Bertram Brockhouse was a professor at McMaster University in Hamilton, Ontario, from 1962 until his retirement in 1984.

### WERNER ISRAEL

Werner Israel has developed theories involving black holes. In 1967 he wrote the first theory to describe black holes as simple objects in the same sense as elementary particles are simple. That is, they are only dependent upon their mass, their charge, and their spin. More recently, Werner Israel co-edited two important texts with Stephen Hawking. From 1958 until 1996, Werner Israel was a professor at the University of Alberta. In 1996 he was appointed adjunct professor of physics and astronomy at the University of Victoria in British Columbia.

### IAN KEITH AFFLECK

World renowned as a brilliant theoretical physicist, Ian Keith Affleck has worked to unite elementary particle physics (quarks, electrons, neutrons, etc.) with condensed matter physics (crystals, semiconductors, gems, etc.). Ian Kieth Affleck has worked at the University of British Columbia in Vancouver since 1987, with the exception of two years he spent in Boston.

### HARRIET BROOKS

Harriet Brooks was the first Canadian female nuclear physicist. She received her master's degree in 1901 under the supervision of Ernest Rutherford at McGill University in Montreal. She also worked under the supervision of Marie Currie before she left the field of physics in 1907. Harriet Brooks was among the first people in the world to discover and experiment with the element radon.

### RICHARD E. TAYLOR

Between 1967 and 1973, Richard Taylor (Stanford), Jarome Friedman (MIT), and Henry Kendall (MIT) used the linear accelerator at Stanford to examine the products of high-energy collisions that broke apart protons and neutrons. Richard Taylor shared the Nobel Prize for physics in 1990 with Friedman and Kendall for confirming quark theory. Richard Taylor is currently a working professor at Stanford University in California.

### WILLIAM UNRUH

William Unruh has made many contributions to the understanding of gravity and cosmology, specifically the workings of black holes. He is most notably responsible for the discovery of the Unruh effect, which states that an accelerating observer will observe blackbody radiation while an inertial observer will not. William Unruh currently holds a fellow position in the cosmology program at the University of British Columbia in Vancouver.



**Practice**

**Open Response**

27. In two or three paragraphs, explain the contributions made by Louis Slotin to the understanding and application of fissionable materials.

---

---

---

---

---

---

---

**SOLUTIONS—MATTER-ENERGY INTERFACE**

1. A	7. A	13. 3.18	19. 1.56	Part C- OR
2. D	8. OR	14. 1 5 1 1 1	20. D	24. D
3. 3, 2, 1	9. C	15. 9, 4, 3, 6	21. 4803	25. OR
4. OR	10. OR	16. A	22. C	26. B
5. A	11. D	17. OR	23. Part A- OR	27. OR
6. OR	12. A	18. C	Part B- OR	

**1. A**

The de Broglie equation is  $\lambda = \frac{h}{m\bar{v}}$

where  $\lambda$  = wavelength

$h$  = Planck's constant

$m$  = mass of the particle

$\bar{v}$  = velocity of the particle

The equation can also be written as  $\lambda = \frac{h}{\bar{p}}$

where  $\bar{p}$  = momentum of the particle

**2. D**

Alpha and beta rays are not electromagnetic in nature; instead, they are rays of particles resulting from nuclear decay. X-rays are not produced during nuclear decay. Gamma rays are electromagnetic radiation and are produced during nuclear decay.

**3. 3,2,1**

Gamma rays can completely penetrate biological tissue, and they have enough energy to completely disrupt chemical bonds in the body. Beta particles can only moderately ionize the atoms they collide with, so they are only a low biological threat, but they do present a greater risk than alpha particles because they can travel greater distances than alpha particles. Alpha particles can travel only very short distances and they can only ionize atoms quite weakly. Therefore they are the lowest biological risk of the three forms of radiation.

**4. Open Response**

An alpha ray is a beam of alpha particles. These particles consist of two protons and two neutrons bound together. While they are an ionizing form of radiation, the mass of the alpha particles prevent them from penetrating very deeply into the human body. In fact, alpha particles can be easily stopped by a sheet of paper. Alpha rays are released from radioactive nuclei in a process known as alpha decay. This same process can also create gamma rays.

A gamma ray is an ionizing form of electromagnetic radiation with extremely high energy. Stopping gamma radiation requires a thick material with a high atomic number to absorb the radiation.

**5. A**

A photocell is a device that converts radiant solar energy into electrical energy. Photocells are based on the principle of the photoelectric effect. This principle states that electrons are emitted from a metallic surface when radiation of sufficient energy shines on its surface. As a result of the flow of electrons, a current flows through the photocell.

**6. Open Response**

Max Planck developed the idea that energy is not continuous, but rather increases in levels, or quanta. His equation  $E = h\nu$  shows that the energy ( $E$ ) of a vibrating molecule is a multiple of the vibration frequency ( $\nu$ ). The multiple is Planck's constant ( $h$ ), which is  $6.626 \times 10^{-34} \text{ J}\cdot\text{s}$

This idea of quantized energy was applied to light by Alberta Einstein, who used Planck's idea to explain the photoelectric effect—electrons emitted from a heated piece of metal. Einstein theorized that electrons are ejected from the surface of the metal only when the light particle, or photon, contains the proper amount of threshold energy. The energy is passed to the electron, which causes it to break free of the attractive forces of the metal.

Planck's and Einstein's findings led to the ideas that electron orbitals have set sizes and energies and that light can behave as a particle or a wave (wave-particle duality of light), since a photon is absorbed and ceases to exist when it hits an electron.

**7. A**

Neils Bohr was the first scientist to explain the structure of an atom using quantum theory.

Einstein postulated the theory of relativity and the quantum theory. He did not explain the structure of the atom using quantum theory. Avogadro introduced the concept of the mole. He did not explain the structure of the atom using quantum theory. Heisenberg developed the principle of uncertainty. He did not explain the structure of the atom using quantum theory.



### 8. Open Response

Since a photon with a wavelength of 435 nm falls into the visible region of the electromagnetic spectrum (it would be violet-coloured light), the electron that created it must have dropped from a higher energy level into the second energy level, since this is the only situation where the emitted photon falls into the visible region.

First, calculate the energy of the emitted photon.

$$E = \frac{hc}{\lambda}$$

$$E = \frac{(4.14 \times 10^{-15} \text{ eVs})(3.00 \times 10^8 \text{ m/s})}{4.35 \times 10^{-7} \text{ m}}$$

$$E = 2.86 \text{ eV}$$

The energy of the emitted photon is equal to the change in energy that caused the change in energy levels. Therefore, calculate the initial energy level of the electron.

$$\Delta E = E_i - E_f \rightarrow E_i = \Delta E + E_f$$

$$E_i = 2.86 \text{ eV} + (-3.40 \text{ eV}) = -0.54 \text{ eV}$$

Therefore, the electron dropped from the fifth energy level to the second when it released the photon of violet-coloured light.

### 9. C

According to Einstein's special theory of relativity, clocks that are moving slow down. This means that the time interval as observed by the first observer would be longer than that observed by the moving observer or  $\Delta t > \Delta t'$ .

### 10. Open Response

Calculate the time it takes for a muon to travel the 10 km to sea level.

$$t = \frac{d}{v} = \frac{10^4 \text{ m}}{(0.99)(3.0 \times 10^8 \text{ m/s})}$$

$$t = 3.37 \times 10^{-5} \text{ s}$$

Calculate the theoretical amount of muons that reach sea level.

$$N = N_0 \times 2^{-\frac{t}{t_1}} = 10^7 \times 2^{-\frac{3.37 \times 10^{-5}}{1.56 \times 10^{-6}}} = 3.15 \text{ muons}$$

Therefore, there should only be approximately 3 muons detected at sea level according to classical physics. This is much less than actually observed.

Since the muons travel at a velocity of  $0.99c$  the distance between the point where the number of muons are first measured and sea level contracts. This length contraction occurs because an observer, moving along with the muons will observe that the distance moves up towards the observer at  $0.99c$ .

$$L' = \frac{L_0}{\gamma} = \left( \sqrt{1 - \frac{0.99^2}{1}} \right) (10^4 \text{ m})$$

$$L' = (1.41 \times 10^{-1}) (10^4 \text{ m})$$

$$L' = 1.41 \times 10^3 \text{ m}$$

Therefore, the time it takes the muons to travel this new distance is given by

$$t = \frac{d}{v} = \frac{1.41 \times 10^3 \text{ m}}{(0.99)(3.0 \times 10^8 \text{ m/s})}$$

$$\therefore t = 4.75 \times 10^{-6} \text{ s}$$

So the total number of muons expected to reach sea level given this relativistic treatment is

$$N = N_0 \times 2^{-\frac{t}{t_1}}$$

$$= 10^7 \times 2^{-\frac{4.75 \times 10^{-5}}{1.56 \times 10^{-5}}} = 1.21 \times 10^6 \text{ muons}$$

The amount of muons calculated matches the measured amount of 1.2 million muons at sea level. Therefore, this experiment shows that Einstein's special theory of relativity.

**Note:** It is also possible to explain the total number of muons at sea level using the concept of time dilation, since moving clocks run more slowly than clocks at rest. The length of time that an observer moving along with the muons would experience is exactly the amount of time after which 1.2 million muons would be detected.

### 11. D

Let  $v$  = speed of the object

From the object's frame of reference, the centre of the galaxy approaches the object at a speed of  $v$ . In 20 years, as measured by the object, the centre will move toward the object by a distance of 20 years  $\times v$ . The radius of the galaxy, as measured from the viewpoint of the object, is therefore 20 years  $\times v$ . The rest length of the radius of the galaxy is  $3 \times 10^{20}$  m.

$$20 \text{ years} \times v = (3 \times 10^{20} \text{ m}) \sqrt{1 - \frac{v^2}{c^2}}$$

$$(6.312 \times 10^8 \text{ s})^2 \times v^2 = (9 \times 10^{40} \text{ m}^2) \left( 1 - \frac{v^2}{c^2} \right)$$

$$v = 0.9999996c$$

### 12. A

From the law of conservation of mass-energy, the combined mass-energy of the reactants must equal the combined mass-energy of the products.

Since the mass of the products (235.86 amu) is less than that of the reactants (236.05 amu), the missing mass, or the mass defect, must have been converted into energy.

**13. 3.18**

$^{235}_{92}\text{U}$  contains 92 protons and  $235 - 92 = 143$  neutrons

$$92 \text{ protons} = 92 \times 1.6726 \times 10^{-27} \text{ kg}$$

$$= 1.5388 \times 10^{-25} \text{ kg}$$

$$143 \text{ neutrons} = 143 \times 1.6749 \times 10^{-27} \text{ kg}$$

$$= 2.3951 \times 10^{-25} \text{ kg}$$

$$\text{theoretical mass of U} = 1.5388 \times 10^{-25} \text{ kg}$$

$$+ 2.3951 \times 10^{-25} \text{ kg} = 3.9339 \times 10^{-25} \text{ kg}$$

$$\text{actual mass of U} = 3.9021 \times 10^{-25} \text{ kg}$$

$$\text{mass defect} = 3.9339 \times 10^{-25} \text{ kg} - 3.9021 \times 10^{-25} \text{ kg}$$

$$\text{mass defect} = 0.0318 \times 10^{-25} \text{ kg}$$

$$\text{mass defect} = 3.18 \times 10^{-27} \text{ kg}$$

**14. 1 5 1 1 1**

C-13 has  $Z = 6$  and  $A = 13$

$$\therefore \#p = 6, \therefore \#n = 7$$

$$\Delta m = 6(1.00728 \text{ u}) + 7(1.00866 \text{ u}) - (13.00335 \text{ u})$$

$$= 1.0102 \times 10^{-1} \text{ u}$$

$$E_b = 1.0102 \times 10^{-1} \text{ u} \times 1.49 \times 10^{-10} \text{ J/u}$$

$$= 1.51 \times 10^{-11} \text{ J}$$

**15. 9,4,3,6**

$$\Delta m = m_{\text{prod}} - m_{\text{react}}$$

$$= (3.01603 \text{ u} + 1.008665 \text{ u}) - 2(2.01210 \text{ u})$$

$$= 4.0247 - 4.0242$$

$$= 5.00 \times 10^{-4} \text{ u}$$

$$\Delta E = 5.00 \times 10^{-4} \text{ u} \times 1.49 \times 10^{-10} \text{ J/u}$$

$$= 7.45 \times 10^{-14} \text{ J}$$

$$E_k = \frac{1}{2} m \bar{v}^2$$

$$\bar{v} = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2(7.45 \times 10^{-14} \text{ J})}{1.6799 \times 10^{-27} \text{ kg}}}$$

$$= 9.43 \times 10^6 \text{ m/s}$$

**16. A**

The subatomic particles that make up protons and neutrons are called quarks.

Gluon particles are bosons, or force-carrying particles.

They carry the strong nuclear force, which combine protons and neutrons together in an atomic nucleus.

Positron particles are the antiparticles of electrons.

They have the same mass as electrons, but the opposite charge. Beta particles are composed of two neutrons and two protons.

**17. Open Response**

A proton is a bound state of two up quarks, with a charge of  $+\frac{2}{3}$  each, and one down quark, with a charge of  $-\frac{1}{3}$ .

Thus a proton has a total charge of  $+1$ .

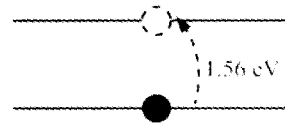
**18. C**

$$\text{Energy required} = E_2 - E_1$$

$$\text{Energy required} = (-3.4) - (-13.6) = 10.2 \text{ eV}$$

**19. 1.56**

$$1.92 \text{ eV} - 0.36 \text{ eV} = 1.56 \text{ eV}$$



Since electrons of 1.55 eV were not absorbed, they were insufficient to move the electrons in the atom to move to the next energy level.

**20. D**

$$\vec{F}_{\text{centripetal}} = \vec{F}_{\text{magnetic}}$$

$$\frac{m\bar{v}^2}{r} = q\bar{v}B$$

$$r = \frac{m\bar{v}}{qB}$$

$$\text{Proton: } r_{\text{proton}} = \frac{m\bar{v}}{qB}$$

Alpha particle: An  $\alpha$ -particle has 4 times the mass of a proton ( $4m$ ) and twice the charge of the proton ( $2q$ )

$$r_{\alpha\text{-particle}} = \frac{(4m)\bar{v}}{(2q)B} = \frac{2m\bar{v}}{qB} = 2r_{\text{proton}}$$

$$2 \times r_{\text{proton}} = 2(3.60 \times 10^{-4} \text{ m}) = 7.20 \times 10^{-4} \text{ m}$$

Therefore, the radius of the alpha particle's path is twice as large as the radius of the proton's path.

**21. 4803**

Use the mass-to-charge ratio expression.

$$\frac{m}{q} = \frac{rB_{\perp}}{\bar{v}}$$

$$B_{\perp} = \frac{m\bar{v}}{qr} = \frac{1.67 \times 10^{-27} \text{ kg} \times 1.38 \times 10^6 \text{ m/s}}{1.60 \times 10^{-19} \text{ C} \times 3.00 \text{ m}}$$

$$B_{\perp} = 4.80 \times 10^{-3} \text{ T}$$

**22. C**

$$\text{Portion that will not decay after 10 years} = \frac{1}{2}$$

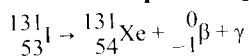
$$\text{Portion that will not decay after 20 years} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\text{Portion of mass that will not have decayed} = \frac{1}{4}$$

$$\therefore \text{Undecayed mass} = \left(\frac{1}{4} \times 12\right) \text{ g} = 3.0 \text{ g}$$



### 23. Part A – Open Response



Note: Students may use the notation of  ${}_{-1}^0\beta$  or  ${}_{-1}^0\text{e}$ , and  ${}^0_0\gamma$  or  $\gamma$ .

### Part B – Open Response

Time (days)	0	8	16	24	32	40
Mass (g)	2.00	1.00	0.50	0.25	0.13	0.06

One half-life means that half the initial amount of iodine-131 decays in 8.00 days. So, at 8 days, there is only 1.00 g left; at 16 days, 0.50 g is left; and so on.

Therefore, to calculate the data for the table, the half-life equation can be used. For example,  $n = \frac{t}{t_{1/2}}$ , then for

$$t = 8 \text{ days,}$$

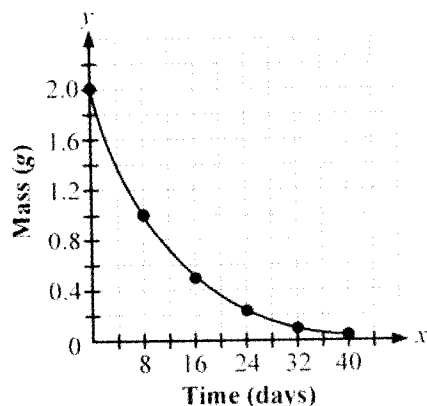
$$N = N_0 \left(\frac{1}{2}\right)^n$$

$$N = 2.00 \text{ g} \left(\frac{1}{2}\right)^1$$

$$N = 1.00 \text{ g}$$

### Part C – Open Response

Mass of Iodine-131 as a Function of Time



### 24. D

Dirac's equation was the first equation that correctly incorporated relativistic considerations into the quantum dynamics of electrons. The original Schrödinger equation was a non-relativistic equation. Schrödinger also found a relativistic equation for particles without quantum spin, now known as the Klein-Gordon equation. There were also attempts to incorporate relativistic effects in the orbits of the old theory by Sommerfeld.

### 25. Open Response

It is now possible to photograph the atom using an electron microscope. The photographs suggest that the electron cloud model explains that electrons move randomly in all directions with different degrees of energy. The greatest density of electron movement is near the nucleus.

### 26. B

Solar cells, which convert the energy of sunlight into electrical energy, are composed of semiconductor materials. The development of semiconductor materials led to revolutionary developments in solar energy.

### 27. Open Response

A correct answer to this question should include at least the following

- He was born on December 1, 1910, in Winnipeg Manitoba.
- After receiving his doctorate from King's College London, Louis Slotin worked as a research associate at the University of Chicago, helping design a cyclotron.
- In 1942, Slotin was asked to join the Manhattan Project, where he performed experiments with uranium and plutonium cores to determine their critical mass values.
- On May 21, 1946, Slotin accidentally began a fission reaction that irradiated him with approximately 2100 rems of neutron and gamma radiation. Slotin was able to stop the fission reaction quickly enough to save the lives of seven other scientists who were in the room with him. He died nine days later on May 30, 1946, the second person to die in a fission related critical accident. The first person to die from a criticality accident was actually irradiated by the same sphere of plutonium, less than a year earlier.



# Unit Test

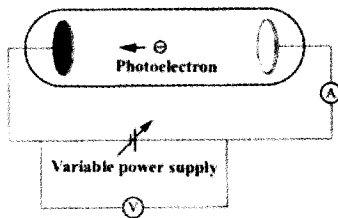


- Which of the following pairs of scientists conducted the experiments that led to the determination of the mass of an electron?
  - Planck and Einstein
  - Rutherford and Bohr
  - Thomson and Millikan
  - Compton and de Broglie

*Use the following information to answer the next question.*

Photoelectrons are emitted from a photoelectric surface when blue light with a frequency of  $6.40 \times 10^{14}$  Hz shines on it. The stopping voltage in the photoelectric cell is measured to be 1.25 V.

Blue light



- What is the maximum kinetic energy of the emitted photoelectrons?
  - $4.91 \times 10^{-19}$  J
  - $2.91 \times 10^{-19}$  J
  - $2.00 \times 10^{-19}$  J
  - $1.28 \times 10^{-19}$  J

**Open Response**

- What is the frequency of a photon carrying  $5 \times 10^{-30}$  J of energy?
 

---



---



---



---

**Open Response**

- Use the quantum theory to explain how the emission spectrum of hydrogen is formed and the reason that hydrogen's absorption spectrum is the negative image of its emission spectrum.

---



---



---



---



---



---

*Use the following information to answer the next question.*

A train moves with a speed of  $0.6c$  and passes a train station platform without slowing down. The observers on the platform observe that the length of the train is 200 m.

- What is the rest length of the train?
  - 128 m
  - 160 m
  - 250 m
  - 313 m

**Open Response**

Use the following information to answer the next question.

A truck driving with a relativistic velocity toward a garage with the door left open. The truck is 4 m long and the garage is only 3 m deep.

6. Calculate the velocity that the truck would need to in order for an observer in the unmoving garage's reference frame to observe that the vehicle fit inside the garage completely.

---

---

---

---

---

---

---

**Numerical Response**

7. What is the ratio to express the relative mass of an atom compared to the mass of hydrogen? \_\_\_\_

**Numerical Response**

Use the following information to answer the next question.

The given measurements indicate that the uranium-235 nucleus has a mass smaller than the mass of the corresponding number of free protons and neutrons. This difference in mass is called the mass defect.

Einstein's of mass-energy equivalence equation,  $E = mc^2$ , can be used to predict the energy that binds a nucleus together by using the mass defect.

mass of uranium-235 nucleus  
 $= 3.9021 \times 10^{-25} \text{ kg}$

mass of proton  $= 1.6726 \times 10^{-27} \text{ kg}$

mass of neutron  $= 1.6749 \times 10^{-27} \text{ kg}$

8. The nuclear binding energy of uranium-235, expressed in scientific notation, is  $b \times 10^x \text{ eV}$ .

The value of  $b$  is \_\_\_\_.

(Record your answer to **three** digits.)

Use the following information to answer the next question.

The given table shows the masses of three unknown particles.

Particle	Mass
<i>a</i>	0.511 MeV/c <sup>2</sup>
<i>b</i>	938 MeV/c <sup>2</sup>
<i>c</i>	$7 \times 10^{-6}$ MeV/c <sup>2</sup>

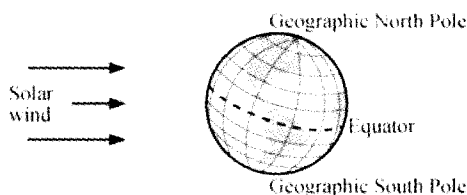
9. Which of the following lists **most likely** identifies the given particles in order of *a*, *b*, *c*?
- A. Electron neutrino, neutron, and electron
- B. Electron, proton, and electron neutrino
- C. Positron, neutron, and antiproton
- D. Electron, proton, and antiproton
10. Which of the following particles will produce a visual spectrum similar to the spectrum produced by a He<sup>+</sup> ion?
- A. H                      B. He
- C. Na<sup>+</sup>                    D. Li<sup>+</sup>

### Open Response

Use the following information to answer the next question.

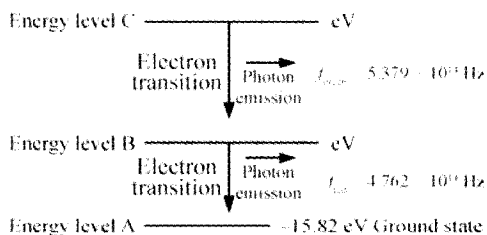
The northern lights that are visible in the sky on many winter nights are produced when free electrons in the solar wind are trapped within Earth's magnetic field.

Diagram 1: Solar Wind and Earth



These free electrons can collide with oxygen atoms in the upper atmosphere. These collisions can cause the oxygen atoms to become excited, meaning the electrons in the atoms' lower energy levels move to higher energy levels. Electron transitions toward the ground state result in the emission of photons, two of which correspond to red and green light. The given diagram shows the three energy levels in an oxygen atom involved in producing green- and red-coloured northern lights.

Diagram II: Selected Electron Energy Levels



In addition to the two photons identified above, a third, unique photon can be emitted by an oxygen atom after it has been excited to energy level C.

11. Determine the minimum speed of a free electron that would excite an oxygen atom from energy level A to energy level B. Clearly state the conservation law that is used in this calculation.

---



---



---



---



---

12. A train moves with a velocity of  $\vec{v}$ , with respect to the ground. A passenger on the train moves with a velocity of  $u'$  with respect to the train. Which of the following equations describes the passenger's velocity relative to the ground,  $u$ , according to Newtonian physics?

A.  $u = \frac{u'}{\vec{v}}$

B.  $u = u' + \vec{v}$

C.  $u = u' - \vec{v}$

D.  $u = \frac{u' + \vec{v}}{z}$

### Numerical Response

Use the following information to answer the next question.

A charged particle with a velocity of  $1.00 \times 10^5$  m/s enters into a magnetic field with a strength of 0.025 T. The path of the particle within the magnetic field is an arc with a radius of 0.083 m.

13. The charge-to-mass ratio of the particle, to two significant digits, is found to be  $a.b \times 10^c$  C/kg. What are the values of  $a$ ,  $b$ , and  $c$ ?

$a = -$

$b = -$

$c = -$

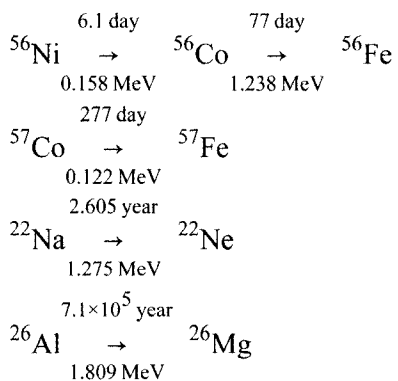
Use the following information to answer the next multipart question.

14. The supernova known as SN1987A reached its maximum brightness, or luminosity (energy release per second), in mid-May, 1987. The luminosity of the supernova decreased afterward.

Decline in Luminosity in Supernova SN1987A

Time (Days)	Luminosity (10 <sup>35</sup> W)
0	1.000
50	0.638
100	0.407
150	0.260
200	0.166
250	0.106
300	0.067
350	0.043

The most likely reason that the luminosity decreased is that luminosity depends on the radioactive decay of isotopes created in the explosion. One source of the luminosity could be the gamma rays that result from any one of the decay chains listed in the following.

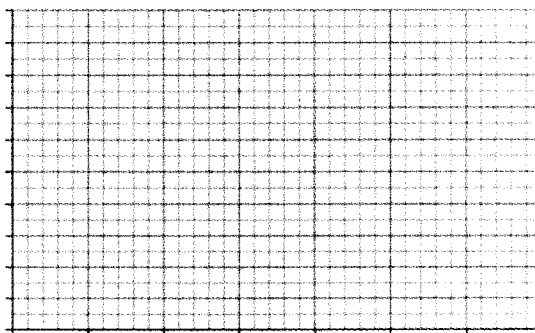


Part A

**Open Response**

Plot a graph of luminosity versus time.

\_\_\_\_\_ (Title)




---



---



---



---



---

Part B

**Open Response**

Determine the half-life of the luminosity, and identify the single decay **most likely** responsible for most of the energy released by the supernova.

---



---



---



---



---

Part C

**Open Response**

The amount of radioactive nickel-56 predicted to have been created in the supernova is about  $1.49 \times 10^{19}$  kg. How many days would it take for the mass of nickel-56 to be reduced to  $1.86 \times 10^{28}$  kg?

---

---

---

---

---

---

---

Part D

**Open Response**

Which of the given decay chains releases the gamma rays with the shortest wavelength? Explain how you identified this decay chain, and calculate the shortest gamma wavelength.

---

---

---

---

---

15. Which of the following types of interaction is **not** described by the Standard Model?
- A. Electromagnetic
  - B. Gravitational
  - C. Strong
  - D. Weak

**Open Response**

16. Using simple diagrams, describe a junction diode and explain the reason it allows a significant amount of current to pass in only one direction.

---

---

---

---

---

---

---

**SOLUTIONS**

1. C	6. OR	11. OR	Part C- OR
2. C	7. 1:1	12. B	Part D- OR
3. OR	8. 1.79	13. 4 8 7	15. B
4. OR	9. B	14. Part A- OR	16. OR
5. C	10. A	Part B- OR	

**1. C**

Maxwell Planck studied blackbody radiation.

He developed the relationship  $E = nhf$ .

Albert Einstein explained the photoelectric effect and the photon of light.

Ernest Rutherford studied the atom and discovered that all the positive mass of the atom was found in a small region in the centre of the atom, which he called the nucleus.

A. H. Compton proved photons have momentum.

Louis de Broglie developed a formula to predict the wavelengths of matter waves.

J. J. Thomson developed the "raisin-bun model" of the atom, and he found the charge-to-mass ratio of the electron using a cathode ray tube, and electric and magnetic fields.

Robert Millikan used gravity and electric fields to measure the charge on the electron.

Therefore, the scientists whose work determined the mass of the electron were J. J. Thomson and Robert Millikan.

**2. C**

The stopping voltage is the minimum voltage required to prevent photoelectrons from reaching the other plate in a photoelectric cell.

$$E_{k \text{ max}} = qV_{\text{stop}}$$

$$= 1.25 \text{ V}(1.60 \times 10^{-19} \text{ C})$$

$$E_{k \text{ max}} = 2.00 \times 10^{-19} \text{ J}$$

**3. Open Response**

$$E = hf$$

$$f = \frac{E}{h}$$

$$= \frac{5 \times 10^{-30} \text{ J}}{6.626 \times 10^{-34} \text{ m}^2 \text{ kg/s}}$$

$$= 7.55 \times 10^3 \text{ Hz}$$

The photon has a frequency of  $7.55 \times 10^3 \text{ Hz}$ .

**4. Open Response**

The experimentally determined Rydberg formula,

describing the wavelengths of light emitted by the hydrogen atom was explained by Niels Bohr in 1913.

Bohr showed that the allowed energy states for an electron in a hydrogen atom were quantized. That is, the electron could absorb or release only specific energies. It was determined that the reason Rydberg's formula worked was that it had successfully described the relationship between the wavelength of the emitted electromagnetic radiation with the change in energy level of the electron within the hydrogen atom.

$$E_n = \frac{E_1}{n^2}$$

Let  $i$  = initial and  $f$  = final

$$\Delta E = E_i - E_f = \frac{E_1}{n_i^2} - \frac{E_1}{n_f^2}$$

The change in energy is the energy released in the form of a photon.

$$\Delta E = E_{\text{ph}} = \frac{hc}{\lambda} = E_1 \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

$$\Rightarrow \frac{1}{\lambda} = \frac{E_1}{hc} \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

Rydberg's formula is

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

$$\text{Therefore, } \frac{E_1}{hc} = R_H = 1.10 \times 10^7 \text{ m}^{-1}$$

The reason that the absorption spectrum and the emission spectrum are the negative images of each other is that the energy of the absorbed or emitted photons must adhere to the allowed energy transitions of the hydrogen atom.

The electrons can only absorb or emit photons with the same energy as would be predicted by the associated change in energy levels. A hydrogen atom can only absorb photons that have enough energy to change the energy state of the electron from one allowed state to another.

Any additional energy is released as another photon after the appropriate amount of energy has been transferred to the electron. In the case of the release of a photon, only photons corresponding to the energy difference between initial and final energy states can be released.

**5. C**

$$\text{The length } L' = L \sqrt{1 - \frac{v^2}{c^2}}$$

$$L = \frac{L'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{200 \text{ m}}{\sqrt{1 - (0.6)^2}}$$

$$L = 250 \text{ m}$$

**6. Open Response**

Rearrange the length contraction formula to solve for  $v$ .

$$L' = \frac{L_0}{\gamma} \text{ where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\therefore L' = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$(L')^2 = (L_0)^2 \left(1 - \frac{v^2}{c^2}\right)$$

$$\frac{(L')^2}{(L_0)^2} = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1 - \frac{(L')^2}{(L_0)^2}$$

$$v^2 = c^2 \left(1 - \frac{(L')^2}{(L_0)^2}\right)$$

$$v = c \sqrt{1 - \frac{(L')^2}{(L_0)^2}}$$

Substitute the known values to calculate  $v$ .

$$L' = 3.0 \text{ m}$$

$$L_0 = 4.0 \text{ m}$$

$$c = 3.0 \times 10^8 \text{ m/s}$$

$$\therefore v = (3.0 \times 10^8 \text{ m/s}) \sqrt{1 - \frac{(3.0 \text{ m})^2}{(4.0 \text{ m})^2}}$$

$$v = (0.66)(3.0 \times 10^8 \text{ m/s}) = 0.66c$$

Therefore, in order for a stationary observer to observe that the truck could fit inside of the garage, the truck would need to travel with a speed of at least  $0.66c$ .

However, to an observer in the reference frame of the truck, the garage is moving toward the truck. This means that from the perspective of an observer in the truck, the garage would experience length contraction. Therefore, the garage would be even smaller than 3 m, so it would be much too small for the truck to fit inside. This apparent paradox is called the ladder paradox.

**7. 1:1**

The atomic weight of an element is the measure of how much mass an atom of the element has. The ratio of the relative mass of an atom compared to the mass of hydrogen can be expressed as the mass of one atom of the element to the mass of one atom of hydrogen.

**8. 1.79**

Use Einstein's mass-energy equivalence

$$E = mc^2$$

$$= (3.18 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2$$

$$E = \frac{(2.862 \times 10^{-10} \text{ J})}{(1.60 \times 10^{-19} \text{ J/eV})}$$

$$E = 1.79 \times 10^9 \text{ eV}$$

**9. B**

$$0.511 \text{ MeV}/c^2 \times 1.78 \times 10^{-30} \text{ kg/MeV}/c^2$$

$$= 9.10 \times 10^{-31} \text{ kg} \Rightarrow \text{the particle is an electron}$$

$$938 \text{ MeV}/c^2 \times 1.78 \times 10^{-30} \text{ kg/MeV}/c^2$$

$$= 1.67 \times 10^{-27} \text{ kg} \Rightarrow \text{proton}$$

Neutrinos are essentially mass less.

**10. A**

Just like hydrogen,  $\text{He}^+$  has one electron. Therefore, their spectra are similar.

The ground state for the electronic configuration of helium is  $1s^2$ . It has two electrons in its outermost orbit whereas  $\text{He}^+$  ion has only one electron. Thus, their spectra will not be similar.

The ground state electronic configuration of sodium is  $1s^2 2s^2 2p^6 3s^1$ . The electronic configuration of a

$\text{Na}^+$  ion is  $1s^2 2s^2 2p^6$  with eight electrons in its

outermost orbit. On the other hand, a  $\text{He}^+$  ion has only one electron. Thus, their spectra will not be similar.

The ground state electronic configuration of lithium is  $1s^2 2s^1$ . The electronic configuration of a  $\text{Li}^+$  ion is

$1s^2$  with two electrons in its outermost orbit. On the other hand, a  $\text{He}^+$  ion has only one electron. Thus, their spectra will not be similar.

**11. Open Response**

$$E_{\text{free electron}} = E_B - E_A = \Delta E_{\text{electron in atom}}$$

$$\frac{1}{2} m \dot{v}^2 = 1.97 \text{ eV}$$

$$\dot{v} = \sqrt{\frac{2(1.97 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{9.11 \times 10^{-31} \text{ kg}}}$$

$$= 8.32 \times 10^5 \text{ m/s}$$

**12. B**

If a train moves with velocity  $\vec{v}$  and a passenger moves with velocity  $u'$  on the train, then the velocity of the passenger, with respect to the ground, is  $u = u' + \vec{v}$ . Relative velocities, under Newtonian physics, are simple Galilean transformations involving nothing more than vector addition.

**13. 4 8 7**

When the particle is in the magnetic field, the magnetic force provides the centripetal force.

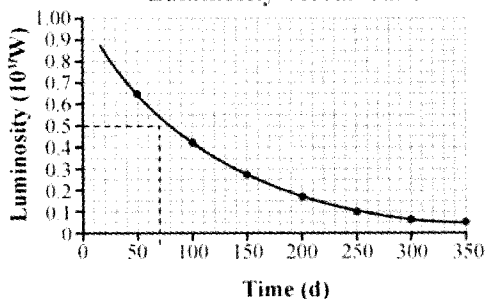
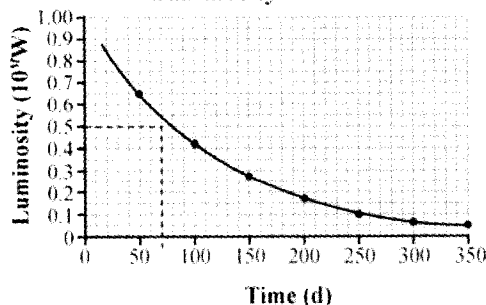
$$F_c = F_m$$

$$\frac{m\vec{v}^2}{r} = q\vec{v}B_{\perp} \Rightarrow \frac{q}{m} = \frac{\vec{v}}{B_{\perp}r}$$

$$\frac{q}{m} = \frac{1.00 \times 10^5 \text{ m/s}}{(0.025 \text{ T})(0.083 \text{ m})}$$

$$\frac{q}{m} = 4.8 \times 10^7 \text{ C/kg}$$

Note: this is the charge-to-mass ratio of an alpha particle.

**14. Part A – Open Response****Luminosity Versus Time****Part B – Open Response****Luminosity Versus Time**

On the graph “Luminosity Versus Time,” draw a horizontal line across from the  $y$ -axis at 0.5 to the curve. Next, draw a vertical line down from the curve to the  $x$ -axis. This shows that when the value of the luminosity is half its original value, the time elapsed was 75 days. Therefore, the half-life of the luminosity was 75 days. This is very close to the expected half-life of cobalt-56 and therefore, the luminosity of this supernova was most likely caused by the decay of cobalt-56.

**Part C – Open Response**

The closest half-life in the table is the decay of cobalt-56 into iron-56 with a half-life of 77.3 days.

$$N_0 = 1.49 \times 10^{29} \text{ kg}$$

$$N = 1.86 \times 10^{28} \text{ kg}$$

$$t \frac{1}{2} = 6.1 \text{ days}$$

$$1.49 \times 10^{29} \div 2 \div 2 \div 2 = 1.86 \times 10^{28} \text{ kg}$$

Therefore 3 half-lives have expired.

$$\text{Total time} = 3 \times 6.1 \text{ d} = 18 \text{ days}$$

**Part D – Open Response**

The emitted photon with the shortest wavelength will have the highest energy. Therefore, the decay chain,

${}_{26}^{26}\text{Al} \xrightarrow{7.1 \times 10^5 \text{ y}} {}_{26}^{26}\text{Mg}$  gives the shortest wavelengths because the energy of 1.809 MeV is the largest energy produced.

$$E = 1.809 \text{ MeV} = 2.894 \times 10^{-13} \text{ J}$$

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{2.894 \times 10^{-13} \text{ J}}$$

$$\lambda = 6.87 \times 10^{-13} \text{ m}$$

**15. B**

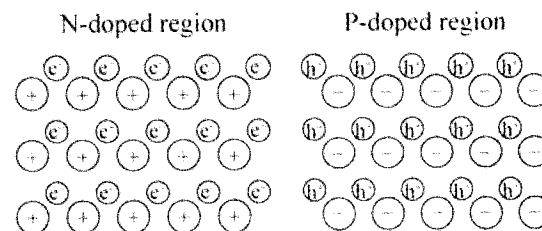
The Standard Model unifies the weak, strong, and electromagnetic interactions into a single quantum field theory. However, it does not explain the gravitation interaction.

**16. Open Response**

A junction diode is formed by creating an n-doped region and a p-doped region beside each other on a semiconductor wafer.

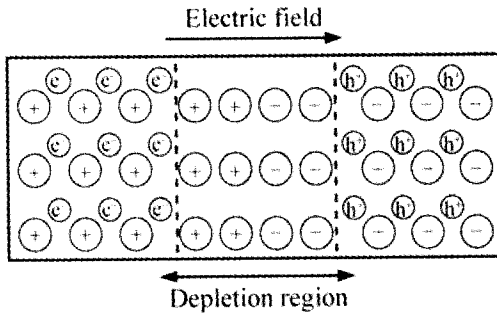
The n-doped region has impurity atoms with five electrons in their outermost electron shell, such as phosphorus. Four of these electrons form covalent bonds with the atoms of the semiconductor crystal, but the remaining electron is available as a free electron for conduction. The loss of the electron makes the impurity atom a positive ion.

The p-doped region has impurity atoms with three electrons in their outermost electron shell, such as boron. An extra electron is captured from the valence bond of a neighbouring atom in the semiconductor crystal. This creates a vacancy, or a *hole*, ( $h^+$ ) that acts as a free charge carrier in its own right. The impurity atom is turned into a negative ion.





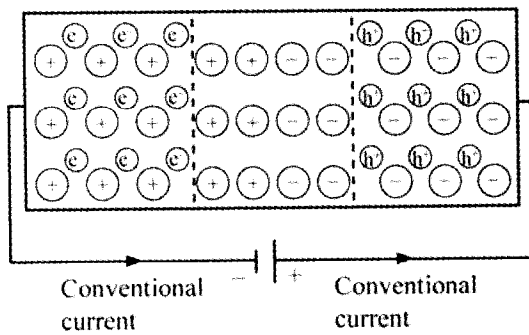
The junction diode may be understood to be created by bringing the n-doped region and the p-doped region together. Near the junction, the free electrons from the n-side very quickly fill the holes on the p-side, leaving a region without any free charges. The ions of opposite signs in the two sides caused an electric field pointing from the n-side to the p-side to form.



The electric field builds up until no further movement of electrons and holes across the junction is possible.

If an external voltage source is introduced with its positive terminal connected to the n-side and its negative terminal to the p-side, it only widens the depletion region and reinforces the junction field, preventing the transport of electrons and holes across the junction. Therefore, a current cannot flow across the junction despite the application of an external potential difference.

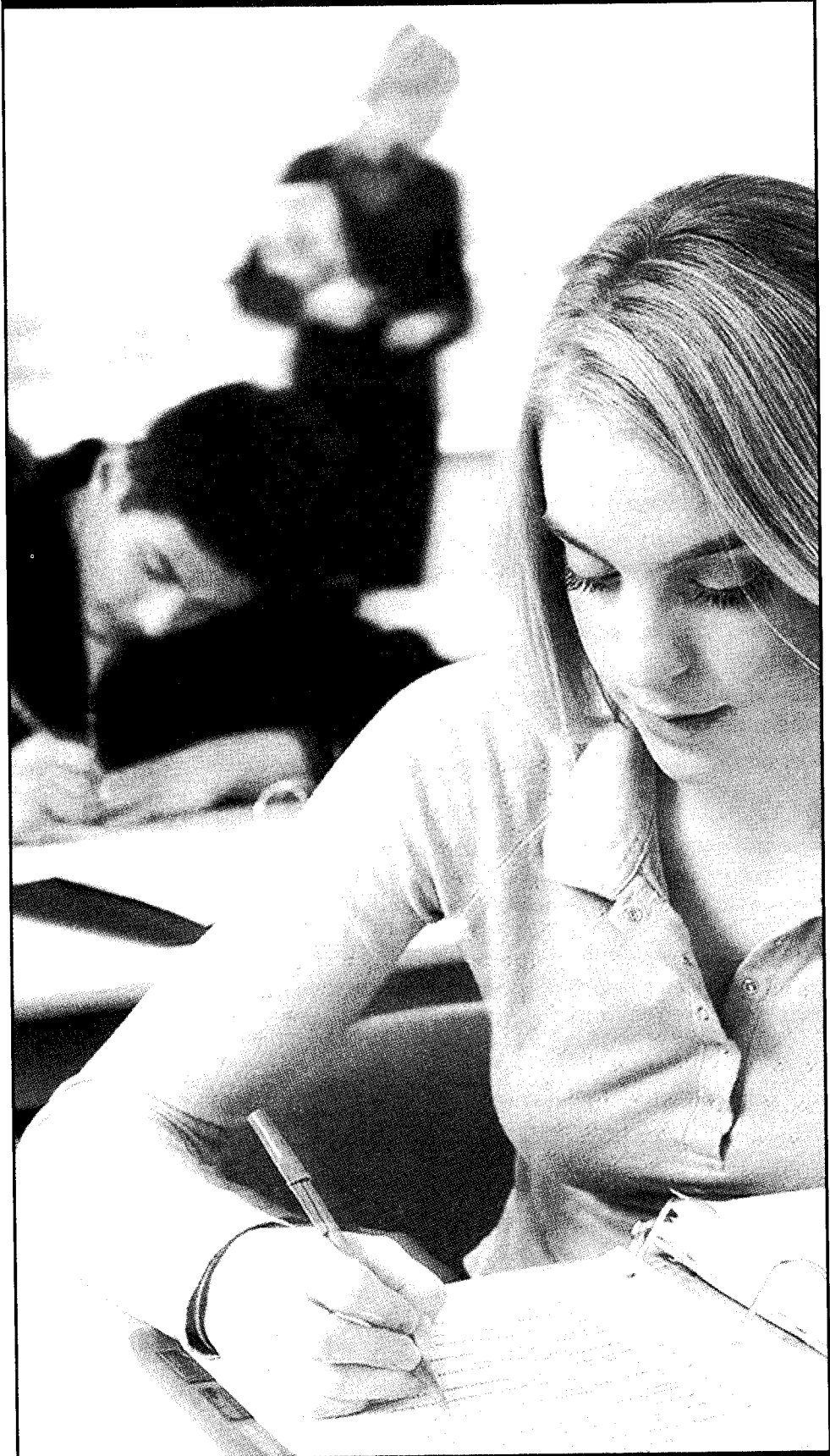
On the other hand, if the positive terminal of the external source is connected to the p-side, and its negative terminal to the n-side, the external field pushes the free electrons on the n-side and the holes on the p-side toward the junction. If the external voltage is sufficient, it can overcome the electric field at the junction and send free electrons and holes across the junction. A current then flows in the circuit.

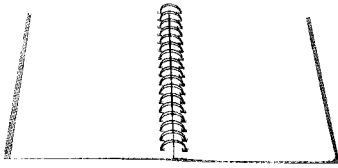


Thus a junction diode allows current to flow through it for only one polarity of the external voltage. This is called its rectifying action.



# Success on Tests





## KEY STRATEGIES FOR SUCCESS ON TESTS

### *Things to Consider When Taking a Test*

- It is normal to feel anxious before you write a test. You can manage this anxiety by:
  - thinking positive thoughts. Visual imagery is a helpful technique to try.
  - making a conscious effort to relax by taking several slow, controlled, deep breaths. Concentrate on the air going in and out of your body.
- Before you begin the test, ask questions if you are unsure of anything.
- Jot down key words or phrases from any oral directions.
- Look over the entire test to assess the number and kinds of questions on the test.
- Read each question closely and reread if necessary.
- Pay close attention to key vocabulary words. Sometimes these are bolded or italicized, and they are usually important words in the question.
- Mark your answers on your answer sheet carefully. If you wish to change an answer, erase the mark completely and then ensure your final answer is darker than the one you have erased.
- On the test booklet, use highlighting to note directions, key words, and vocabulary that you find confusing or that are important to answering the question.
- Double-check to make sure you have answered everything before handing in your test.

When taking tests, the easy words are often overlooked. Failure to pay close attention to these words can result in an incorrect answer. One way to avoid this is to be aware of these words and to underline, circle, or highlight these words while you are taking the test.

Even though some words are easy to understand, they can change the meaning of the entire question, so it is important that you pay attention to them. Here are some examples.

<b>all</b>	<b>always</b>	<b>most likely</b>	<b>probably</b>	<b>best</b>	<b>not</b>
<b>difference</b>	<b>usually</b>	<b>except</b>	<b>most</b>	<b>unlikely</b>	<b>likely</b>

### *Example*

1. Which of the following items is **not** considered abiotic?
  - A. Wind
  - B. Bacteria
  - C. Sunlight
  - D. Precipitation

### ***Helpful Strategies for Answering Multiple-Choice Questions***

A multiple-choice question provides some information for you to consider and then asks you to select a response from four choices. Each question has one correct answer. The other answers are distractors, which are incorrect.

Below are some strategies to help you when answering multiple-choice questions.

- Quickly skim through the entire test. Find out how many questions there are and plan your time accordingly.
- Read and reread questions carefully. Underline key words and try to think of an answer before looking at the choices.
- If there is a graphic, look at the graphic, read the question, and go back to the graphic. Then, you may want to underline the important information from the question.
- Carefully read the choices. Read the question first and then each answer that goes with it.
- When choosing an answer, try to eliminate those choices that are clearly wrong or do not make sense.
- Some questions may ask you to select the best answer. These questions will always include words like **best**, **most appropriate**, or **most likely**. All of the answers will be correct to some degree, but one of the choices will be better than the others in some way. Carefully read all four choices before choosing the answer you think is the best.
- If you do not know the answer or if the question does not make sense to you, it is better to guess than to leave it blank.
- Do not spend too much time on any one question. Make a mark (\*) beside a difficult question and come back to it. If you are leaving a question to come back to later, make sure you also leave the space on the answer sheet.
- Remember to go back to the difficult questions at the end of the test; sometimes clues are given throughout the test that will provide you with answers.
- Note any negative words like **no** or **not** and be sure your choice fits the question.
- Before changing an answer, *be sure* you have a very good reason to do so.
- Do not look for patterns on your answer sheet.

## *About Science Tests*

### **What You Need to Know about Science Tests**

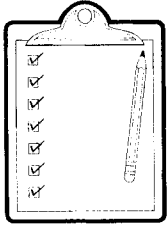
To do well on a science test, you need to understand and apply your knowledge of scientific concepts. Reading skills can also make a difference in how well you perform. Reading skills can help you follow instructions and find key words, as well as read graphs, diagrams, and tables. Math skills are also important to success on science exams. Formulas are often used to solve scientific problems. Practicing with formulas is the best way to become good at them. They can also help you solve science problems.

Science tests usually have two types of questions: questions that ask for understanding of scientific concepts and questions that test how well you can solve scientific problems.

### **How You Can Prepare for the Science Test**

Below are some strategies that are particular to preparing for and writing science tests.

- Know how to use your calculator and, if it is allowed, use your own for the test.
- Note-taking is a good way to review and study important information from your class notes and textbook.
- Sketch a picture of the problem, procedure, or term. Drawing is helpful for learning and remembering concepts.
- Check your answer to practice questions that require formulas by working backward to the beginning. You can find the beginning by going step-by-step in reverse order.
- When answering questions with graphics (pictures, diagrams, tables, or graphs), read the test question carefully.
  - Read the title of the graphic and any key words.
  - Read the test question carefully to figure out what information you need to find in the graphic.
  - Go back to the graphic to find the information you need.
- Always pay close attention when pressing the keys on your calculator. Repeat the procedure a second time to be sure you pressed the correct keys.



## **TEST PREPARATION COUNTDOWN**

There is little doubt that if you develop a plan for studying and test preparation, you *will* perform well on tests.

Below is a general plan to follow seven days before you write a test.

### ***Countdown: 7 Days before the Test***

1. Use “Finding Out About the Test” to help you make your own personal test preparation plan.
2. Review the following information:
  - areas to be included on the test
  - types of test items
  - general and specific test tips
3. Start preparing for the test at least 7 days before the test. Develop your test preparation plan and set time aside to prepare and study.

### ***Countdown: 6, 5, 4, 3, 2 Days before the Test***

1. Review old homework assignments, quizzes, and tests.
2. Rework problems on quizzes and tests to make sure you still know how to solve them.
3. Correct any errors made on quizzes and tests.
4. Review key concepts, processes, formulas, and vocabulary.
5. Create practice test questions for yourself and then answer them. Work out many sample problems.

### ***Countdown: The Night before the Test***

6. The night before the test is for final preparation, which includes reviewing and gathering material needed for the test before going to bed.
7. Most important is getting a good night’s rest and knowing you have done everything possible to do well on the test.

### ***Test Day***

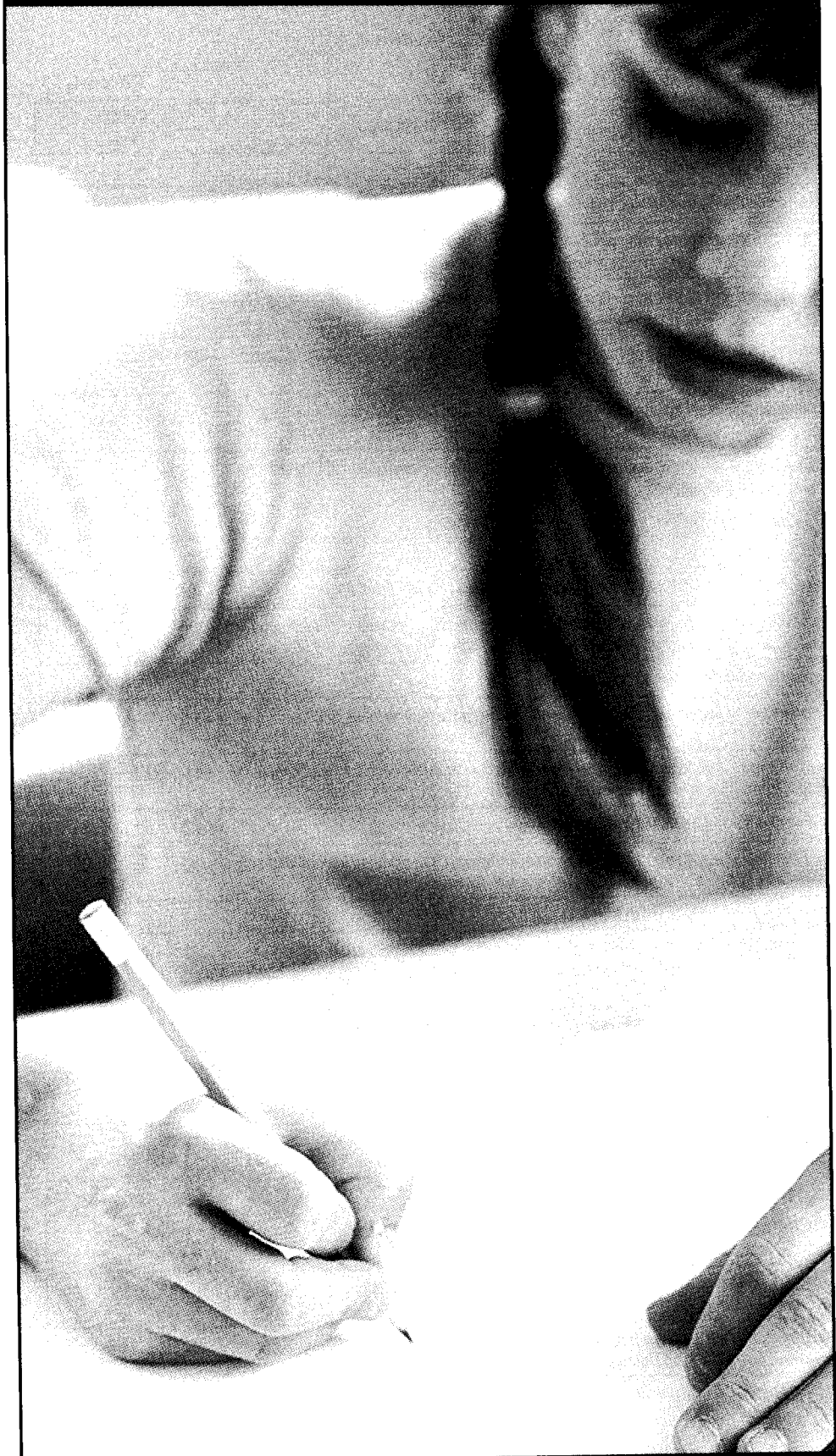
8. Eat a healthy and nutritious breakfast.
9. Ensure you have all the necessary materials.
10. Think positive thoughts: “I can do this.” “I am ready.” “I know I can do well.”
11. Arrive at your school early so you are not rushing, which can cause you anxiety and stress.

***SUMMARY OF HOW TO BE SUCCESSFUL DURING THE TEST***

The following are some strategies you may find useful for writing your test.

- Take two or three deep breaths to help you relax.
- Read the directions carefully and underline, circle, or highlight any important words.
- Survey the entire test to understand what you will need to do.
- Budget your time.
- Begin with an easy question or a question you know you can answer correctly rather than following the numerical question order of the test.
- If you cannot remember how to answer a question, try repeating the deep breathing and physical relaxation activities first. Then, move to visualization and positive self-talk to get you going.
- Write down anything you remember about the subject on the reverse side of your test paper. This activity sometimes helps you to remind yourself that you *do* know something and you *are* capable of writing the test.
- Look over your test when you have finished and double-check your answers to be sure you did not forget anything.

# Practice Test



## Practice Test

### Table of Correlations

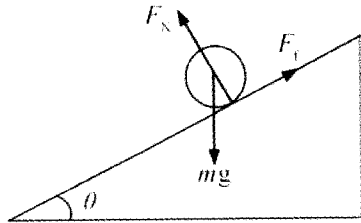
Specific Expectation	Practice Test
<b>12.1.1</b> Understanding Basic Concepts	
<b>12.1.1.1</b> <i>define and describe the concepts and units related to dynamics</i>	1
<b>12.1.1.3</b> <i>analyse and predict, in quantitative terms, and explain the motion of a projectile with respect to the horizontal and vertical components of its motion</i>	4, 5, 6
<b>12.1.1.4</b> <i>analyse and predict, in quantitative terms, and explain uniform circular motion in the horizontal and vertical planes with reference to the forces involved</i>	7, 8
<b>12.1.1.5</b> <i>distinguish between inertial and accelerating (non-inertial) frames of reference, and predict velocity and acceleration in a variety of situations</i>	9
<b>12.1.1.6</b> <i>describe Newton's law of universal gravitation, apply it quantitatively, and use it to explain planetary and satellite motion</i>	10, 11, 12
<b>12.1.1.2</b> <i>analyse and predict, in quantitative terms, and explain the linear motion of objects in horizontal, vertical, and inclined planes</i>	2, 3
<b>12.1.3</b> Relating Science to Technology, Society and the Environment	
<b>12.2.1</b> Understanding Basic Concepts	
<b>12.2.1.1</b> <i>define and describe the concepts and units related to momentum and energy</i>	14
<b>12.2.1.3</b> <i>analyse situations involving the concepts of mechanical energy, thermal energy and its transfer (heat), and the laws of conservation of momentum and of energy</i>	16
<b>12.2.1.4</b> <i>distinguish between elastic and inelastic collisions</i>	17
<b>12.2.1.5</b> <i>analyse and explain common situations involving work and energy, using the work-energy theorem</i>	18
<b>12.2.1.7</b> <i>analyse isolated planetary and satellite motion and describe it in terms of the forms of energy and energy transformations that occur</i>	19, 20
<b>12.2.1.8</b> <i>state Hooke's law and analyse it in quantitative terms</i>	22
<b>12.2.1.2</b> <i>analyse, with the aid of vector diagrams, the linear momentum of a collection of objects, and apply quantitatively the law of conservation of linear momentum</i>	15
<b>12.2.2</b> Developing Skills of Inquiry and Communication	
<b>12.2.2.1</b> <i>investigate the laws of conservation of momentum and of energy in one and two dimensions by carrying out experiments or simulations and the necessary analytical procedures</i>	21
<b>12.2.2.2</b> <i>design and conduct an experiment to verify the conservation of energy in a system involving kinetic energy, thermal energy and its transfer (heat), and gravitational and elastic potential energy</i>	23
<b>12.2.3</b> Relating Science to Technology, Society and the Environment	
<b>12.2.3.1</b> <i>analyse and describe, using the concepts and laws of energy and of momentum, practical applications of energy transformations and momentum conservation</i>	24
<b>12.3.1</b> Understanding Basic Concepts	
<b>12.3.1.1</b> <i>define and describe the concepts and units related to electric, gravitational, and magnetic fields</i>	25
<b>12.3.1.3</b> <i>apply Coulomb's law and Newton's law of universal gravitation quantitatively in specific contexts</i>	27
<b>12.3.1.4</b> <i>compare the properties of electric, gravitational, and magnetic fields by describing and illustrating the source and direction of the field in each case</i>	28
<b>12.3.1.5</b> <i>apply quantitatively the concept of electric potential energy in a variety of contexts, and compare the characteristics of electric potential energy with those of gravitational potential energy</i>	29

Specific Expectation	Practice Test
<b>12.3.1.6</b> <i>analyse in quantitative terms, and illustrate using field and vector diagrams, the electric field and the electric forces produced by a single point charge, two point charges, and two oppositely charged parallel plates</i>	30
<b>12.3.1.7</b> <i>describe and explain, in qualitative terms, the electric field that exists inside and on the surface of a charged conductor</i>	31
<b>12.3.1.2</b> <i>state Coulomb's law and Newton's law of universal gravitation, and analyse and compare them in qualitative terms</i>	26
<b>12.3.2</b> Developing Skills of Inquiry and Communication	
<b>12.3.2.1</b> <i>determine the net force on, and resulting motion of, objects and charged particles by collecting, analysing, and interpreting quantitative data from experiments or computer simulations involving electric, gravitational, and magnetic fields</i>	32
<b>12.3.2.2</b> <i>analyse and explain the properties of electric fields and demonstrate how an understanding of these properties can be applied to control or alter the electric field around a conductor</i>	33
<b>12.3.3</b> Relating Science to Technology, Society and the Environment	
<b>12.3.3.1</b> <i>explain how the concepts of a field developed into a general scientific model, and describe how it affected scientific thinking</i>	34
<b>12.4.1</b> Understanding Basic Concepts	
<b>12.4.1.1</b> <i>define and explain the concepts and units related to the wave nature of light</i>	35
<b>12.4.1.3</b> <i>describe the phenomenon of wave interference as it applies to light in qualitative and quantitative terms, using diagrams and sketches</i>	36
<b>12.4.1.4</b> <i>describe and explain the phenomenon of wave diffraction as it applies to light in quantitative terms, using diagrams</i>	37, 38
<b>12.4.2</b> Developing Skills of Inquiry and Communication	
<b>12.4.2.1</b> <i>identify the theoretical basis of an investigation, and develop a prediction that is consistent with that theoretical basis</i>	39
<b>12.4.2.4</b> <i>analyse and interpret experimental evidence indicating that light has some characteristics and properties that are similar to those of mechanical waves and sound</i>	41
<b>12.4.2.2</b> <i>identify the interference pattern produced by the diffraction of light through narrow slits (single and double slits) and diffraction gratings, and analyse it in qualitative and quantitative terms</i>	40
<b>12.4.3</b> Relating Science to Technology, Society and the Environment	
<b>12.4.3.2</b> <i>describe and explain the design and operation of technologies related to electromagnetic radiation</i>	42
<b>12.5.1</b> Understanding Basic Concepts	
<b>12.5.1.1</b> <i>define and describe the concepts and units related to the present-day understanding of the nature of the atom and elementary particles</i>	43
<b>12.5.1.2</b> <i>describe the principle forms of nuclear decay and compare the properties of alpha particles, beta particles, and gamma rays in terms of mass, charge, speed, penetrating power, and ionizing ability</i>	44
<b>12.5.1.4</b> <i>describe and explain in qualitative terms the Bohr model of the (hydrogen) atom as a synthesis of classical and early quantum mechanics</i>	45
<b>12.5.1.6</b> <i>apply quantitatively the laws of conservation of mass and energy, using Einstein's mass-energy equivalence</i>	47
<b>12.5.1.7</b> <i>describe the Standard Model of elementary particles in terms of the characteristic properties of quarks, leptons, and bosons, and identify the quarks that form familiar particles such as the proton and neutron</i>	48
<b>12.5.1.5</b> <i>state Einstein's two postulates for the special theory of relativity and describe related thought experiments</i>	46

Specific Expectation	Practice Test
<b>12.1.2</b> Developing Skills of Inquiry and Communication	
<b>12.1.2.4</b> <i>investigate, through experimentation, the relationships among centripetal acceleration, radius of orbit, and the period and frequency of an object in uniform circular motion; analyse the relationships in quantitative terms; and display the relationships using a graph</i>	13
<b>12.5.3</b> Relating Science to Technology, Society and the Environment	
<b>12.5.3.1</b> <i>outline the historical development of scientific views and models of matter and energy, from Bohr's model of the hydrogen atom to present-day theories of atomic structure</i>	53
<b>12.5.2</b> Developing Skills of Inquiry and Communication	
<b>12.5.2.1</b> <i>collect and interpret experimental data in support of a scientific theory</i>	49, 50
<b>12.5.2.3</b> <i>analyse images of the trajectories of elementary particles to determine the mass-versus-charge ratio</i>	52
<b>12.5.2.2</b> <i>conduct thought experiments as a way of developing an abstract understanding of the physical world</i>	51

**Practice Test**

- The state of rest or motion of a body is
  - relative.
  - positive.
  - negative.
  - constant.
- A sphere of mass  $m$  rolls without slipping on an inclined plane with an inclination of  $\theta$ .

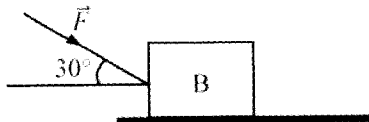


What is the linear acceleration of the sphere?

- $\frac{1}{4}g \sin \theta$
- $\frac{5}{7}g \sin \theta$
- $\frac{6}{7}g \sin \theta$
- $\frac{7}{5}g \sin \theta$

**Numerical Response**

- A force of 125 N acts on a block of mass 2.50 kg, as shown. The coefficient of friction between the block and the surface is 0.46.



Force on block acting at 30 degrees to horizontal

The acceleration of the block is \_\_\_  $\text{m/s}^2$ .

- How are the horizontal and vertical components of velocity related, for a projected particle?

F.  $v_x = \frac{1}{2} v_y$

G.  $v_x = 2.432 v_y$

H.  $v_y = \frac{1}{2} v_x$

J.  $v_x$  is independent of  $v_y$

**Numerical Response**

- A zoologist fires a tranquilizer dart horizontally with a velocity of 15 m/s directly at a monkey sitting in a tree 20 m away. The dart passes below the monkey by a distance of \_\_\_ m.

(Round and record your answer to two digits.)

**Open Response**

- A boy launches a paper airplane with a velocity of 4 m/s into the air, at an angle of  $75^\circ$  with the horizontal. It is assumed that the airplane does not face any air resistance. The airplane was launched by the boy from a height of 80 cm above the ground.

Assuming projectile motion, what is the maximum height reached by the airplane?

---



---



---



---

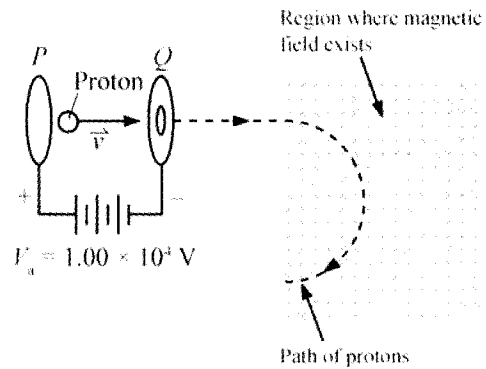


---

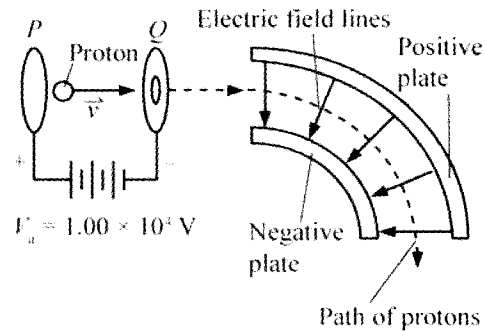
7. A fighter plane travelling at a speed of 900 km/h is moving in a vertical circle of radius 2000 m. Given that the mass of the plane is 16 000 kg, what force does the surrounding air exert on the plane at the lowest point of its path ( $g = 9.8 \text{ m/s}^2$ )?
- A.  $5.72 \times 10^4 \text{ N}$
  - B.  $6.06 \times 10^5 \text{ N}$
  - C.  $6.56 \times 10^5 \text{ N}$
  - D.  $7.03 \times 10^5 \text{ N}$

**Numerical Response**

8. A mass spectrometer uses either a magnetic field or an electric field to deflect charged particles.



*Magnetic field mass spectrometer*



*Electric field mass spectrometer*

In the electric field mass spectrometer shown, the radius of curvature of a proton's path is 10.0 m. The proton experiences an electrostatic force, expressed in scientific notation, of  $b \times 10^{-x} \text{ N}$ . The value of  $b$  is \_\_\_\_.

(Record your answer to three digits.)

9. A particle of mass  $m$  is viewed from an inertial frame of reference and is observed to move in a circle of radius  $r$  with a uniform speed  $v$ . The centrifugal force operating on the particle is
- A.  $\frac{mv^2}{r}$  toward the centre of the circle
  - B.  $\frac{mv^2}{r}$  along the tangent of the circle
  - C. infinite
  - D. zero
10. Which scientist discovered the law of universal gravitation?
- F. Galileo
  - G. Archimedes
  - H. Blaise Pascal
  - J. Sir Isaac Newton

**Numerical Response**

11. Radius of Mercury =  $2.44 \times 10^6$  m;  
 mass of Mercury =  $3.30 \times 10^{23}$  kg.

Calculate the speed of a satellite orbiting  $5.2 \times 10^5$  m above the surface of Mercury in the form of  $a.bc \times 10^d$  m/s and give your answer as  $abcd$ . \_\_\_\_, \_\_\_\_, \_\_\_\_, and \_\_\_\_

**Open Response**

12. The planet Mars has an orbital radius around the sun of  $2.28 \times 10^{11}$  m, and it orbits the sun once every 687 days. Using this data, calculate the mass of the sun.

---

---

---

---

---

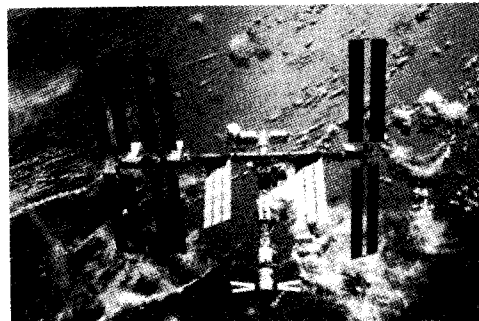
---

---

---

**Open Response**

13. The International Space Station (ISS) orbits Earth at an altitude of 390 km and completes one orbit every 92 min.



If the mass of the station were doubled, how would this affect its orbital speed?

---

---

---

---

---

---

---

---

14. Work is negative when the directions of the force and the motion are
- F. opposite
  - G. the same
  - H. perpendicular
  - J. at an angle of  $120^\circ$

**Numerical Response**

15. An iron ball of mass 5 kg, moving with a velocity of 6 m/s toward the east, collides with another iron ball of mass 3 kg, which was initially at rest. After the collision, the velocity of the 5-kg iron ball becomes 5.2 m/s, the velocity of the 3-kg ball becomes 3.5 m/s and the 3-kg ball moves at angle  $\theta$  to the southeast.

What is the value of  $\theta$  in degrees? \_\_\_\_°

16. The concept of mechanical energy deals with the idea that
- F. mechanical energy is the amount of energy saved by a mechanical device.
  - G. mechanical energy is the sum of potential and kinetic energy.
  - H. potential energy and kinetic energy are always equal.
  - J. mechanical energy is a vector quantity.

17. Two satellites of equal masses,  $m$ , orbit Earth, of mass  $M$  with the same circular orbit of radius  $r$ , but in opposite directions, and eventually they collide.

What is the total mechanical energy immediately after a completely inelastic collision?

- A.  $-\frac{2GMm}{r}$
- B.  $-\frac{GMm}{r}$
- C.  $\frac{2GMm}{r}$
- D.  $\frac{GMm}{r}$

18. When the kinetic energy of a body decreases, the work done on the body must be
- F. negative.
  - G. positive.
  - H. infinite.
  - J. zero.

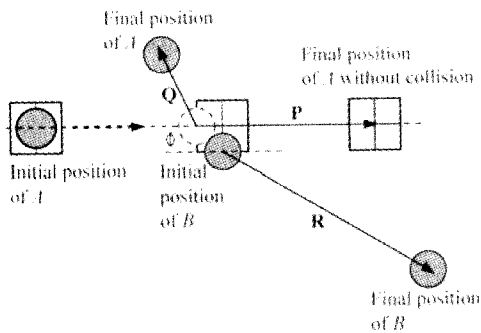
19. An artificial satellite is moving in a circular orbit around Earth. The speed of the satellite is equal to half the magnitude of the escape velocity from Earth. Given that Earth's radius is 6400 km, what is the height of the satellite above Earth's surface?
- A. 3200 km
  - B. 3600 km
  - C. 4800 km
  - D. 6400 km

**Numerical Response**

20. Assume that the mass of Earth has doubled but the radius is still 6400 km. A satellite moves in a circular orbit around Earth with a speed equal to one quarter the magnitude of the escape velocity of this new, heavier Earth. How high, in kilometres, is the satellite above Earth's surface? \_\_\_\_ km

**Open Response**

21. A simple experiment to verify the conservation of momentum in two dimensions without an air table uses two identical metal discs, A and B, that move on a large sheet of chart paper pinned to a horizontal surface. Both discs experience the same coefficient of kinetic friction on the paper sheet.
- A striker mechanism is available to start disc A moving with nearly the same velocity each time. First, disc A is used without B. The initial position of disc A and the position at which it stops moving are noted on the piece of paper.
- Then disc B is placed so as to collide with A, as shown in the diagram. After disc A leaves the striker, it collides with B, and the two discs scatter to their final positions, as shown below. The direction and magnitude of vectors P, Q, and R are recorded.



Three vectors,  $\vec{L}$ ,  $\vec{M}$  and  $\vec{N}$ , are constructed with magnitudes of  $\sqrt{P}$ ,  $\sqrt{Q}$ , and  $\sqrt{R}$ , with the same orientation as P, Q, and R, respectively.

Explain why the vectors  $\vec{L}$ ,  $\vec{M}$ , and  $\vec{N}$  form the sides of a triangle. Show any relevant calculations.

---



---



---



---



---



---

**Numerical Response**

22. A 750-N force compresses a spring by 0.150 m. The spring constant for this spring is \_\_\_ N/m.
- (Record your answer to three significant digits.)

**Numerical Response**

23. A group of students investigate the motion of a falling plastic ball using an ultrasonic motion detector connected to a computer interface. They obtain a set of values of the height of the ball and the velocity of the ball at different times. They then correctly plot the experimental data with the square of the velocity on the x-axis and the height of the ball on the y-axis. The acceleration due to gravity is  $9.8 \text{ m/s}^2$ .

The slope of the best-fit line through the plotted points, written to two significant digits, is \_\_\_  $\text{s}^2/\text{m}$ .

**Open Response**

24. Briefly describe the workings of a pendulum clock.

---



---



---



---



---



---

25. Which of the following statements **best** describes a scenario using charges that is similar to the increase in gravitational energy that occurs when a mass is raised at constant speed in a uniform gravitational field?

- A. A small, negatively charged sphere is placed between two parallel, oppositely charged plates and moved at constant speed toward the negative plate.
- B. A small, positively charged sphere is placed between two parallel, oppositely charged plates and moved at constant speed toward the negative plate.
- C. A small, negatively charged sphere is placed near another small, negatively charged sphere and moved toward it at constant speed.
- D. A small, positively charged sphere is placed near a small, negatively charged sphere and moved toward it at constant speed.

**Open Response**

26. State one similarity and one difference between Coulomb's Law and Newton's Law of Universal Gravitation.

---



---



---



---

27. Two masses  $2.5 \times 10^3$  kg and  $5.0 \times 10^3$  kg are situated at a distance of  $1.5 \times 10^2$  m.

What is the force of attraction between the two masses?

- A.  $3.7 \times 10^{-11}$  N
- B.  $5.5 \times 10^{-11}$  N
- C.  $3.7 \times 10^{-8}$  N
- D.  $5.5 \times 10^{-8}$  N

28. In order to determine the direction of an electric field surrounding a source charge, it is necessary to place a

- F. negative test charge in various locations around the source charge and measure the electrical force on the test charge
- G. positive test charge in various locations around the source charge and measure the electrical force exerted on the test charge
- H. neutral test charge in various locations around the source charge and measure the electrical force exerted on this test charge
- J. positive test charge in the field if the source charge is negative, or negative test charge in the field if the source charge is positive, and measure the force of attraction in various locations around the source

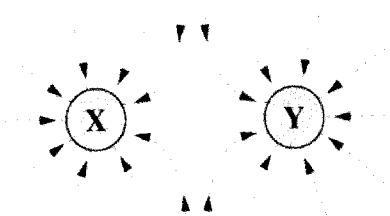
**Numerical Response**

29. An electron has a charge of  $1.60 \times 10^{-19}$  C. As the electron moves from one plate of a parallel plate capacitor, to the other, it gains  $4.50 \times 10^{-18}$  J of potential energy.

What is the electric potential difference between the plates, in volts? \_\_\_\_ V

**Numerical Response**

30.



*Electric field lines near two charged spheres*

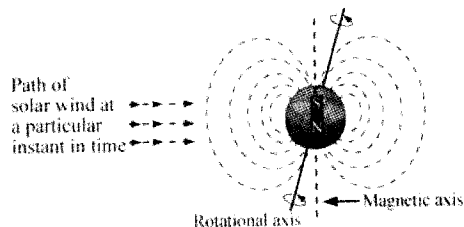
The types of charge present on X and Y are, respectively, \_\_\_\_ and \_\_\_\_.  
Enter 1 for positive and 2 for negative.

31. The electric field inside a hollow charged conductor
- A. is zero.
  - B. is not zero.
  - C. varies with the radial distance.
  - D. is equal to its value at the surface.

**Numerical Response**

**32. Earth's Magnetic Field**

The solar wind consists of particles emitted by the sun. Some of these particles are charged; therefore, when they enter Earth's magnetic field, they experience a magnetic force. A stream of charged particles travelling at a speed of  $8.00 \times 10^5$  m/s encounters Earth's magnetic field, as shown below, at an altitude where the field has a magnitude of  $1.10 \times 10^{-7}$  T.



Assume that the velocity of the solar wind particles is perpendicular to the magnetic field. The radius of the circular path that protons in a solar wind follow, expressed in scientific notation, is  $a.bc \times 10^d$  m. The values of  $a$ ,  $b$ ,  $c$ , and  $d$  are \_\_\_\_, \_\_\_\_, \_\_\_\_, and \_\_\_\_.  
(Record your four-digit answer.)

**Open Response**

33. Explain why lightning rods have pointed tips rather than being uniformly shaped.

---



---



---



---



---

34. 1. A compass needle was observed to deflect each time a nearby circuit was switched on.  
 2. Alpha particles should only be deflected by a small angle as they pass through a thin foil of gold if the plum-pudding atomic model is correct.  
 3. Light exists in discrete particle-like packets known as photons.

Hans Christian Oersted is associated with \_\_\_?

- F. statement 1
- G. statement 3
- H. statements 1 and 2
- J. statements 2 and 3

35. The phenomenon of the spreading of a light wave into its spectral components when incident on a medium is called  
 A. interference      B. polarization  
 C. dispersion      D. diffraction

**Open Response**

36. Describe the setup for Young's double-slit experiment using a clearly labelled diagram.

---

---

---

---

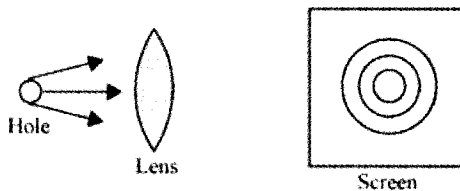
---

---

---

---

37.



The given figure shows the experimental setup for the circular aperture diffraction experiment. Bright and dark circular rings are obtained on the screen.

- The diffraction pattern of a circular aperture diffraction experiment consists of concentric dark and bright rings because of the  
 A. rotational symmetry of the system  
 B. diffraction of light at the circular lens  
 C. translational symmetry of the system  
 D. finite distance between the hole and the screen

**Numerical Response**

38. Calculate the angle of deviation of the second-order maximum produced by directing monochromatic light ( $\lambda = 3.75 \times 10^2 \text{ nm}$ ) through a diffraction grating in which the slits are  $5.10 \times 10^{-6} \text{ m}$  apart.

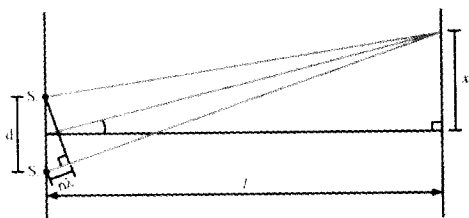
39. A strand of hair is held taut horizontally and placed in the path of a laser beam.

In which direction would you expect to observe the diffraction pattern, due to the strand of hair, spread out?

- A. neither horizontally nor vertically
- B. both horizontally and vertically
- C. horizontally
- D. vertically

**Open Response**

40. A student shines a helium-neon laser with a wavelength of 632 nm onto a diffraction grating with 150 lines per mm to get an interference pattern on a dark screen 2.00 m away. In order to make predictions about the diffraction pattern that is observed, it is possible to use the basic double slit diffraction set up with the minor difference that the distance between the slits is the reciprocal of the grating's lines per mm value.



Using the information given, calculate the distance from the centre of the central maximum of the diffraction pattern from which a third-order maximum would be expected to be found.

---

---

---

---

---

---

---

**Open Response**

41. High frequency sound waves, like those produced by a violin or a flute, sound much different than low frequency sound waves, such as those produced by a base drum. Using visible light, give an example of a high frequency wave and a low frequency wave.

---

---

**Open Response**

42. Thermal imaging detects what kind of radiation emitted by objects and their surroundings?

---

---

43. The mass defect can be defined as the
- A. sum of the masses of the bound atoms
  - B. mass of the protons present in the atoms
  - C. mass of the neutrons present in the atoms
  - D. difference in the masses of the bound atom and its constituents
44. Alpha particles are not harmful unless the source is inhaled or swallowed, because they
- F. are affected by electric fields
  - G. have high penetrating power
  - H. have low penetrating power
  - J. carry a positive charge

**Open Response**

45. Consider the case of a hydrogen-like atom with a ground state energy of  $-25.5 \text{ eV}$ , and a radius of its ground state of  $6.50 \times 10^{-11} \text{ m}$ .

Calculate the energy and radius of this atom when it is excited to its fourth energy state, and also calculate the wavelength of electromagnetic radiation that is released when an electron falls from the fourth energy level to the ground state.

---

---

---

---

---

---

---

---

**Open Response**

46. Einstein's special theory of relativity set an upper bound for the velocity of any object. It can sometimes be helpful to examine what could happen if this upper bound were changed to a more meaningful velocity, like  $100 \text{ km/h}$ .

If you were driving your car down a major highway at  $90 \text{ km/h}$  and your vehicle's rest length was  $5.0 \text{ m}$  long, how long would an observer standing on the side of the highway measure your car to be?

---

---

---

---

---

---

---

---

**Numerical Response**

47. The reactants in a nuclear reaction have a mass  $0.010 \text{ g}$  larger than the mass of the products. The energy released by this nuclear reaction is  $a.b \times 10^{cd} \text{ J}$ . The values for  $a$ ,  $b$ ,  $c$ , and  $d$  are? \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_ (Give your complete answer to four digits.)
48. Which of the following particles consists of two up quarks and one down quark?  
 F. Proton                      G. Meson  
 H. Neutron                     J. Electron
49. A hydrogen-like atom has a ground state energy of  $-22.0 \text{ eV}$ . If an electron in the ground state interacts with a photon of  $20.0 \text{ eV}$ , the electron will absorb  
 A. only  $19.6 \text{ eV}$  and will move to energy level  $n = 3$   
 B. only  $16.5 \text{ eV}$  and will remain in the ground state  
 C. all  $20.0 \text{ eV}$  and will move to energy level  $n = 3$   
 D.  $0 \text{ eV}$  and will remain in the ground state

**Numerical Response**

50. An electron in a hydrogen atom transitions from  $n = 3$  to  $n = 2$ , where  $n$  is the energy level of the electron. What is the wavelength of the resulting photon in nanometres to one decimal place? \_\_\_\_\_ nm

**Open Response**

51. Describe a thought experiment involving a line of charge and a charged particle to show that electric and magnetic fields may be transformed into each other by a change of reference frame.

---

---

---

---

---

---

---

52. A double-ionized particle enters a magnetic field of 2 T and moves in a circular path of radius 0.5 m and with a speed of  $10^7$  m/s.

The particle is **most likely**

F.  $\text{He}^{2+}$ .                      G.  $\text{F}^{2+}$ .

H.  $\text{Mg}^{2+}$ .                      J.  $\text{S}^{2+}$ .

53. Which of the following situations is **not** explained by quantum electrodynamics?
- A. The interaction of electrons and positrons
  - B. The pair production of electrons and positrons
  - C. The properties of interactions mediated by gluons
  - D. The properties of interactions mediated by photons



## SOLUTIONS—PRACTICE TEST

1. A	12. OR	23. 0.051	34. F	45. OR
2. G	13. OR	24. OR	35. C	46. OR
3. 39.0	14. G	25. A	36. OR	47. 9,0,1,1
4. J	15. 58	26. OR	37. A	48. F
5. 8.7	16. G	27. C	38. 8.46	49. A
6. OR	17. A	28. G	39. D	50. 654.5
7. C	18. F	29. 28.1	40. OR	51. OR
8. 3.20	19. D	30. 2,2	41. OR	52. G
9. D	20. 19200	31. A	42. OR	53. C
10. J	21. OR	32. 7,5,9,4	43. D	
11. 2,7,3,3	22. 500	33. OR	44. H	

## 1. A

There is nothing in the universe in a state of absolute rest or in absolute motion. The concept of absolute rest or motion is completely insignificant. The state of a body is described with respect to a frame fixed on Earth. Earth is travelling around the sun at a speed of nearly 30.6 km/s, and both the sun and the entire solar system are moving around in our galaxy. Hence, nothing exists in an absolute state of rest or motion.

## 2. G

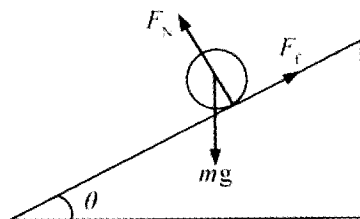
Let the radius of the sphere be  $r$ . The forces acting on the sphere are shown in the given figure.

They are:

- weight ( $mg$ )
- normal force ( $F_N$ )
- friction ( $F_f$ )

Let the linear acceleration of the sphere down the plane be  $a$ .

The equation for the linear motion of the centre of mass is  $mg \sin \theta - F_f = ma$  (i).



As the sphere rolls without slipping, its angular acceleration about the centre is  $\frac{a}{r}$ .

The equation of rotational motion about the centre of mass is  $F_f r = \left(\frac{2}{5}mr^2\right)\left(\frac{a}{r}\right) \Rightarrow F_f = \frac{2}{5}ma$  (ii).

From (i) and (ii),

$$a = \frac{5}{7}g \sin \theta.$$

## 3. 39.0

The horizontal component of the force  $F$  is  $F \cos(\theta) = (125 \text{ N})\cos 30^\circ = 108.3 \text{ N}$ .

The force of friction is

$$F_f = \mu F_N = \mu mg = (0.46)(2.50 \text{ kg})(9.81 \text{ m/s}^2) = 11.3 \text{ N}.$$

The acceleration of the mass is

$$\vec{a} = \frac{\vec{F}}{m} = \frac{108.3 \text{ N} - 11.3 \text{ N}}{2.50 \text{ kg}} = 39.0 \text{ m/s}^2.$$

## 4. J

The horizontal component of the velocity of the particle is given by:  $v_x = v \cos \theta$ , while its vertical component is given by:  $v_y = v \sin \theta$ . The velocity along the horizontal direction is constant, hence, the horizontal motion of the particle is independent of its vertical motion. Since the  $x$  and  $y$  components of the velocity are perpendicular to one another, a force that acts only in the vertical direction will effect the vertical velocity, but will have no effect on the horizontal velocity.

## 5. 8.7

Horizontal motion:

$$t = \frac{\vec{d}}{\vec{v}} = \frac{20 \text{ m}}{15 \text{ m/s}} = 1.33 \text{ s}$$

Vertical motion:

$$\vec{d} = \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

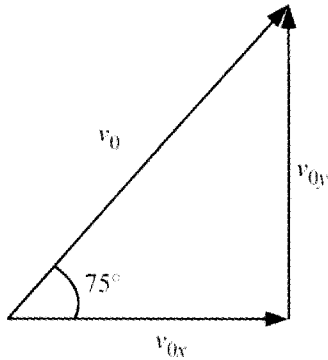
$$\vec{v}_i = 0$$

$$\begin{aligned} \Rightarrow \vec{d} &= \frac{1}{2} \vec{a} t^2 \\ &= \frac{1}{2} (-9.81 \text{ m/s}^2)(1.33 \text{ s})^2 \\ &= -8.7 \text{ m} \end{aligned}$$

Or 8.7 m below the monkey.

### 6. Open Response

The initial velocity ( $v_0$ ) of the paper airplane is 4 m/s. Its initial vertical velocity is ( $v_{iy}$ ) and the initial horizontal velocity is ( $v_{ix}$ ).



$$\begin{aligned} \therefore v_{iy} &= v_0 \sin 75^\circ \\ &= 4 \sin 75^\circ \\ &= 3.86 \text{ m/s} \end{aligned}$$

At the maximum height, there is no vertical motion for the airplane. Thus, its final vertical velocity at the maximum height is zero.

$$\therefore v_{fy} = 0$$

Let the maximum height reached by the airplane be  $d$ .

$$\begin{aligned} (v_{fy})^2 &= (v_{iy})^2 + 2ad \\ \therefore 0 &= (3.86)^2 + 2(-9.8)d \\ 14.8996 &= 19.6d \\ d &= 0.76 \text{ m} \\ &= 76 \text{ cm} \end{aligned}$$

Thus, the maximum height of the airplane from the ground is

$$\begin{aligned} 80 + 76 &= 156 \text{ cm} \\ &= 1.56 \text{ m} \end{aligned}$$

### 7. C

At the lowest point of the path, the acceleration is vertically upward (toward the centre of the circular path), and its magnitude is  $\frac{v^2}{r}$ , where  $v$  is velocity and  $r$  is the radius of the plane's path.

The forces on the plane are:

- the weight  $Mg$ , acting downward
- the force  $F$  exerted by the air, acting upward

Therefore:

$$\begin{aligned} F - Mg &= \frac{Mv^2}{r} \\ \Rightarrow F &= M \left( g + \frac{v^2}{r} \right) \\ v &= 900 \text{ km/h} \\ &= 900 \times \frac{5}{18} = 250 \text{ m/s} \\ F &= 16\,000 \left( 9.8 + \frac{250^2}{2000} \right) \\ &= 6.56 \times 10^5 \text{ N} \end{aligned}$$

### 8. 3.20

In the electric field mass spectrometer, the curving force is accomplished by an electric field rather than by a magnetic field. There is no information provided to allow you to solve directly for the electrical force, but the centripetal force can be found, and since the centripetal force is supplied by the electric force, this will be the value of the electric force.

$$\begin{aligned} \Delta E_c &= \Delta E_K \\ qV &= \frac{1}{2}mv^2 \\ \Rightarrow v^2 &= \frac{2qV}{m} \\ v &= \sqrt{\frac{2qV}{m}} \\ v &= \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(1.00 \times 10^4 \text{ V})}{10.0 \text{ m}}} \end{aligned}$$

$$v = 1.38 \times 10^6 \text{ m/s}$$

Now, in order to calculate the electrical force, calculate the centripetal force required to keep the particle at the desired radius.

$$\begin{aligned} F_c &= \frac{mv^2}{r} \\ F_c &= \frac{(1.67 \times 10^{-27} \text{ kg})(1.38 \times 10^6 \text{ m/s})^2}{10.0 \text{ m}} \\ F_c &= 3.20 \times 10^{-16} \text{ N} \end{aligned}$$

### 9. D

The centrifugal force is a **pseudo** force that only exists if the reference frame is non-inertial.

In this scenario, there is no centrifugal force on the particle because the reference frame is inertial, hence, the centrifugal force on the particle is zero.

### 10. J

Sir Isaac Newton discovered the law of universal gravitation, which states that every body in the universe attracts every other body.

### 11. 2,7,3,3

$$\begin{aligned} v &= \sqrt{\frac{Gm_e}{r}} \\ &= \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(3.30 \times 10^{23} \text{ kg})}{2.44 \times 10^6 \text{ m} + 5.2 \times 10^5 \text{ m}}} \\ &= 2.73 \times 10^3 \text{ m/s} \end{aligned}$$

**12. Open Response**

$$\begin{aligned} \bar{v} &= \frac{2\pi r}{T} \\ &= \frac{2\pi(2.28 \times 10^{11} \text{ m})}{(687 \text{ days})(24 \text{ h/day})(3600 \text{ s/h})} \\ &= 2.41 \times 10^4 \text{ m/s} \\ \bar{v} &= \sqrt{\frac{Gm_s}{r}} \\ \bar{v}^2 &= \frac{Gm_s}{r} \\ m_s &= \frac{\bar{v}^2 r}{G} \\ &= \frac{(2.41 \times 10^4 \text{ m/s})^2 (2.28 \times 10^{11} \text{ m})}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} \\ &= 1.99 \times 10^{30} \text{ kg} \end{aligned}$$

**13. Open Response**

$$\begin{aligned} \vec{F}_c &= \vec{F}_g \\ \frac{m_s \bar{v}^2}{r} &= \frac{Gm_E m_s}{r^2} \\ \bar{v}^2 &= \frac{Gm_E}{r} \\ \bar{v} &= \sqrt{\frac{Gm_E}{r}} \end{aligned}$$

As  $m_s$  is not a factor,  $m_s$  does not affect  $\bar{v}$ . Therefore, if the mass of the station were doubled, its speed would not be affected.

**14. G**

Work is done when its point of application moves in the direction of the force. When a mass falls from the top of a tower, work is done by the force of gravity since the direction of the force and that of the motion are the same. This work is termed negative work.

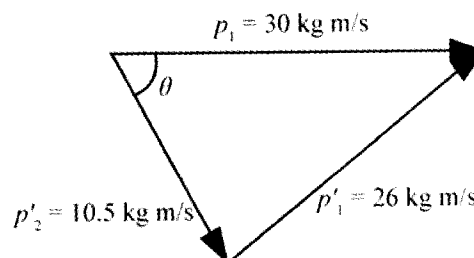
**15. 58**

$m_1$  = Mass of the first iron ball  
 $v_{1i}$  = Initial velocity of the 5-kg iron ball  
 Initial momentum of the 5-kg iron ball  
 $\vec{p}_1 = m_1 v_{1i}$   
 $= 5 \text{ kg}(6 \text{ m/s})$   
 $= 30 \text{ kg m/s}$   
 $m_2$  = Mass of the second iron ball = 3 kg  
 Initial momentum( $\vec{p}_2$ ) of the 3-kg ball at rest = 0  
 (Since it is at rest, velocity,  $v_{2i} = 0$ )  
 $\bar{v}_{1f}$  = Velocity of the 5-kg iron ball after collision  
 The final momentum of the 5-kg iron ball is given as  
 $\vec{p}'_1 = m_1 \bar{v}_{1f}$   
 $= 5 \text{ kg}(5.2 \text{ m/s})$   
 $= 26 \text{ kg m/s}$   
 $\bar{v}_{2f}$  = Velocity of the 3-kg iron ball after collision

Final momentum of the 3-kg iron ball =  $\vec{p}'_2$

$$\begin{aligned} \vec{p}'_2 &= m_2 \bar{v}_2 \\ &= 3(3.5) \\ &= 10.5 \text{ kg m/s} \end{aligned}$$

In a collision, the total momentum of the system remains conserved in all directions. A vector triangle is constructed using the fact that the total momentum of both the balls is conserved.



$$\therefore \vec{p}_1 + \vec{p}_2 = \vec{p}'_1 + \vec{p}'_2$$

Using the cosine law,

$$\begin{aligned} (26)^2 &= (10.5)^2 + (30)^2 - 2(10.5)(30)\cos\theta \\ \therefore \cos\theta &= \frac{(10.5)^2 + (30)^2 - (26)^2}{2(10.5)(30)} \\ &= 0.530 \\ \theta &= \cos^{-1}(0.53) \\ \theta &\approx 58^\circ \end{aligned}$$

**16. G**

The definition of the mechanical energy of an object is the sum of its kinetic and potential energies.

**17. A**

After an inelastic collision, the K.E. of both satellites = 0, and the P.E. of the combined mass

$$= -\frac{GM(2m)}{r} = -\frac{2GMm}{r}$$

Therefore, the total energy =  $-\frac{2GMm}{r}$ .

**18. F**

When the kinetic energy of a body decreases, the work done must be against the direction of motion of the body. Thus, the work done is negative.

19. D

The escape velocity  $(v_c) = \sqrt{\frac{2GM}{R}}$

For a satellite of mass  $m$ , travelling around Earth in an orbit of radius  $r$ , the velocity is given by:

$$v = \sqrt{\frac{GM}{r}}$$

$$V = \frac{1}{2} v_c = \sqrt{\frac{GM}{2R}}$$

$$\Rightarrow \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{2R}}$$

$$\Rightarrow r = 2R$$

The height of the satellite above Earth's surface is:

$$h = r - R$$

$$= 2R - R$$

$$= R$$

$$= 6400 \text{ km}$$

20. 19200

It is first necessary to calculate the new velocity of the satellite orbiting Earth.

$$v_E = \sqrt{\frac{2GM_E'}{r_E}}, \text{ but the new mass of Earth is } M_E' = 2M_E.$$

Now the velocity with which the satellite is orbiting Earth may be calculated.

$$v_s = \frac{v_E}{4} = \frac{\sqrt{\frac{4GM_E}{r_E}}}{4} = \sqrt{\frac{GM_E}{4r_E}}$$

Equating the force due to gravity and the centripetal force it is possible to calculate the height above Earth's centre of mass that the satellite is orbiting.

$$F_g = F_c$$

$$\frac{Gm_s M_E}{r^2} = \frac{m_s v_s^2}{r}$$

$$\Rightarrow r = \frac{GM_E}{v_s^2} = \frac{GM_E}{\frac{GM_E}{4r_E}} = 4r_E$$

So the satellite is orbiting Earth at a distance of four Earth radii from the planet's centre of mass. Therefore, the satellite is travelling at a height of  $3r_E$ , or 19 200 km, above Earth's surface.

21. Open Response

Suppose the mass of each disc is  $m$ , and the coefficient of kinetic friction between the discs and the surface is  $\mu$ . Let the velocity of disc  $A$  at its position just before the collision be  $u$ . If the striker mechanism is reliable, this is the same when disc  $B$  is used and when it is not. The magnitude  $P$  is the distance covered from this position until  $A$  comes to rest due to kinetic friction. The force of kinetic friction on  $A$  is given by  $F_{k \text{ fr}} = \mu F_N = \mu mg$  and produces a negative acceleration on it.

According to Newton's second law, the acceleration can be found as

$$\vec{a} = -\frac{F_N}{m} = -\vec{g}\mu.$$

Using this value for the deceleration due to the kinetic friction in the equation of motion,

$$v^2 = u^2 + 2ad,$$

you get

$$0 = u^2 - 2g\mu P$$

because the velocity of the disc is  $v = 0$ .

Thus, the magnitude of the initial velocity of disc  $A$  is given as

$$u = \sqrt{2g\mu P}.$$

Similarly, if the velocities of  $A$  and  $B$  just after the collision are  $v_A$  and  $v_B$ , their magnitudes are

$$v_A = \sqrt{2g\mu Q} \text{ and } v_B = \sqrt{2g\mu R}.$$

The conservation of momentum during the collision implies that the total vector momentum is the same before and after the collision:

$$m\vec{u} + 0 = m\vec{v}_A + m\vec{v}_B.$$

After cancelling  $m$ , this gives

$$\vec{u} = \vec{v}_A + \vec{v}_B.$$

Geometrically this means that the vectors  $\vec{u}$ ,  $\vec{v}_A$ , and  $\vec{v}_B$  form the sides of a triangle.

Since  $\vec{u}$ ,  $\vec{v}_A$ , and  $\vec{v}_B$  have the same directions as  $\vec{P}$ ,  $\vec{Q}$ , and  $\vec{R}$ , and magnitudes  $\sqrt{2g\mu P}$ ,  $\sqrt{2g\mu Q}$ , and  $\sqrt{2g\mu R}$ , respectively, it follows that the vectors  $\vec{L}$ ,  $\vec{M}$ , and  $\vec{N}$ , being of the same proportion, will also form the sides of a triangle.

22. 500

$$\vec{F} = kx$$

$$k = \frac{\vec{F}}{x} = \frac{75.0 \text{ N}}{0.150 \text{ m}}$$

$$k = 500 \text{ N/m}$$

23. 0.051

The sum of the potential energy and the kinetic energy remains a constant.

$$mgh + \frac{1}{2}mv^2 = \text{constant}$$

$$\Rightarrow h = \frac{\text{constant} - v^2}{2g}$$

When experimental values approximately satisfying this equation are plotted with  $h$  along the vertical axis, and  $v^2$  along the horizontal axis, the slope is just the proportionality factor:

$$-\frac{1}{2g} = -\frac{1}{2 \times 9.8 \text{ m/s}^2} = -0.051 \text{ s}^2/\text{m}.$$

**24. Open Response**

A pendulum clock uses the oscillations of a pendulum, or a mass at the end of a rod, to measure the passage of time. A pendulum left oscillating by itself will eventually lose its mechanical energy, due to friction and air resistance, and come to a stop. An external source is needed to keep the pendulum oscillating and to supply energy to any other moving parts, like the hands of the clock face. The source of energy most commonly used in a pendulum clock is a coil spring that can be wound manually. When the coil is wound, the work done in winding it is stored as elastic potential energy. This elastic potential energy is then slowly converted to kinetic energy in the clock through a system of gears and axles connected to the moving parts of the clock.

It is important that the energy is released in steps synchronized with the oscillations of the pendulum. This is ensured by a part called the escapement, which allows a gear to move only on each successive swing of the pendulum. At the same time, the escapement transfers energy to the pendulum to keep it going. In pendulum clocks this is the part of the clock that causes the typical ticking sound.

**25. A**

Work must be done against the field in both scenarios. When a positive charge moves toward a negative charge or surface, no work is done. As well, the fields in both scenarios need to be uniform. The electric field around a point charge is not uniform. An electric field between two parallel plates is uniform. The scenario that is similar to the increase in gravitational energy is the scenario in which a negatively charged sphere is lifted toward the negative plate.

**26. Open Response**

With both laws, the force experienced by the objects decreases by the inverse square of the radius of the distance between the objects. The electrostatic force described in Coulomb's Law can be either attractive or repulsive, while the force of gravity can only be attractive.

**27. C**

Newton's universal law of gravitation states that

$$\vec{F}_g = \frac{Gm_1m_2}{r^2},$$

where,

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$\vec{F}_g = \frac{6.67 \times 10^{-11} \times 2.5 \times 10^3 \times 5.0 \times 10^3}{(1.5 \times 10^2)^2} \text{ N}$$

$$\vec{F}_g = 3.7 \times 10^{-8} \text{ N}$$

**28. G**

An electric field has been defined in terms of the electrical force acting on a positive test charge placed in various locations within the field.

**29. 28.1**

$$V = \frac{\Delta E_p}{q} = \frac{4.50 \times 10^{-18} \text{ J}}{1.60 \times 10^{-19} \text{ C}}$$

$$V = 28.1 \text{ V}$$

**30. 2,2**

An electric field line gives the direction that an imaginary positive test charge would travel when released and free to move.

Since the field lines point toward X and Y, both X and Y must themselves be negative charges that attract positive charges.

**31. A**

As the electric potential on all the points on the conductor is the same, the electric field inside a hollow charged conductor is zero.

**32. 7,5,9,4**

In this question, assume that the protons remain in a perfectly circular path as they are deflected by Earth's magnetic field. This means that the net force on the protons is zero; therefore:

$$\vec{F}_{\text{centripetal}} = \vec{F}_{\text{magnetic}}$$

$$\frac{m\vec{v}^2}{r} = q\vec{v}B_{\perp}$$

$$r = \frac{m\vec{v}}{qB_{\perp}}$$

$$= \frac{(1.67 \times 10^{-27} \text{ Kg})(8.00 \times 10^5 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(1.10 \times 10^{-7} \text{ T})}$$

$$r = 7.59 \times 10^4 \text{ m}$$

The values of *a*, *b*, *c*, and *d* are 7, 5, 9 and 4.

**33. Open Response**

Charges on conducting objects will accumulate at points when moved by induction due to another charge, such as the charge that builds up in the clouds during an electrical storm. On lightning rods, this creates a stronger electric field at the tip than in the surrounding areas, allowing charge to either dissipate through the tip into the atmosphere, or attracting the lightning to itself and sending it to the ground instead of allowing it to strike the building the lightning rod is protecting.

**34. F**

Statement 1 is a description of Oersted's accidental discovery that electric currents produce magnetic fields. Statement 2 is a description of Rutherford's discovery of the orbital nature of atomic charges. Statement 3 is a description of the most important aspect of Einstein's explanation of the photoelectric effect.

35. C

When a light wave is incident on a medium, it spreads into its spectral components of different wavelengths. This phenomenon is known as dispersion.

**Distractor Rationale**

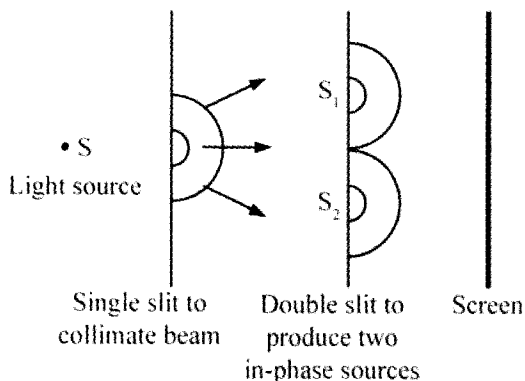
D. Diffraction is the spreading of light waves around an edge.

A. The redistribution of light energy caused by the superposition of waves is known as interference.

B. The redirection of the plane of vibrations of a transverse wave is known as the polarization of a wave.

36. Open Response

Young's double-slit experiment used the following set up:



37. A

The appearance of a circular concentric series of dark and bright rings shows that there is a rotational symmetry in the given system. The light rays diffract at each point on the circumference of the circular hole. The convex lens is also symmetric, causing the diffracted rays to converge in concentric circles. As a result of the rotational symmetry of the aperture and the convex lens, a concentric series of dark and bright rings is formed in the circular aperture diffraction experiment.

**Distractor Rationale**

C. The circular aperture diffraction experiment does not have linear or translational symmetry.

B. The light is diffracted at the hole in the experiment. The circular lens causes the light to converge at certain points on the screen, forming the bright rings in the given experiment.

D. The formation of alternate bright and dark circles in the circular aperture diffraction experiment does not depend on the distance between the aperture and the screen.

38. 8.46

$$\lambda = \frac{d \sin \theta}{n}$$

$$\sin \theta = \frac{n\lambda}{d}$$

$$= \frac{(2)(3.75 \times 10^{-7} \text{ m})}{5.10 \times 10^{-6} \text{ m}}$$

$$\sin \theta = 0.147$$

$$\theta = 8.46^\circ$$

39. D

Since a strand of hair is much longer than it is wide, the light from the laser beam would only be expected to exhibit a diffraction pattern in one dimension. Since the hair is being held out horizontally, the diffraction pattern would be expected to spread out in a vertical direction. This corresponds to the direction in which the width of the hair is positioned.

40. Open Response

$$n\lambda = \frac{xd}{l} \Rightarrow x = \frac{n\lambda l}{d}$$

$$n = 3$$

$$\lambda = 632 \text{ nm}$$

$$l = 2.00 \text{ m}$$

In order to calculate  $d$ , take the reciprocal of 150 lines per mm.

$$d = \frac{1 \times 10^{-3} \text{ m}}{150 \text{ lines}} = 6.67 \times 10^{-6} \text{ m}$$

$$x = \frac{(3)(632 \times 10^{-9} \text{ m})(2.00 \text{ m})}{6.67 \times 10^{-6} \text{ m}}$$

$$x = 0.569 \text{ m} = 5.69 \times 10^{-1} \text{ m}$$

The third-order maximum is expected to appear 0.569 m away from the centre of the central maximum.

41. Open Response

Blue light is a high frequency, visible light wave; while red light is a low frequency visible light wave.

42. Open Response

Infrared radiation is the electromagnetic radiation associated with heat. A thermal imaging device images the infrared radiation that an object emits.

43. D

The mass defect is the difference in the masses between the bound atom and its constituent parts.

**44. H**

Alpha ( $\alpha$ ) particles have very little penetrating power. When a person is exposed to  $\alpha$ -emission, the particles cannot easily penetrate the skin and thus do not cause harm. If a person inhales or swallows  $\alpha$ -particles, however, the particles' low penetrating power causes them to become trapped inside the body, resulting in enormous damage due to their high mass and strong absorption. Alpha particles are the most destructive form of ionizing radiation.

**45. Open Response**

$$E_n = \frac{E_1}{n^2} = \frac{-25.5 \text{ eV}}{16} = -1.59 \text{ eV}$$

$$r_n = n^2 r_1 = (16)(6.50 \times 10^{-11} \text{ m})$$

$$r_4 = 1.04 \times 10^{-9} \text{ m}$$

The energy of an electron in the fourth energy level of this atom is  $-1.59 \text{ eV}$ , with a radius of  $1.04 \times 10^{-9} \text{ m}$ .

Next, calculate the change in energy as the electron drops from the fourth energy level to its ground state.

$$\Delta E = E_4 - E_1 = (-1.59 \text{ eV}) - (-25.5 \text{ eV})$$

$$\Delta E = 23.9 \text{ eV}$$

Now, since  $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$  you can calculate the energy that was released as a photon in joules.

$$23.9 \text{ eV} \left( \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) \\ = 3.82 \times 10^{-18} \text{ J}$$

Next, calculate the wavelength of a photon given its energy.

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} \\ = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{3.82 \times 10^{-18} \text{ J}}$$

$$\lambda = 5.20 \times 10^{-8} \text{ m} = 52.0 \text{ nm}$$

The wavelength of the released photon due to a transition from the fourth energy state to the ground state of its valence electron is  $52.0 \text{ nm}$ .

**46. Open Response**

This is a simple case in which all that needs to be done is to swap the value of  $c$  for the  $100 \text{ km/h}$ .

$$L' = \frac{L_0}{\gamma} = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

In this case  $c = 100 \text{ km/h}$ .

$$\therefore L' = (5 \text{ m}) \sqrt{1 - \frac{90^2}{100^2}} = (5 \text{ m})(0.436)$$

$$L' = 2.2 \text{ m}$$

So the car will appear to the observer on the side of the highway as measuring  $2.2 \text{ m}$  long. This would be the same as travelling at  $0.9c$  when the speed of light is the upper velocity limit.

**47. 9,0,1,1**

$$E = mc^2 \\ = (1.0 \times 10^{-5} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \\ = 9.0 \times 10^{11} \text{ J}$$

**48. F**

Protons consist of two up quarks and one down quarks; neutrons consist of two down quarks and one up quark. Quarks are a type of fermion. Electrons are a type of lepton. They are not made up of quarks.

**49. A**

$$n = 4; E_4 = \frac{E_1}{4^2} = \frac{-22.0 \text{ eV}}{16} = -1.38 \text{ eV}$$

$$n = 3; E_3 = \frac{E_1}{3^2} = \frac{-22.0 \text{ eV}}{9} = -2.44 \text{ eV}$$

$$n = 2; E_2 = \frac{E_1}{2^2} = \frac{-22.0 \text{ eV}}{4} = -5.5 \text{ eV}$$

$$n = 1; E_1 = -22.0 \text{ eV}$$

$$E_2 - E_1 = -5.5 \text{ eV} - (-22.0 \text{ eV}) = 16.5 \text{ eV}$$

$$E_3 - E_1 = -2.44 \text{ eV} - (-22.0 \text{ eV}) = 19.6 \text{ eV}$$

$$E_4 - E_1 = -1.38 \text{ eV} - (-22.0 \text{ eV}) = 20.6 \text{ eV}$$

The electron will absorb enough energy to move to energy state three, but not enough to move to energy state four. Since the electron moves between energy levels in an all-or-nothing manner, and cannot get to the fourth level, it will only absorb enough energy to stabilize at the third energy level, and the remaining  $0.4 \text{ eV}$  will be emitted in the form of a low energy photon.

**50. 654.5**

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\frac{1}{\lambda} = (1.10 \times 10^7 \text{ m}^{-1}) \left( \frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\frac{1}{\lambda} = (1.10 \times 10^7 \text{ m}^{-1}) \left( \frac{5}{36} \right)$$

$$\frac{1}{\lambda} = 1.53 \times 10^6 \text{ m}^{-1}$$

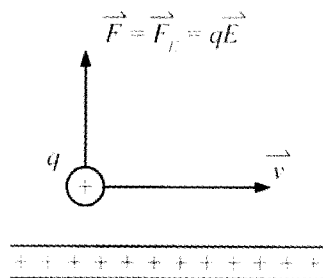
$$\therefore \lambda = 6.545 \times 10^{-7} \text{ m} = 654.5 \text{ nm}$$

The photon emitted from the hydrogen atom represents the first line of the Balmer series. This photon has a wavelength of  $654.5 \text{ nm}$ , which is seen by the human eye as red.

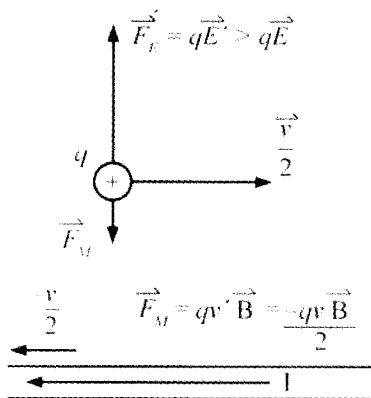
**51. Open Response**

Imagine a small positive charge,  $q$ , moving with a velocity,  $v$ , parallel to a long line of positive charge, whose charge per unit length is  $\lambda$ .

For an observer at rest relative to the line of charge, only the charged particle appears to move. The line of charge produces an electric field around it whose strength depends on the charge per unit length. Here, the particle only interacts with an electric field, which repels it away from the line.



However, for an observer moving with half the velocity of the small charge relative to the line of charge, the line of charge appears to move in the opposite direction with a velocity of  $\frac{v}{2}$ . The line then appears to carry a current in the direction of its relative motion. This current produces a magnetic field around it, which attracts the particle toward the line. As the length of the line contracts in this frame of reference, its charge density increases. This increases the electrostatic repulsion.



Here, the charged particle interacts both with an electric field that is stronger than in the other frame, and a magnetic field that opposes the effect of the electric field. In fact, the two effects balance each other so that the physical effect on the charged particle remains the same. This shows that magnetic forces may be considered a relativistic correction to electrostatic forces.

52. G

Since the particle is moving along a circular path in a magnetic field,

$$\vec{F}_{\text{magnetic}} = \vec{F}_{\text{centripetal}}$$

$$qvB = \frac{mv^2}{r}$$

$$m = \frac{qBr}{v}$$

Since it is a double-ionized particle,  $q = 2e$

$$m = \frac{2eBr}{v} = \frac{2(1.6 \times 10^{-19} \text{ C})(2 \text{ T})(0.5 \text{ m})}{10^7 \text{ m/s}} = 3.2 \times 10^{-26} \text{ kg}$$

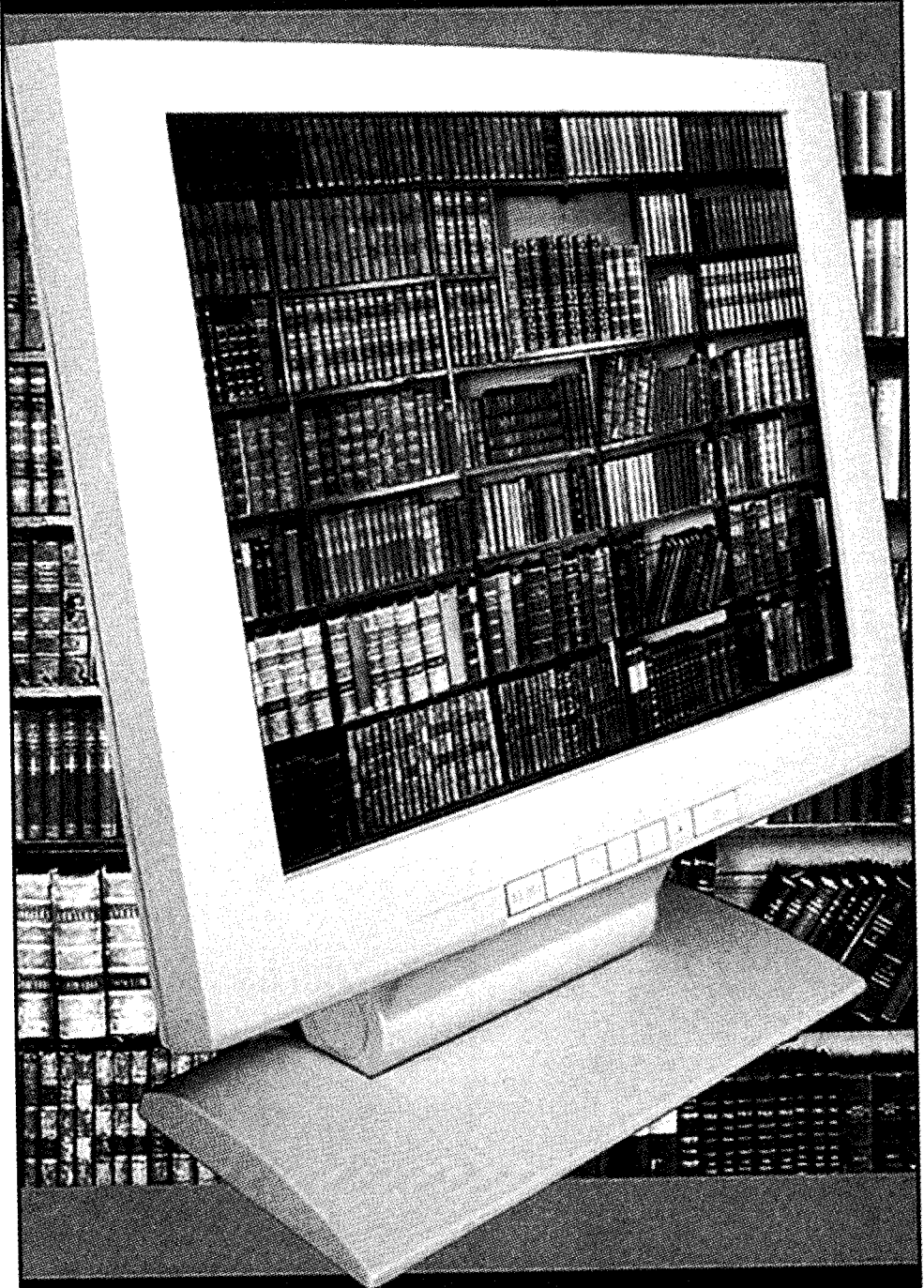
Mass number of the particle

$$= \frac{\text{mass of double-ionized particle}}{\text{mass of proton}} = \frac{3.2 \times 10^{-26} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} = 19.16$$

Since the mass number of fluorine is 19, the particle must be  $\text{F}^{2+}$ .

53. C

Quantum electrodynamics is a field theory of electrons, positrons, and photons. Gluons are carriers of strong nuclear interactions, which involve quarks.



# Appendices

# PHYSICS DATA TABLES

## KINEMATICS

$$\vec{v} = \frac{\vec{d}}{t}$$

$$\vec{F} = m\vec{a}$$

$$\vec{a} = \frac{\vec{v}_k - \vec{v}_i}{t}$$

$$W = Fd$$

$$E_k = \frac{1}{2}mv^2$$

$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$E_p = m\vec{g}h \text{ (gravitational)}$$

$$\vec{d} = \left( \frac{\vec{v}_i + \vec{v}_k}{2} \right) t$$

$$P = \frac{W}{t} \text{ or } \frac{\Delta E}{t}$$

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}d$$

## MOMENTUM AND IMPULSE

$$\vec{p} = m\vec{v}$$

$$\vec{F}t = m\Delta\vec{v}$$

## FORCES AND FIELDS

$$\vec{F}_g = m\vec{g}$$

$$\vec{B} = \mu_0 In$$

$$\vec{F}_c = \frac{mv^2}{r}$$

$$\frac{\vec{F}}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

$$\vec{E} = \frac{kq}{r^2}$$

$$V = \frac{kq}{r}$$

$$\vec{F}_g = \frac{Gm_1 m_2}{r^2}$$

$$\vec{E} = \frac{\vec{F}_c}{q} = \frac{V}{d}$$

$$F_m = B_{\perp} Il$$

$$\vec{g} = \frac{Gm}{r^2}$$

$$\vec{F} = -kx$$

$$\vec{F}_m = q\vec{v}B_{\perp}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}$$

$$E_p = \frac{1}{2}kx^2$$

$$V = \frac{\Delta E}{q}$$

$$\vec{F}_c = \frac{kq_1 q_2}{r^2}$$

$$W = q\vec{E}d$$

## ELECTROMAGNETIC RADIATION

$$c = \lambda f$$

$$f = \frac{1}{T}$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

$$\lambda = \frac{dx}{nl}$$

$$\lambda = \frac{d \sin \theta}{n}$$

$$\sin \theta = \frac{m\lambda}{w} \text{ (for a min)}$$

$$\sin \theta = \frac{\left(m + \frac{1}{2}\right)\lambda}{w} \text{ (for a max)}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$E = hf$$

$$E = \frac{hc}{\lambda}$$

$$E_{k \text{ max}} = hf - W$$

$$W = hf_0$$

$$E_{k \text{ max}} = qV_{\text{stop}}$$

$$\vec{p} = \frac{h}{\lambda} = \frac{hf}{c}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

## ATOMIC PHYSICS

$$E = mc^2$$

$$E = hf$$

$$E = \frac{hc}{\lambda}$$

$$E_{k \text{ max}} = hf - W$$

$$W = hf_0$$

$$E_{k \text{ max}} = qV_{\text{stop}}$$

$$n\lambda = 2\pi r_n$$

$$E_n = \frac{E_1}{n^2}$$

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$r_n = r_1 n^2$$

$$\lambda = \frac{h}{mv}$$

$$\vec{p} = h\lambda = \frac{hf}{c}$$

$$N = N_0 \left( \frac{1}{2} \right)^n$$

## SPECIAL RELATIVITY

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L' = \frac{L_0}{\gamma} = \left( \sqrt{1 - \frac{v^2}{c^2}} \right) L_0$$

$$\Delta t' = \gamma \Delta t = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

## CONSTANTS

Acceleration due to gravity near Earth	$g = 9.81 \text{ m/s}^2$
Gravitational field near Earth	$g = 9.81 \text{ N/kg}$
Gravitational constant	$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$
Index of refraction (air)	$n = 1.00$
Speed of light (air or vacuum)	$c = 3.00 \times 10^8 \text{ m/s}$
Coulomb's constant	$k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$
Elementary charge	$e = 1.60 \times 10^{-19} \text{ C}$
Electron volt	$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$
Hydrogen atom (Bohr model)	
1st orbit radius	$r_1 = 5.29 \times 10^{-11} \text{ m}$
1st orbit energy	$E_1 = 2.18 \times 10^{-18} \text{ J (13.6 eV)}$
Planck's constant	$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$
Rydberg's constant for hydrogen	$R_H = 1.10 \times 10^7 \text{ m}^{-1}$

<b>Particles</b>	<b>Rest Mass</b>	<b>Charge</b>
alpha particle	$6.65 \times 10^{-27} \text{ kg}$	$3.20 \times 10^{-19} \text{ C}$
electron	$9.11 \times 10^{-31} \text{ kg}$	$-1.60 \times 10^{-19} \text{ C}$
neutron	$1.67 \times 10^{-27} \text{ kg}$	0
proton	$1.67 \times 10^{-27} \text{ kg}$	$1.60 \times 10^{-19} \text{ C}$

**PREFIXES**

<b>Name</b>	<b>Symbol</b>	<b>Multiplier</b>	<b>Name</b>	<b>Symbol</b>	<b>Multiplier</b>
yotta-	Y	$10^{24}$	deci-	d	$10^{-1}$
terra-	T	$10^{12}$	centi-	c	$10^{-2}$
giga-	G	$10^9$	milli-	m	$10^{-3}$
mega-	M	$10^6$	micro-	$\mu$	$10^{-6}$
kilo-	k	$10^3$	nano-	n	$10^{-9}$
hecto-	h	$10^2$	pico-	p	$10^{-12}$
deca-	da	$10^1$	femto-	f	$10^{-15}$

# Periodic Table of the Elements

## KEY

1	<b>H</b> 1.01 hydrogen	28	<b>Ni</b> 58.71 nickel
3	<b>Li</b> 6.94 lithium	4	<b>Be</b> 9.01 beryllium
11	<b>Na</b> 22.99 sodium	12	<b>Mg</b> 24.31 magnesium

## LEGEND

METALS

NON-METALS

METALLOIDS

2	<b>He</b> 4.00 helium	10	<b>Ne</b> 20.17 neon	18	<b>Ar</b> 39.95 argon	36	<b>Kr</b> 83.80 krypton	54	<b>Xe</b> 131.30 xenon	86	<b>Rn</b> (222) radon																						
5	<b>B</b> 10.81 boron	6	<b>C</b> 12.01 carbon	7	<b>N</b> 14.01 nitrogen	8	<b>O</b> 16.00 oxygen	9	<b>F</b> 19.00 fluorine	16	<b>S</b> 32.06 sulfur	34	<b>Se</b> 78.96 selenium	52	<b>Te</b> 127 tellurium	84	<b>Po</b> (209) polonium																
13	<b>Al</b> 26.98 aluminum	14	<b>Si</b> 28.09 silicon	15	<b>P</b> 30.97 phosphorus	16	<b>S</b> 32.06 sulfur	17	<b>Cl</b> 35.45 chlorine	32	<b>Ge</b> 72.59 germanium	33	<b>As</b> 74.92 arsenic	34	<b>Se</b> 78.96 selenium	35	<b>Br</b> 79.90 bromine	53	<b>I</b> 126.90 iodine	85	<b>At</b> (210) astatine												
19	<b>K</b> 39.10 potassium	20	<b>Ca</b> 40.08 calcium	21	<b>Sc</b> 44.96 scandium	22	<b>Ti</b> 47.90 titanium	23	<b>V</b> 50.94 vanadium	24	<b>Cr</b> 52.00 chromium	25	<b>Mn</b> 54.94 manganese	26	<b>Fe</b> 55.85 iron	27	<b>Co</b> 58.93 cobalt	28	<b>Ni</b> 58.71 nickel	29	<b>Cu</b> 63.55 copper	30	<b>Zn</b> 65.38 zinc	49	<b>In</b> 114.82 indium	50	<b>Sn</b> 118.69 tin	81	<b>Tl</b> 204.37 thallium	82	<b>Pb</b> 207.19 lead		
37	<b>Rb</b> 85.47 rubidium	38	<b>Sr</b> 87.62 strontium	39	<b>Y</b> 88.91 yttrium	40	<b>Zr</b> 91.22 zirconium	41	<b>Nb</b> 92.91 niobium	42	<b>Mo</b> 95.94 molybdenum	43	<b>Tc</b> 98.91 technetium	44	<b>Ru</b> 101.07 ruthenium	45	<b>Rh</b> 102.91 rhodium	46	<b>Pd</b> 106.40 palladium	47	<b>Ag</b> 107.87 silver	48	<b>Cd</b> 112.41 cadmium	80	<b>Hg</b> 200.59 mercury	83	<b>Bi</b> 208.98 bismuth	84	<b>Po</b> (209) polonium				
55	<b>Cs</b> 132.91 cesium	56	<b>Ba</b> 137.33 barium	57	<b>La</b> 138.91 lanthanum	58	<b>Ce</b> 140.12 cerium	59	<b>Pr</b> 140.91 praseodymium	60	<b>Nd</b> 144.24 neodymium	61	<b>Pm</b> (145) promethium	62	<b>Sm</b> 150.35 samarium	63	<b>Eu</b> 151.96 europium	64	<b>Gd</b> 157.25 gadolinium	65	<b>Tb</b> 158.93 terbium	66	<b>Dy</b> 162.50 dysprosium	67	<b>Ho</b> 164.93 holmium	68	<b>Er</b> 167.26 erbium	69	<b>Tm</b> 168.93 thulium	70	<b>Yb</b> 173.04 ytterbium	71	<b>Lu</b> 174.97 lutetium
87	<b>Fr</b> (223) francium	88	<b>Ra</b> 226.03 radium	89	<b>Ac</b> (227) actinium	90	<b>Th</b> 232.04 thorium	91	<b>Pa</b> 231.04 protactinium	92	<b>U</b> 238.03 uranium	93	<b>Np</b> 237.05 neptunium	94	<b>Pu</b> 244 plutonium	95	<b>Am</b> 243 americium	96	<b>Cm</b> 247 curium	97	<b>Bk</b> 247 berkelium	98	<b>Cf</b> 251 californium	99	<b>Es</b> 252 einsteinium	100	<b>Fm</b> 257 fermium	101	<b>Mn</b> 288 mendelevium	102	<b>Nv</b> 289 nobelium	103	<b>Lr</b> 260 lawrencium

---

---

---

---