MBF 3C Foundations for College Mathematics 11 Student Handbook

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Student	Name:	
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Edited by: Scott McEwen

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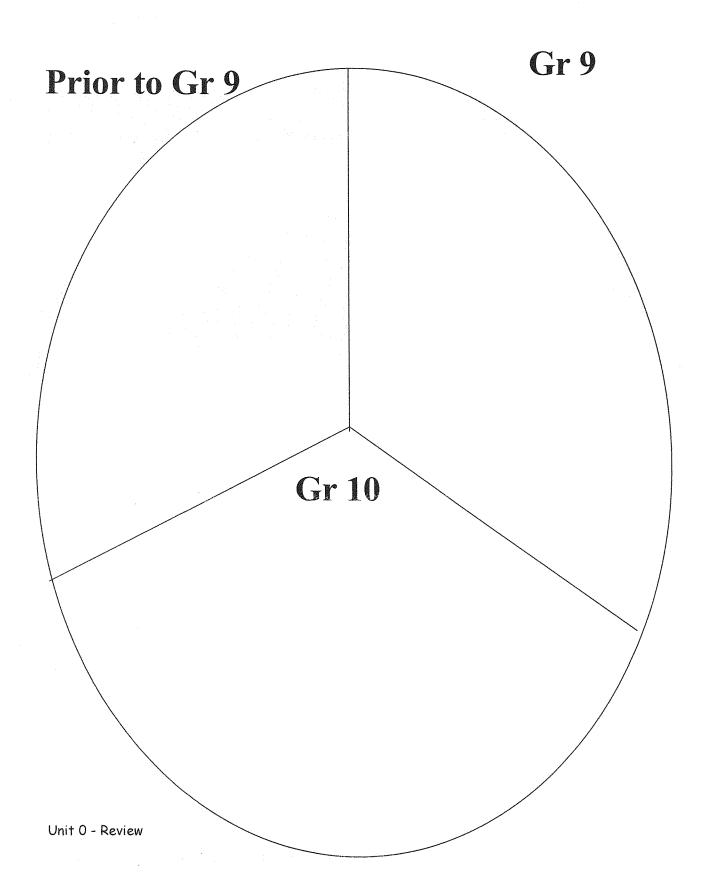
Final Exam

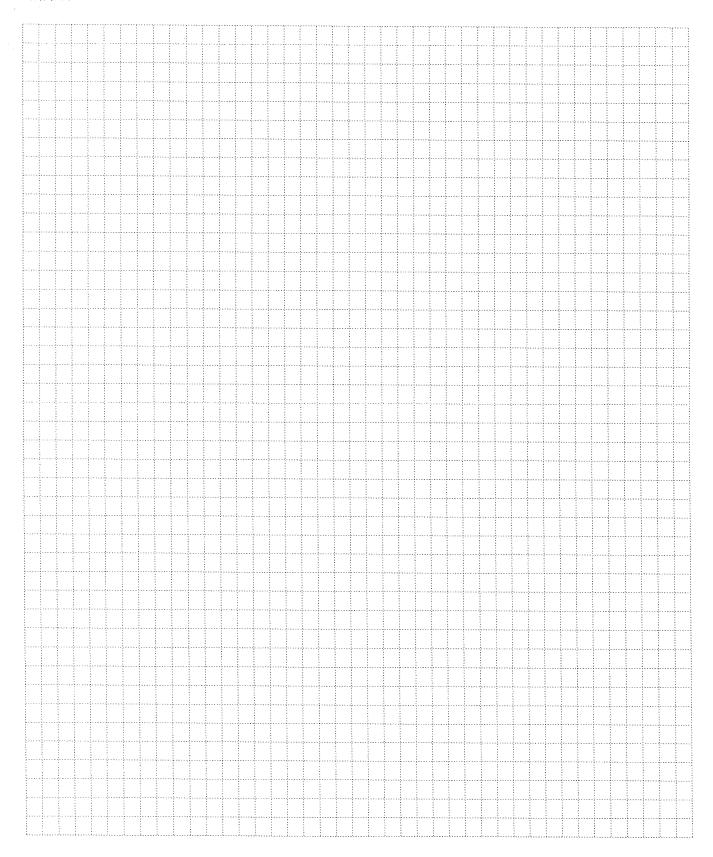
Evaluation Rubric(s)

MBF 3C Unit 0 - Mathematics Review Unit

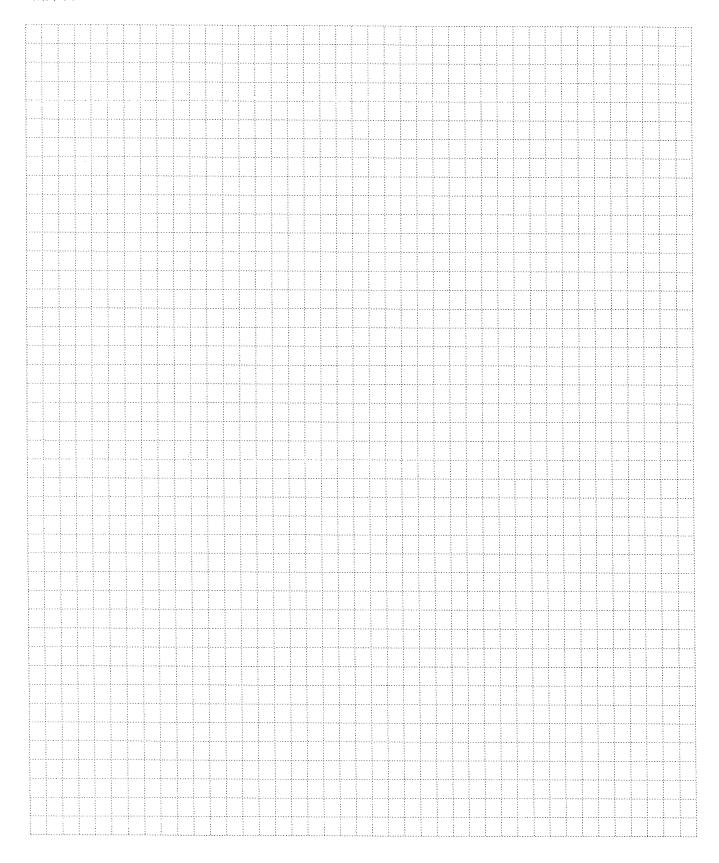
Day	Date	Lesson Title	Handouts	Homework Assigned
1		General Review of Grades 7 - 10	U00A01	U00A03
			U00A02	
2		Review of Ratios	U00A04	Supplemental Worksheet (not provided)
3		Review of Solving Equations	U00A05	Supplemental
			U00A06	Worksheet (not provided)
4		Review of Similar Triangles	U00A07	Supplemental Worksheet (not provided)
5		Flex Period		
TOTAL	DAYS:		J	5

Math I Know!







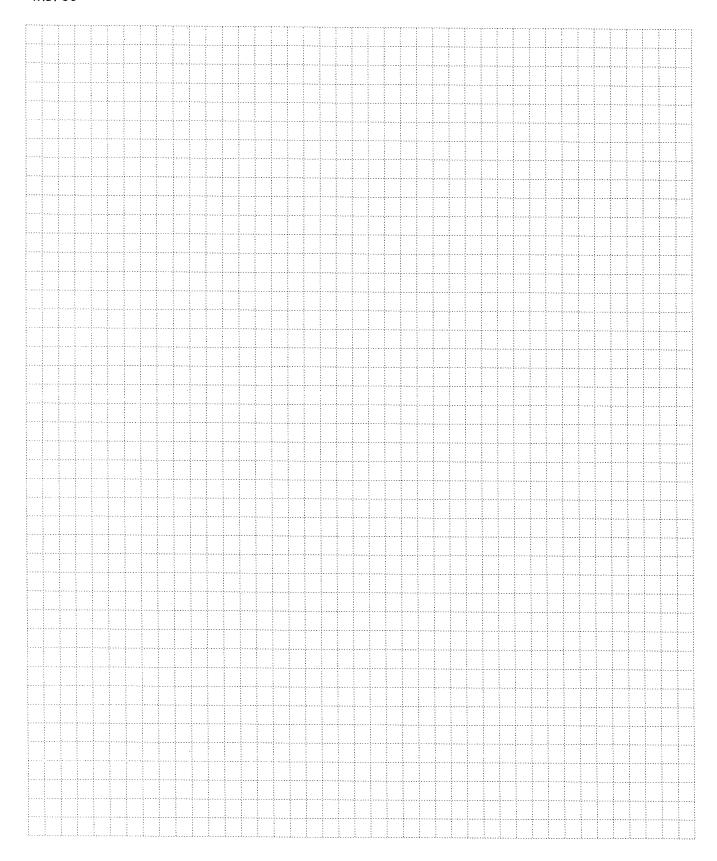


U00A03

Math MATCHING

Match the letter from Column B with the most appropriate number in Column A. Be certain to \underline{show} ALL \underline{work}

	Column A	Column B
1.	Expand and simplify $2(x + 3)$	A. 24.4 m
2.	Use the Pythagorean Theorem to solve for c given a = 8 and b = 6	B. 15.56 m
3.	Expand and simplify $(x + 3)(x + 2)$	C. 2x + 6
4.	Use the Pythagorean Theorem to solve for c given a = 20m and b = 14m	D. 7.5 m
5.	If you build a skateboard ramp whose ratio of height to base must be 2:3, what is the base if the height is 5 m?	E. 17.68 m
6.	A kite is 15 m up in the sky. The angle the string makes with the ground is 50 degrees. How long is the kite string?	F. $x^2 + 5x + 6$
7.	A kite is 10 m up in the sky. The angle the string makes with the ground is 40 degrees. How long is the kite string?	<i>G</i> . 19.58 m
8.	Bob has a kite and his string is 25 m long. The angle the kite makes with the ground is 45 degrees. How far horizontally is the kite from Bob?	H. 10



The Golden Ratio

Station # 1: Investigating The Golden Ratio

On the line segment AB, draw a point C such that the ratio of the short part of the segment AC to the long part CB equals the ratio of the long part CB to the entire segment AB.

Ie:
$$\frac{AC}{BC} = \frac{BC}{AB}$$

Calculate the value of the ratio as a decimal?

A				7

Record your various trials in the table below.

AB	AC	BC	$\frac{AC}{BC}$	$\frac{BC}{AB}$
4.781.6847.6641				

MBF3*C* **U00A04-P2**

Station # 2: Creating a Golden Rectangle

- Step 1 Create a square that is 1 inch by 1 inch.
- Step 2 Add a square of the same size to form a new rectangle.
- Step 3 Continue adding squares whose side lengths are the <u>length of the longer side</u> to form a new rectangle.

Complete the table below recording the lengths of the rectangle and calculate the value of the ratio of the longest side of the rectangle to the shortest side of the rectangle as a decimal.

Diagram	Length of the Longest Side of the Rectangle (L)	Length of the Shortest Side of the Rectangle (S)	L : S (as a decimal)
◆ old • new			

Station # 3: Ratio War

78	r					, 4				
I	1/2	6	tr	H A	0	FI	1	M	C	
-4	98		2#	88	•	z z	₹/	2 Z	L.B	

- Deal from a deck of cards (only Ace to 10 for each suit) to each of the players.
- Each player turns over a card for the first term of the ratio and a card for the second term of the ratio and places them on the mat provided.
- The player with the larger ratio wins the turn.

Player 1 – 1 ^{S1} term of ratio	Player $2 - 1^{ST}$ term of ratio		
1			
	1		
Player 1 – 2 nd term of ratio	Player 2 – 2 nd term of ratio		
Player 1 – 2 nd term of ratio	Player 2 – 2 nd term of ratio		
Player 1 – 2 nd term of ratio	Player 2 – 2 nd term of ratio		
Player 1 – 2 nd term of ratio	Player 2 – 2 nd term of ratio		
Player 1 – 2 nd term of ratio	Player 2 – 2 nd term of ratio		
Player 1 – 2 nd term of ratio	Player 2 – 2 nd term of ratio		
Player 1 – 2 nd term of ratio	Player 2 – 2 nd term of ratio		
Player 1 – 2 nd term of ratio	Player 2 – 2 nd term of ratio		
Player 1 – 2 nd term of ratio	Player 2 – 2 nd term of ratio		
Player 1 – 2 nd term of ratio	Player 2 – 2 nd term of ratio		

Human beauty is based on the Divine Proportion

Station # 4: Circumference of Wrist vs. Circumference of Thumb

- a) Measure the circumference of your wrist in inches.
- b) Measure the circumference of your thumb in centimeters.
- c) What do you notice about the measurements?
- d) What is this ratio?

Station # 5: Width of Nostrils vs. Width of Nose

- a) Measure the width (from the outside of one nostril to the outside of the other) in centimeters.
- b) Measure the width of your nose (at the widest part) in centimeters.
- c) What do you notice about the measurements?
- d) What is this ratio?

Station # 6: Width of Nostrils vs. Distance from Upper Lip to Bottom of Nose

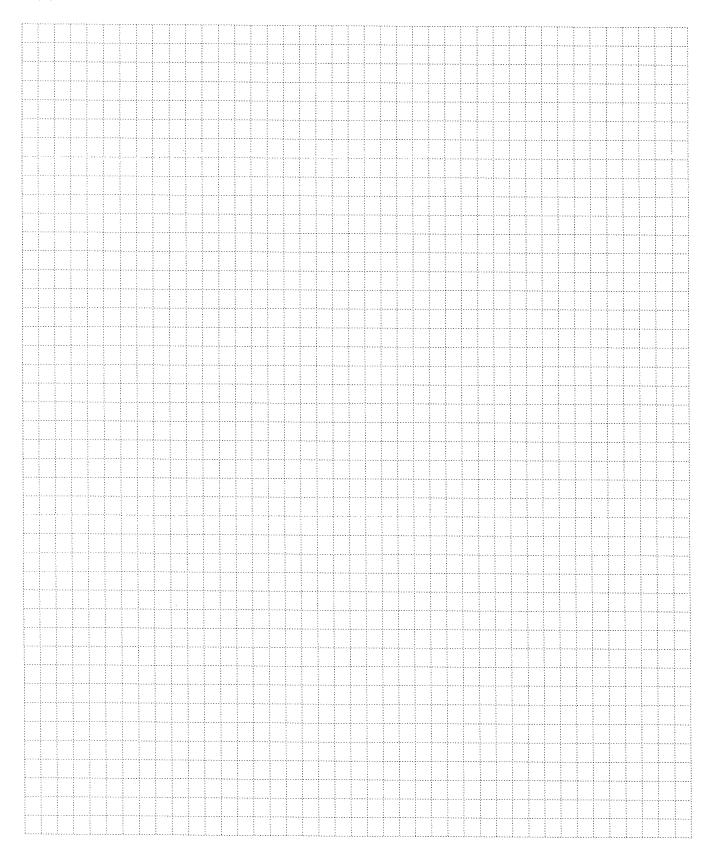
- a) Using the measurement of part (a) from Station # 5
- b) Measure the distance from the top of your upper lip to the bottom of your nose in centimeters.
- c) What do you notice about the measurements?
- d) What is this ratio?

Station # 7: Height of Body vs. Height from your head to the tip of your fingers

- a) Measure how tall you are in centimeters.
- b) Measure the distance from the top of your head to the tip of your fingers when your arms are at your sides (soldier style) in centimeters.
- c) What do you notice about the measurements?
- d) What is this ratio?

Other Interesting Ratios

- Top of your head to your navel vs. Measurement from part (b) in Station # 7
- Top of your head to your navel vs. Width of your shoulders
- Top of your head to your navel vs. Length of your forearm (from the inside of your elbow to your finger tips)
- Length of your forearm vs. Distance from the top of your head to the base of your chin.

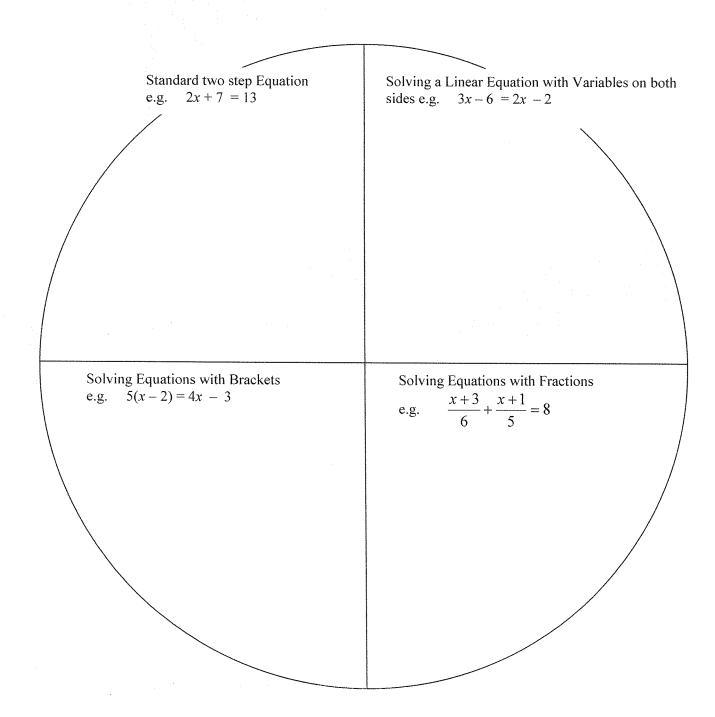


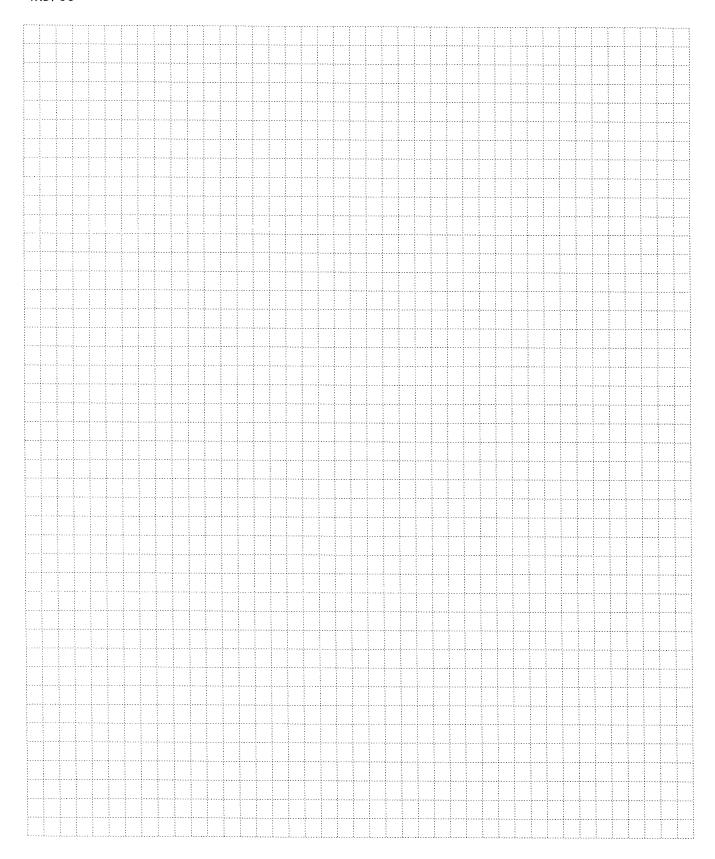
K.W.L. - Chart for Equations

What I \underline{K} now - What I \underline{W} ant to Know - What I have \underline{L} earned

Concept, Term, or Diagram	K What do I KNOW?	W What do I WANT to Know?	L What have I LEARNED?
Equations			

<u>Solve the equations</u>. Indicate on the Concept Circle the methods/steps you used to solve the equations.





Solving and Checking Equations

Solve and Check the following Equations.

1)
$$15 = 23 - 4x$$

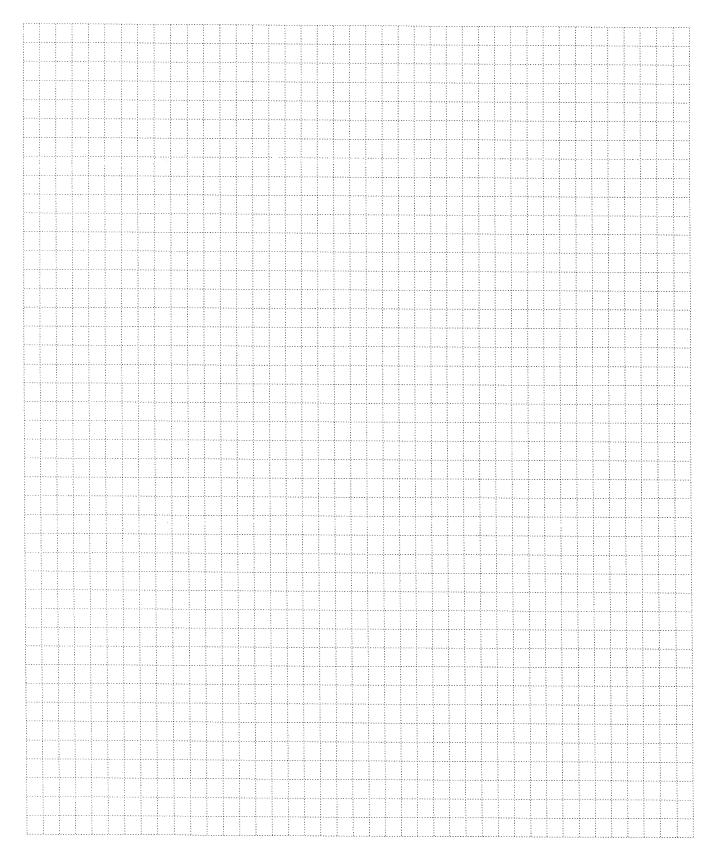
2)
$$3(x-4)=12$$

3)
$$2-5x=-1-4x$$

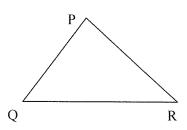
4)
$$3(1-x) = -2(1-x)$$

5)
$$\frac{x}{6} - 5 = \frac{1}{2}x + \frac{1}{3}x - 4$$

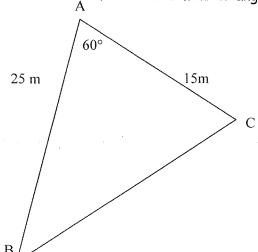
Answers 1.)
$$x = 2$$
 2.) $x = 8$ 3.) $x = 3$ 4.) $x = 1$ 5.) $x = 6$



1. Name the triangle, the sides and the angles.



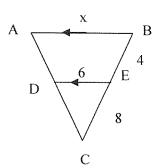
2. Find the value of \boldsymbol{x} in the similar triangles



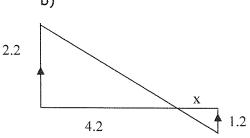
x cm x cm x cm x cm

3. Determine if the triangles are similar, and if they are state how you know this, and find the value of \times .

a)



b)



Answers:

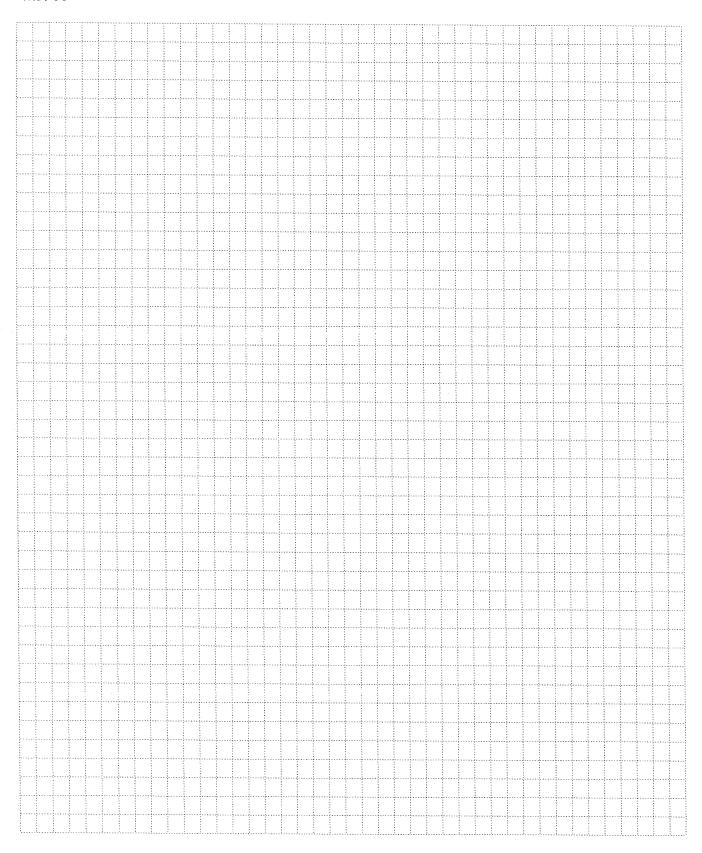
- 1. One such naming is Triangle PQR (remember it doesn't matter which way you name it as long as you are consistent), sides p, q, and r and angles P, Q, and R
- 2. x = 10

3a) x = 9

3b) x = 2.29

MBF3C Unit 1 - Trigonometry

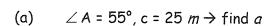
Section	Date	Date Lesson Title		Homework Assigned
1		Review of Pythagorean Theorem Labelling Triangles Primary Trigonometric Ratios		Supplemental Worksheet (not provided)
2		Review Trigonometry - Solving for Sides	U01A01	Supplemental Worksheet (not provided)
3		Review Trigonometry - Solving for Angles	U01A02	Supplemental Worksheet (not provided)
4		Trigonometry in the Real World	U01A03-P1 U01A03-P2	U01A04-P1 U01A04-P2 U01A04-P3
5		Sine Law	U01A05-P1 U01A05-P2	U01A06
6		Cosine Law		U01A07
7		Making Decisions	U01A08	U01A09
8		Real World Problems		U01A10
9		More Real World Problems		U01A11
10		Review Day/Practice Test	U01A14	U01A15
11		Flex Period		
12		Test Day		

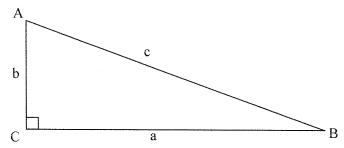


MBF3C U01A01

Solving for Side Lengths in Right Triangles

1. Based on the following diagram (not drawn to scale) use the values given to find the missing/indicated side:





(b)
$$\angle A = 65^{\circ}$$
, $c = 32 \ cm \rightarrow find b$

(c)
$$\angle B = 15^{\circ}$$
, $c = 42 m \rightarrow \text{ find } b$

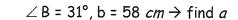
(d)
$$\angle B = 35^{\circ}$$
, $c = 55 \text{ cm} \rightarrow \text{find } a$

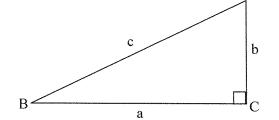
2. Based on the following diagram (not drawn to scale) use the values given to find the missing/indicated side:

(a)
$$\angle A = 75^{\circ}$$
, b = 52 $m \rightarrow$ find a

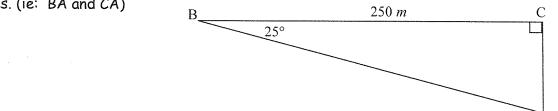
(b)
$$\angle A = 64^{\circ}$$
, $a = 23 \ cm \rightarrow \text{ find } b$

(c)
$$\angle B = 18^{\circ}$$
, $a = 24 m \rightarrow find b$

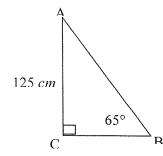




3. Given the following diagram (not drawn to scale) solve for the lengths of the missing sides. (ie: BA and CA)



Given the following diagram (not drawn to scale) solve for the lengths of the missing 4. sides. (ie: AB and CB)



(d)

Solutions: (Note: answers should be within a decimal place depending on accuracy of numbers used.)

1. (a)
$$\alpha = 20.5m$$

(b)
$$b = 13.5cm$$

(c)
$$b = 10.9 m$$

(c)
$$b = 10.9m$$
 (d) $a = 45.1cm$

$$2. (a) a = 194.1m$$

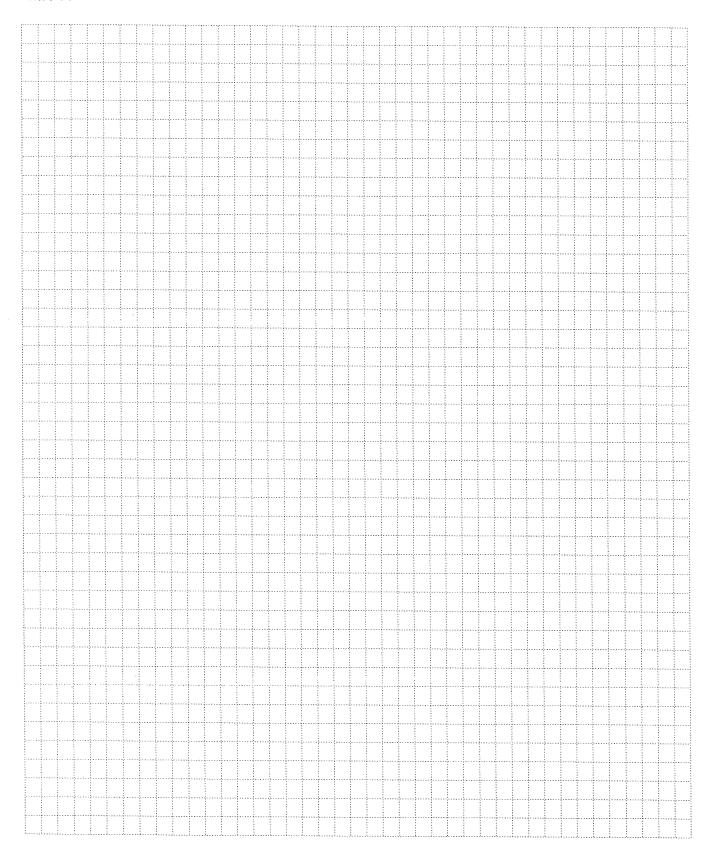
(b)
$$b = 11.2 cm$$

(c)
$$b = 7.8m$$

(d)
$$a = 96.5cm$$

3.
$$b = 116.6m c = 275.8m$$

$$4. a = 58.3cm c = 137.9cm$$



Diagrams are not drawn to scale. Round angle measures to the nearest degree. The side length answers should be rounded to one decimal place.

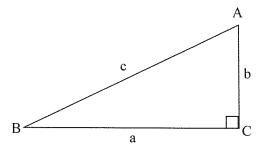
1. Based on the following diagram, use the values given to find the missing/indicated side:

(a)
$$a = 58 \text{ cm}, c = 124 \text{ cm} \rightarrow \text{find } \angle A$$

(b)
$$b = 75 m, c = 215 m \rightarrow find \angle A$$

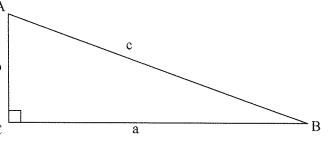
(c)
$$b = 64 m$$
, $c = 225 m \rightarrow find \angle B$

(d)
$$a = 45 \text{ cm}, c = 238 \text{ cm} \rightarrow \text{find } \angle B$$

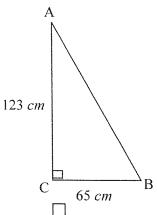


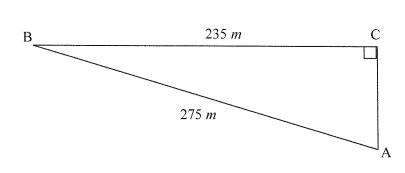
2. Based on the following diagram, use the values given to find the missing/indicated side:

- (a) $a = 55 \text{ cm}, b = 137 \text{ cm} \rightarrow \text{find } \angle A^A$
- (b) $a = 235 m, b = 68 m \rightarrow find \angle A$
- (c) $a = 212 m, b = 100 m \rightarrow find \angle B$
- (d) $\alpha = 30 \text{ cm}, b = 285 \text{ cm} \rightarrow \text{find } \angle B^{C}$



3. Using the diagram below on the left solve for the measure of the missing angles.





4. Using the diagram above on the <u>right</u> solve for both the measure of the missing angles and the length of the missing side.

Solutions:

- 1. (a) $\angle A = 28^{\circ}$
- (b) $\angle A = 70^{\circ}$
- (c) $\angle B = 17^{\circ}$
- (d) $\angle B = 79^{\circ}$

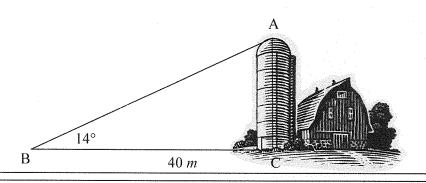
- 2. (a) $\angle A = 22^{\circ}$
- (b) $\angle A = 74^{\circ}$
- (c) $\angle B = 25^{\circ}$
- (d) $\angle B = 84^{\circ}$

- 3. $\angle A = 28^{\circ}, \angle B = 62^{\circ}$
- 4. $\angle A = 59^{\circ}$, $\angle B = 31^{\circ}$, b = 142.8 m

Problem 1:

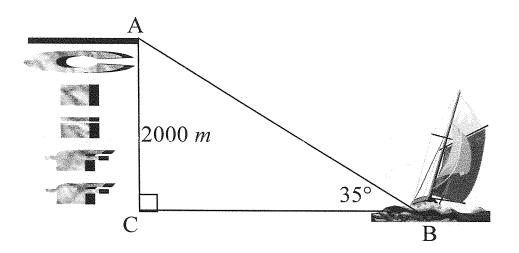
While walking to school you pass a barn with a silo. Looking up to the top of the silo you estimate the angle of elevation to the top of the silo to be about 14° . You continue walking and find that you were around

40 m from the silo. Using this information and your knowledge of trigonometric ratios calculate the height of the silo.



Problem 2:

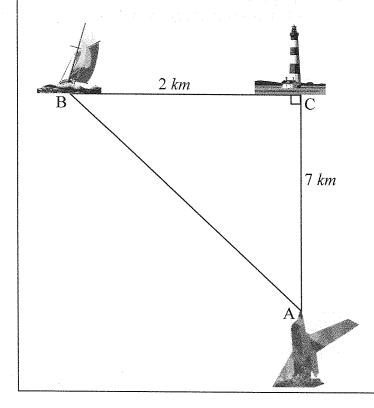
A sailboat is approaching a cliff. The angle of elevation from the sailboat to the top of the cliff is 35° . The height of the cliff is known to be about 2000 m. How far is the sailboat away from the base of the cliff?



Unit 1 Trigonometry

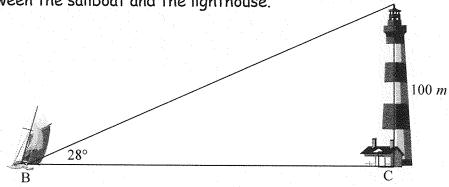
Problem 3:

A sailboat that is 2 km due west of a lighthouse sends a signal to the lighthouse that it is in distress. The lighthouse quickly signals a rescue plane that is 7 km due south of the lighthouse. What heading from due north should the plane take in order to intercept the troubled sailboat?

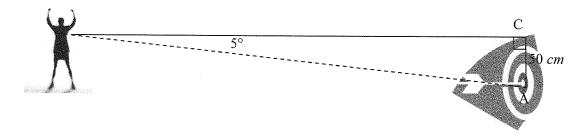


Round \angle 's to whole degrees; length answers should be rounded to 1 decimal place and include units.

1. The top of a lighthouse is 100 m above sea level. The angle of elevation from the deck of the sailboat to the top of the lighthouse is 28°. Calculate the distance between the sailboat and the lighthouse.



2. An archer shoots and gets a bulls-eye on the target. From the archer's eye level the angle of depression to the bulls-eye is 5°. The arrow is in the target 50 cm below the archer's eye level. Calculate the distance the arrow flew to hit the target (the dotted line).



Note: For the following questions you will need to create your own diagrams.

Draw them carefully and refer to the written description to ensure you find the correct solution.

3. Two islands A and B are 3 km apart. A third island C is located due south of A and due west of B. From island B the angle between islands A and C is 33°. Calculate how far island C is from island A and from island B.

4. The foot (bottom) of a ladder is placed 1.5 m from a wall. The ladder makes a 70° angle with the level ground. Find the length of the ladder. (Round to one decimal place.)

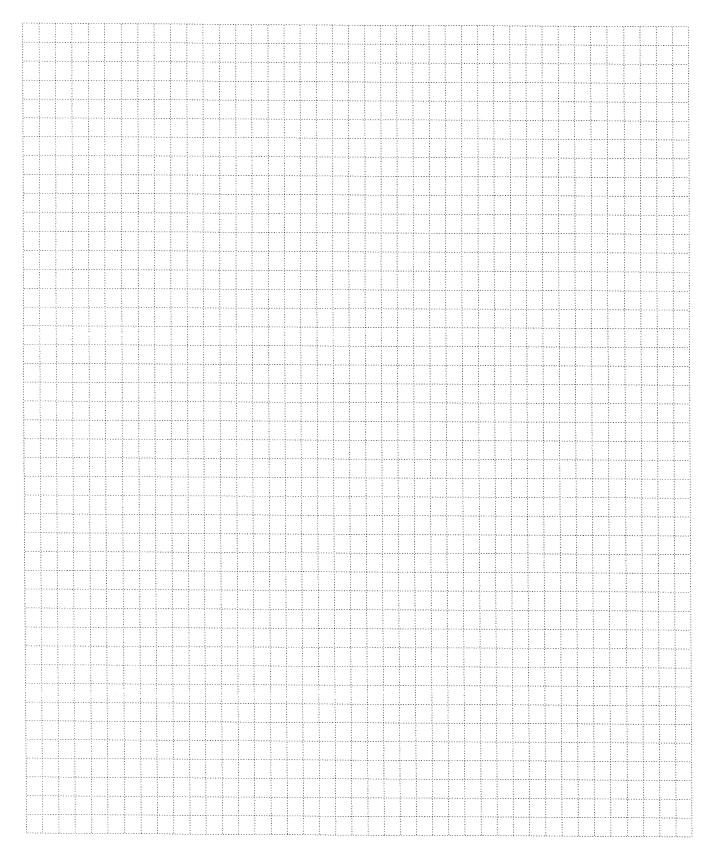
5. A tow truck raises the front end of a car 0.75 m above the ground. If the car is 2.8 m long what angle does the car make with the ground?

6. A construction engineer determines that a straight road must rise vertically 45 m over a 250 m distance measured along the surface of the road (this represents the hypotenuse of the right triangle). Calculate the angle of elevation of the road.

Solutions:

1. 188.1 m 2. 573.7 cm 3. Distance A to C: 1.6 km Distance B to C: 2.5 km

4. 4.4 *m* 5. 16° 6. 10°

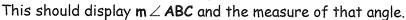


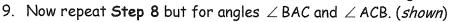
U01A05-P1

Investigation: The Sine Law (using Geometer's Sketchpad)

Load Geometer's Sketchpad.

- Start with a new document (default).
- 2. Select the Straightedge Tool (4th button down the toolbar)
- 3. Draw three lines making sure that each new line starts from a previous line and that the last point returns to the first completing the triangle. (shown right)
- 4. Switch to the selection tool (1st button on the toolbar)
- 5. Select and right-click on each vertex and from the short-cut menu select "Show Label" (also shown right)
- Next select any line and from the Measure menu (or from the right-click short-cut) select "Length".
 This should display m AB (shown)
- 7. Repeat Step 6 for the other lines, making sure to unselect before selecting a new line. (If anything else is selected length may not appear on the menu.)
- 8. Next select in the following order the vertices: A, B then C then click the Measure menu and choose "Angle".

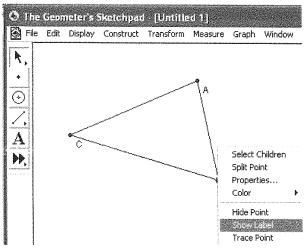


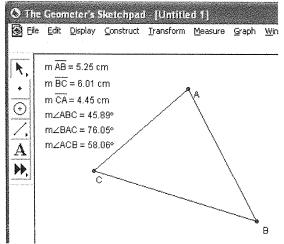


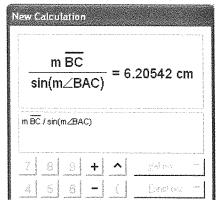
- 10. If you select any point you can drag the point to a new location and all of the measurements update automatically. (You can also select and move an entire line.)
- 11. Try this and adjust the position of the triangle to leave more room below our measurements.
- 12. We will now add some calculations namely the values for the Sine Law:

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

- 13. To do this select the Measure menu and select "Calculate...". A new dialogue box appears
- 14. shown right) where we will enter our calculation.

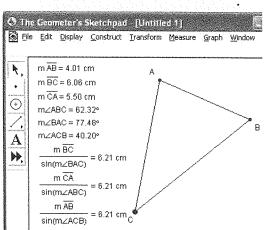




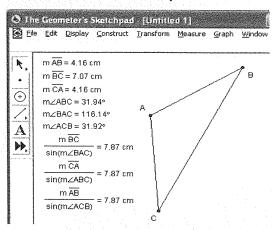


- 15. First click on the measurement for side a (in this case it is m \overline{BC}), then click on the division sign and type "si" for the sine function, next click on the measurement for $\angle A$ (in this case it is m $\angle BAC$ (depending on the size of your triangle you will see different results.) Click OK
- 16. This will add a new measurement to your document, repeat step 15 for side b and side c. For side b use $m \ \overline{CA}$ and $sin(m \angle ABC)$ for side c use $m \ \overline{AB}$ and $sin(m \angle ACB)$. Calculations are shown in the bottom diagrams.
- 17. Now change the position of your vertices; this will change the lengths and angles in your triangle make note of all three of the calculation boxes for the Sine Law.

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$



(two variations shown below)

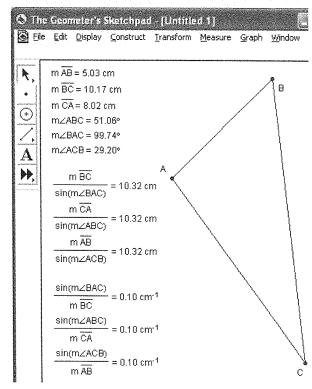


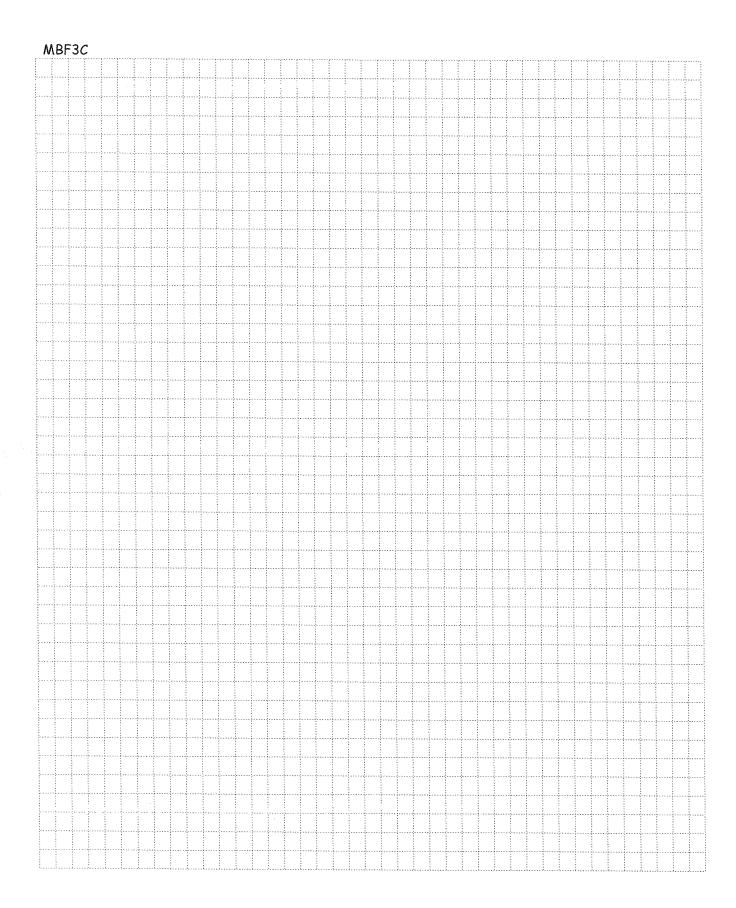
18. Next create three more calculations for the other version of the Sine Law:

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$
 (shown right)

- 19. Experiment with more positions of the triangle vertices.
- 20. Notice that the set of three values in either version of the Sine Law remain the same. This shows that the ratio of any side to the sine of the corresponding angle in a triangle remains equal to the ratio of any other side to the sine of the corresponding angle. Either

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$





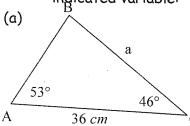
1. Solve for the given variable (correct to 1 decimal place) in each of the following:

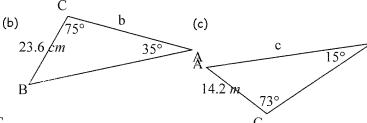
(a)
$$\frac{a}{\sin 35^{\circ}} = \frac{10}{\sin 40^{\circ}}$$

(b)
$$\frac{65}{\sin 75^{\circ}} = \frac{b}{\sin 48^{\circ}}$$

$$\frac{75}{\sin 55^{\circ}} = \frac{c}{\sin 80^{\circ}}$$

2. For each of the following diagrams write the equation you would use to solve for the indicated variable: Γ





(c)

- 3. Solve for each of the required variables from Question #2.
- 4. For each of the following triangle descriptions you should make a sketch and then find the indicated side rounded correctly to one decimal place.
- (a) In $\triangle ABC$, given that $\angle A = 57^{\circ}$, $\angle B = 73^{\circ}$, and AB = 24 cm. Find the length of AC
- (b) In $\triangle ABC$, given that $\angle B = 38^{\circ}$, $\angle C = 56^{\circ}$, and BC = 63 cm. Find the length of AB
- (c) In $\triangle ABC$, given that $\angle A = 50^{\circ}$, $\angle B = 50^{\circ}$, and AC = 27 m. Find the length of AB
- (d) In $\triangle ABC$, given that $\angle A = 23^{\circ}$, $\angle C = 78^{\circ}$, and AB = 15 cm. Find the length of BC
- (e) In $\triangle ABC$, given that $\angle A = 55^{\circ}$, $\angle B = 32^{\circ}$, and BC = 77 cm. Find the length of AC
- (f) In $\triangle ABC$, given that $\angle B = 14^{\circ}$, $\angle C = 78^{\circ}$, and AC = 36 m. Find the length of BC

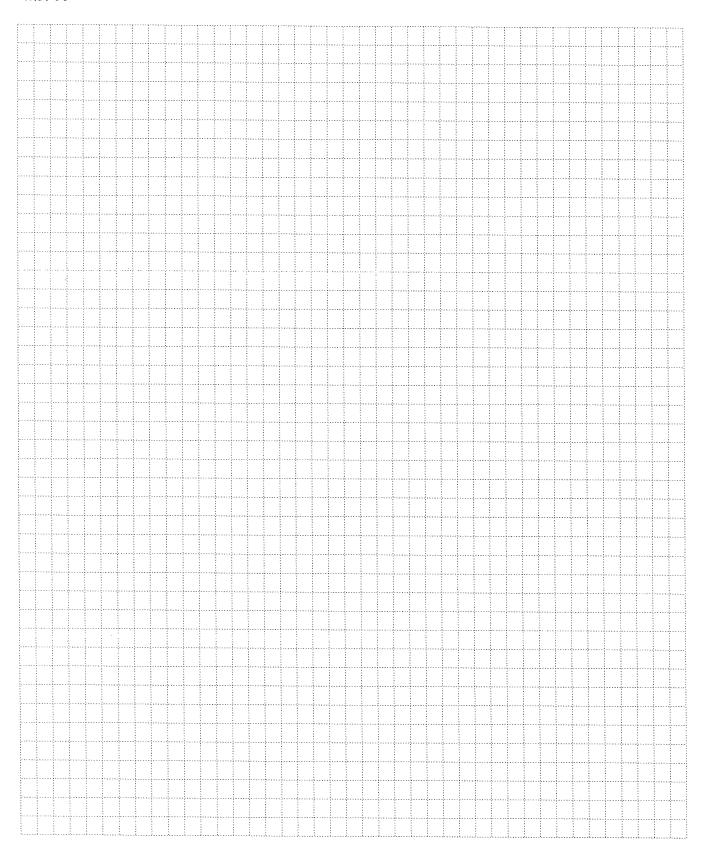
Solutions:

1. (a) 8.9 units (b) 50.0 units (c) 90.2 units

2. (a)
$$\frac{a}{\sin 53^{\circ}} = \frac{36}{\sin 81^{\circ}}$$
 (b) $\frac{23.6}{\sin 35^{\circ}} = \frac{b}{\sin 70^{\circ}}$ (c) $\frac{14.2}{\sin 15^{\circ}} = \frac{c}{\sin 73^{\circ}}$

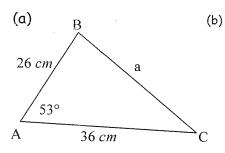
- 3. (a) 29.1 cm (b) 38.7 cm (c) 52.5 m
- 4. (a) 30.0 cm (b) 52.4 cm (c) 34.7 m (d) 6.0 cm (e) 49.8 cm (f) 148.7 m

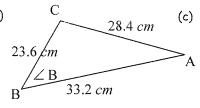
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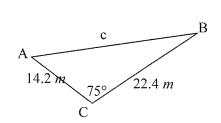


The Cosine Law Homework

1. For each of the following diagrams write the equation you would use to solve for the indicated variable:





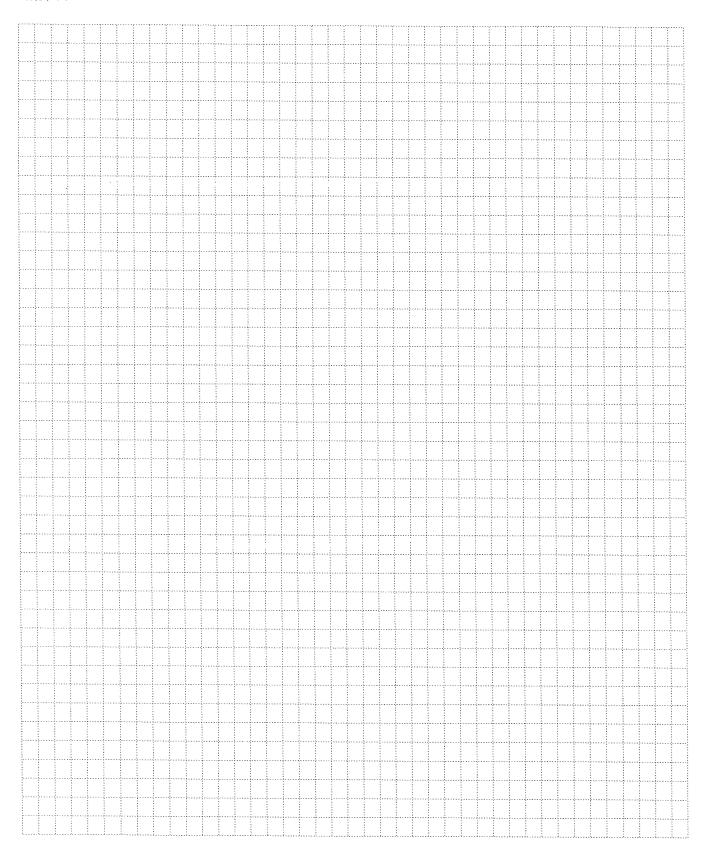


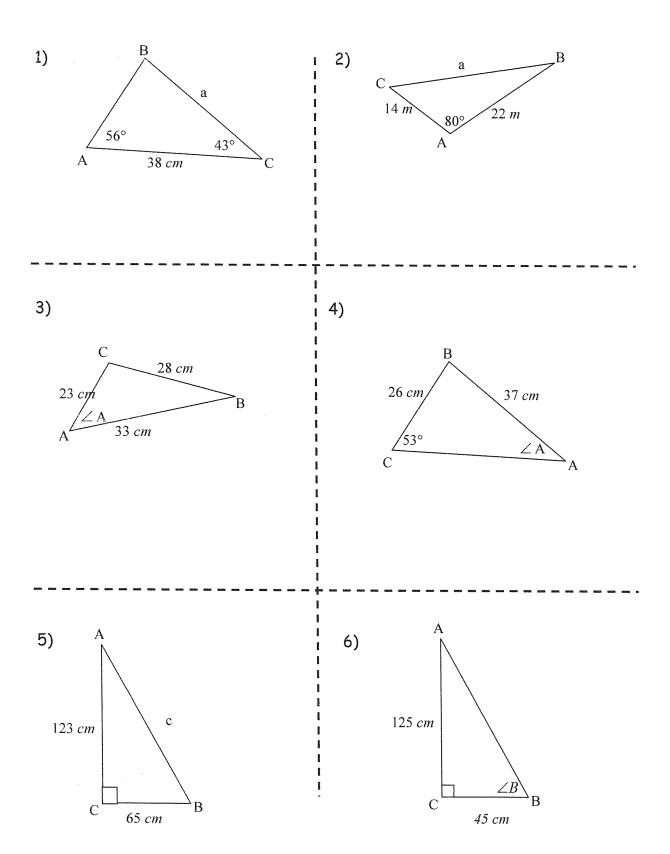
- 2. Solve for each of the required variables from Question #1.
- 3. For each of the following triangle descriptions you should make a sketch and then find the indicated value.
- (a) In $\triangle ABC$, given that AB = 24 cm, AC = 34 cm, and $\angle A = 67^{\circ}$. Find the length of BC
- (b) In $\triangle ABC$, given that AB = 15 m, BC = 8 m, and $\angle B = 24^{\circ}$. Find the length of AC
- (c) In $\triangle ABC$, given that AC = 10 cm, BC = 9 cm, and $\angle C = 48^{\circ}$. Find the length of AB
- (d) In $\triangle ABC$, given that $\angle A = 24^{\circ}$, AB = 18.6 m, and AC = 13.2 m. Find the length of BC
- (e) In $\triangle ABC$, given that AB = 9 m, AC = 12 m, and BC = 15 m. Find the measure of $\angle B$.
- (f) In $\triangle ABC$, given that AB = 18.4 m, BC = 9.6 m, and AC = 10.8 m. Find the measure of $\angle A$.

Solutions:

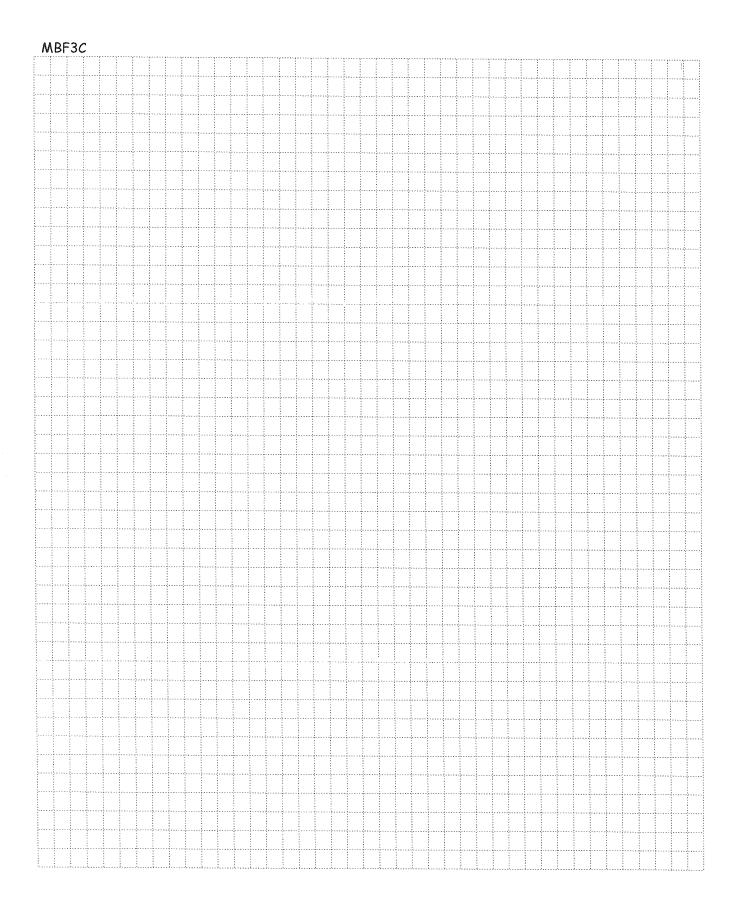
- 1. (a) $a^2 = (36)^2 + (26)^2 2(36)(26) \cdot \cos 53^\circ$
 - (b) $(28.4)^2 = (23.6)^2 + (33.2)^2 2(23.6)(33.2) \cdot \cos \angle B$
 - (c) $c^2 = (22.4)^2 + (14.2)^2 2(22.4)(14.2) \cdot \cos 75^\circ$
- 2. (a) 29.1 cm (b) 57° (c) 23.2 m
- 3. (a) 33.1 cm (b) 8.4 m (c) 7.8 cm (d) 8.5 m (e) 53° (f) 24°

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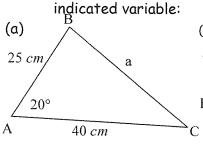


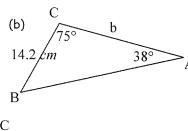
Unit 1 Trigonometry

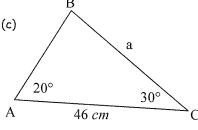


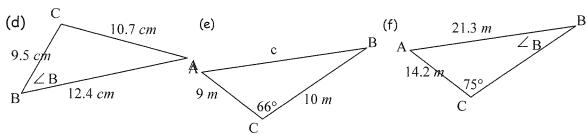
Round \angle 's to whole degrees; length answers should be rounded to 1 decimal place and include units.

For each of the following diagrams write the equation you would use to solve for the 1.









- 2. Solve for each of the required variables from Question #1.
- 3. For each of the following triangle descriptions you should make a sketch and then completely solve each triangle.
- In $\triangle ABC$, given that $\angle A = 38^{\circ}$, $\angle C = 85^{\circ}$, and c = 32 cm. (a)
- In $\triangle ABC$, given that $\angle A = 24^{\circ}$, b = 12.5 m, and c = 13.2 m. (b)
- (c) In $\triangle ABC$, given that a = 17 m, b = 18 m, and c = 26 m.
- (d) In $\triangle ABC$, given that $\angle A = 52^{\circ}$, $\angle B = 47^{\circ}$, and a = 25 m.
- In $\triangle ABC$, given that $\angle B = 43^{\circ}$, $\angle C = 73^{\circ}$, and b = 19 m. (e)
- (f) In $\triangle ABC$, given that a = 32 m, b = 30 m, and c = 28 m.

Solutions:

1. (a)
$$\alpha^2 = (40)^2 + (25)^2 - 2(40)(25) \cdot \cos 20^\circ$$

(b)
$$\frac{14.2}{\sin 38^{\circ}} = \frac{b}{\sin 67^{\circ}}$$

(c)
$$\frac{a}{\sin 20^\circ} = \frac{46}{\sin 130^\circ}$$

(d)
$$(10.7)^2 = (9.5)^2 + (12.4)^2 - 2(9.5)(12.4) \cdot \cos \angle B$$

(e)
$$c^2 = (10)^2 + (9)^2 - 2(10)(9) \cdot \cos 66^\circ$$

(f)
$$\frac{\sin \angle B}{14.2} = \frac{\sin 75^{\circ}}{21.3}$$

2. (a) 18.6 cm (b) 21.2 cm (c) 20.5 cm
3. (a)
$$\angle$$
B = 57°, a = 19.8 cm, b = 26.9 cm

(d)
$$57^{\circ}$$
 (e) 10.4 m (f) 40° (b) $a = 5.4 \text{ m}$, $\angle B = 70^{\circ}$, $\angle C = 86^{\circ}$

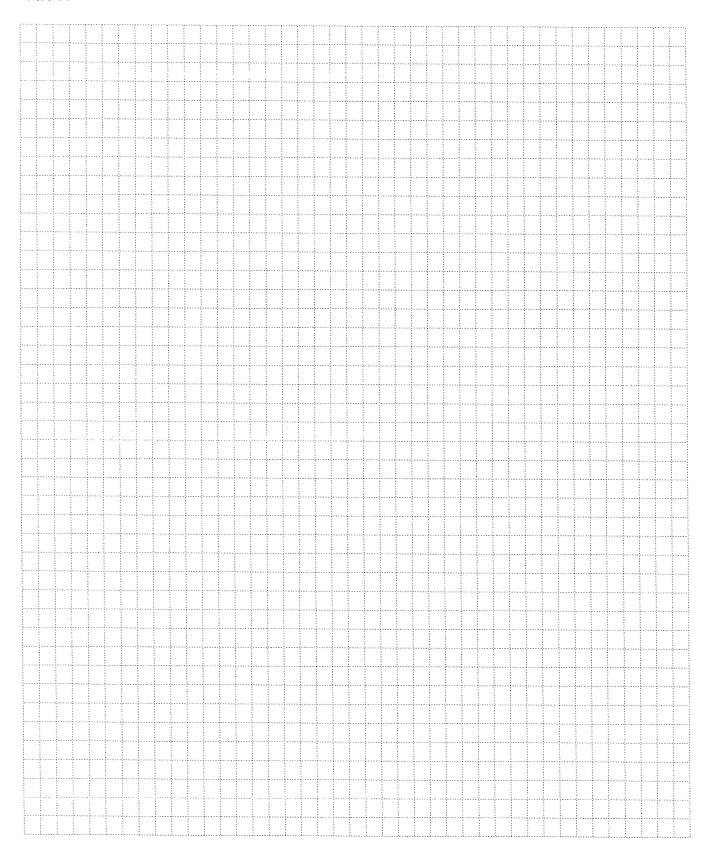
(a)
$$\angle B = 57^{\circ}$$
, $a = 19.8 \text{ cm}$, $b = 26.9 \text{ cm}$
(c) $\angle A = 41^{\circ}$, $\angle B = 43^{\circ}$, $\angle C = 96^{\circ}$

(d)
$$\angle C = 81^{\circ}$$
, $b = 23.2 m$, $c = 31.3 m$

(e)
$$\angle A = 64^{\circ}$$
, $a = 25.0 \text{ m}$, $c = 26.6 \text{ m}$

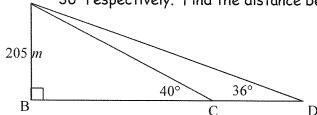
(f)
$$\angle A = 67^{\circ}$$
, $\angle B = 60^{\circ}$, $\angle C = 53^{\circ}$

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If diagrams are not included in any of the following questions it is advisable to sketch a diagram to aid in your solution to the problem. Round \angle 's to a whole degrees; length answers should be rounded to 1 decimal place and include units.

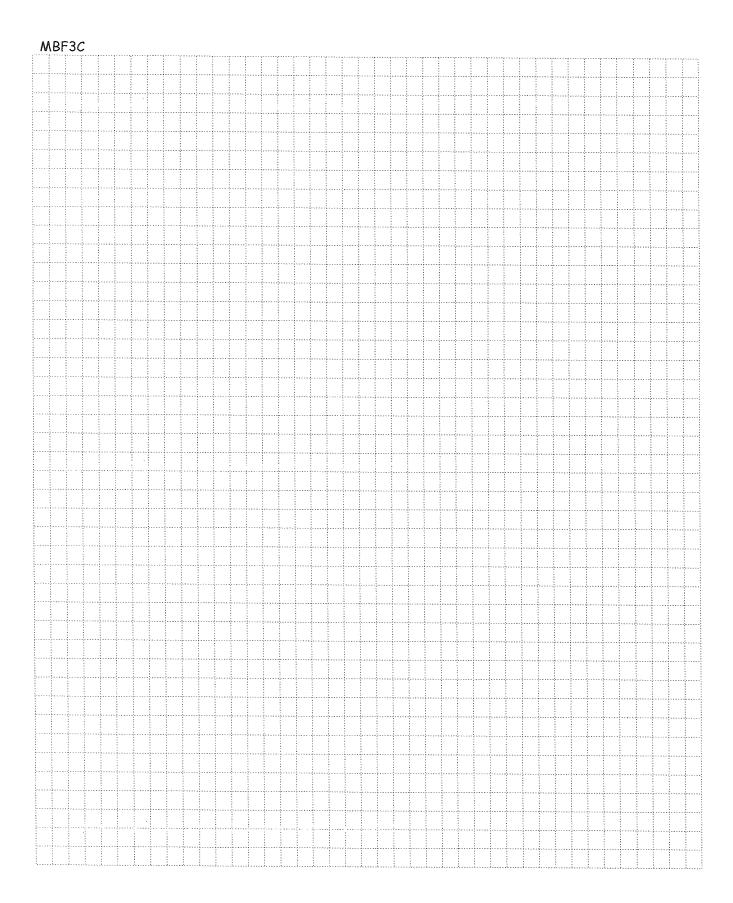
- 1. A squash player hits the ball 2.3 m to the side wall. The ball rebounds at an angle of 100° and travels 3.1 m to the front wall. How far is the ball from the player when it hits the front wall? (Assume the player does not move after the shot.)
- 2. A smokestack, AB, is 205m high. From two points C and D on the same side of the smokestack's base B, the angles of elevation to the top of the smokestack are 40° and 36° respectively. Find the distance between C and D. (Diagram included.)



- 3. Trina and Mazaheer are standing on the same side of a Red Maple tree. The angle of elevation from Mazaheer to the tree top is 67° and the angle of elevation from Trina to the tree top is 53°. If Mazaheer and Trina are 9.3 feet apart and Mazaheer is closer to the tree than Trina, how tall is the tree?
- 4. Two roads separate from a village at an angle of 37°. Two cyclists leave the village at the same time. One travels 7.5 km/h on one road and the other travels 10.0 km/h on the other road. How far apart are the cyclists after 2 hours?
- 5. A pilot is flying from Thunder Bay, Ontario to Dryden, Ontario, a distance of approximately 320 km. As the plane leaves Thunder Bay, it flies 20° off-course for exactly 80 km.
- (a) After flying off-course, how far is the plane from Dryden?
- (b) By what angle must the pilot change her course to correct the error?

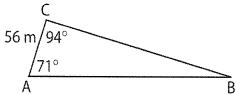
Solutions:

- 1. 4.2 m 2. 37.8 m 3. 28.3 feet 4. 12.1 km
- 5. (a) 246.4 km (b) approximately 26° turn towards Dryden.

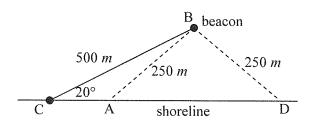


If diagrams are not included in any of the following questions it is advisable to sketch a diagram to aid in your solution to the problem. Round \angle 's to whole degrees; length answers should be rounded to 1 decimal place and include units.

- 1. To calculate the height of a tree, Marie measures the angle of elevation from a point A to be 34° . She then walks 10 feet directly toward the tree, and finds the angle of elevation from the new point B to be 41° . What is the height of the tree?
- 2. To measure the distance from a point A to an inaccessible point B, a surveyor picks out a point C and measures \angle BAC to be 71°. He moves to point C, a distance of 56 m from point A, and measures \angle BCA to be 94° How far is it from A to B? (Diagram below.)



- 3. A radar tracking station locates an oil tanker at a distance of 7.8 km, and a sailboat at a distance of 5.6 km. At the station, the angle between the two ships is 95°. How far apart are the ships?
- 4. Two islands A and B are 5 km apart. A person took a vacation from island B and travelled 7 km to a third island C. At island B the angle separating island A and island C was 34°. While on this vacation the person decided to visit island A. Calculate how far the person will have to travel to get to island A from island C.
- 5. The light from a rotating offshore beacon can illuminate effectively up to a distance of 250 m. From a point on the shore that is 500 m from the beacon, the sight line to the beacon makes an angle of 20° with the shoreline. What length of shoreline is effectively illuminated by the beacon? (i.e. solve for the length of AD in the diagram below.)

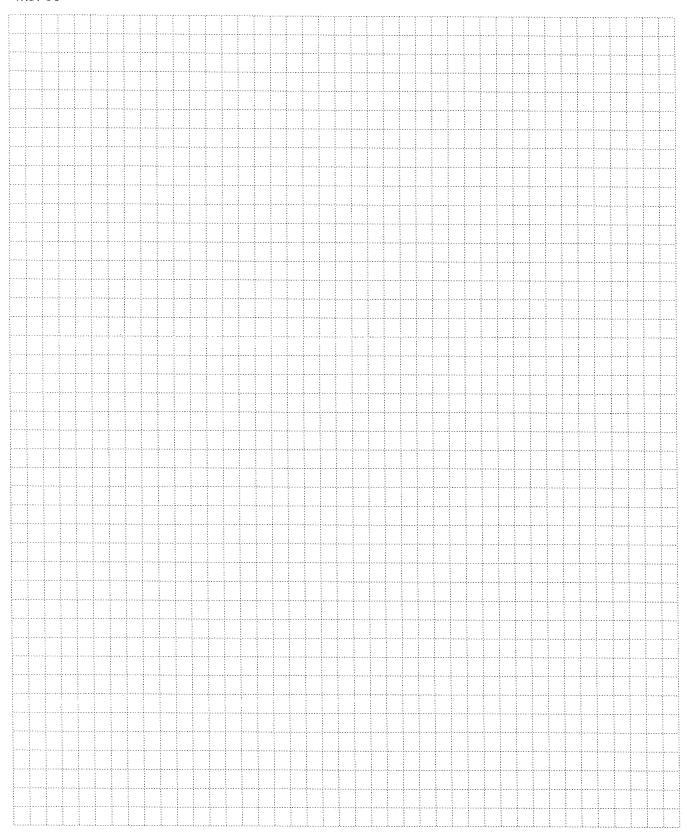


Solutions:

- 1. 30.1 feet 2. 215.8 m
 - 215.8 *m* 3. 10.0 *km*
- 4. 4.0 km
- 5. Hint: When you solved for $\angle CAB$ the angle 43.20 actually is the value for angle(s) $\angle ADB$ and $\angle DAB$ ($\triangle ABD$ is isosceles since AB = BD $\therefore \angle ADB = \angle DAB$) and the result of 43.20 is too small for $\angle CAB$ (using the SAT we get a value of 136.80) so the length of shareling that is effective

small for $\angle CAB$. (using the SAT we get a value of 136.80) so the length of shoreline that is effectively illuminated by the beacon is 364.5 m.

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U01A12

Making a Clinometer

A clinometer is a device used to measure the angle of inclination between two points.

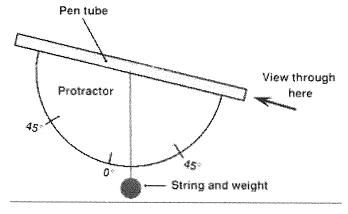
The following items are needed to make a simple clinometer:

- 1. Protractor: The plastic type found in drawing sets and used by Math departments are fine, but larger ones can be easier to read.
- 2. Narrow tube: A thin tube such as the body of a ballpoint pen. This will be the 'sight' mounted on the protractor.
- 3. String or thread: About 30 cm (1 foot) of thin but strong string.
- 4. Weight: Anything small and heavy which can be attached to one end of the string to keep it taught, such as a metal nut or fishing weight.
- 5. Glue or sticking tape: To assemble your clinometer. Glue must NOT dissolve in water!

Method

- 1. Glue or tape one end of the string firmly to the center of the protractor where all the lines meet. This will be half way along its flat base. Make sure that the string can hang down freely when the protractor is held with the flat side at the top.
- 2. Attach (glue or tape) the 'sight' tube along the straight edge of the protractor.
- 3. Attach your weight to the free end of the string and check again that the string can move freely as you tilt the protractor up and down.

That's all there is to it. You now have a clinometer which, hopefully, looks something like the one below.



Note: The angle between 0° and the string is the angle of inclination.

Clinometer Exercises

Exercise 1: (Determine the height of the classroom)

In your group, one of you stand back from the wall a few meters and look through the tube of the clinometer to the top of the wall. The other people will need to record the angle of inclination from the clinometer, the height of the centre of the clinometer, and the distance from the wall to the person using the clinometer. With this information draw and label a diagram of this situation and use trigonometry to find the height of the wall. Check your answer by measuring the height of the wall.

Exercise 2: (Determine the width of a classroom wall)

This can be done in many ways but the method you will use here will involve adapting the clinometer to find a horizontal angle. In your group, one of you go to one end of the wall that you are going to measure and then back up along the other wall a few meters (measure and record this distance). Hold the clinometer horizontally with the tube side of the clinometer against the wall. Now the tube of the clinometer is directed to one end of the wall. Another person in the group will need to try and direct the weight and string to the other end of the wall and record the angle between the string and tube.

- (a) With this information draw and label a diagram of this situation and use trigonometry to find the width of the wall. Check your answer by measuring the width of the wall.
- (b) Instead of backing up from one end of the wall, try to solve this problem by backing up from any spot on the wall and use the clinometer to find the angle between the ends of the wall.
- (c) Can you think of a better way to adapt the clinometer to find horizontal angles?

Clinometer Tasks

Task 1: Finding the Height of an object without approaching it

You are an unknown distance from a wall. Your task is to determine the height of the wall without measuring it. You have a metre stick and a clinometer. You can move around, but you are unable to approach the wall.

- (a) Create a detailed plan to determine the height of the wall.
- (b) Use your plan to determine the height of the wall.

Task 2: Finding the Width of an object without approaching it

You are an unknown distance from a wall. Your task is to determine the width of the wall without measuring it. You have a metre stick and a clinometer. You will be given the height of the wall. You can move around, but you are unable to approach the wall.

- (a) Create a detailed plan to find the width of the wall.
- (b) Use your plan to determine the width of the wall.

Trig Tasks

Task 1:

Jack and Jill are standing on Parliament Hill looking at the Peace Tower. The travel guide they have indicates that the height of the tower is 92 m. Using only a clinometer how could they calculate the distance between them and the base of the tower? Make a list of steps that would find the answer. Guess a reasonable angle measure and use the steps to estimate the distance. Be sure to draw a diagram.

Task 2:

Jack and Jill are standing in front of the Supreme Court House on Wellington Street. They are standing 20 m from one edge of the Court House and 24 m from the other edge of the Court House. Using only a clinometre determine the width of the Court House. *Make a list of steps that would find the answer. Guess a reasonable angle measure and use the steps to estimate the distance. Be sure to draw a diagram.*

Task 3:

Jack and Jill are standing in front of the new War Museum. They want to know the height of the tallest point of the building. They have a 15 m tape measure as well as a clinometre. The only problem is that they can't get close to the building due to construction. Devise a plan to find the height. Make a list of steps that would find the answer. Be sure to draw a diagram.

TRIGONOMETRY REVIEW

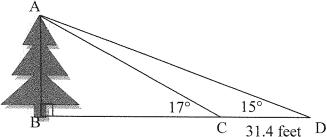
TOPICS:

- 1. Right angle triangles
 - Primary trig ratios SOH CAH TOA
 - Pythagorean Theorem
- 2. Acute triangles
 - Sine Law
 - Cosine Law
- 3. Solving triangles choose a strategy
- 4. Application problems choose a strategy

REVIEW QUESTIONS:

<u>Problem 1:</u> In $\triangle ABC$, given that $\angle B = 43^{\circ}$, $\angle C = 73^{\circ}$, and b = 19 m. Solve the triangle

<u>Problem 2:</u> Jillian stood at a distance admiring a magnificent Douglas Fir. Jillian measured the angle of elevation to the top of the tree and found it to be 15° . Jillian then walked 31.4 feet closer to the tree. This time the angle of elevation to the top of the tree was 17° . Calculate the height of the tree to the nearest tenth of a metre



TRIGONOMETRY REVIEW

<u>Problem 3</u>: Jill and her friends built an outdoor hockey rink. Their hockey goal line is 5 feet wide. Jill shoots a puck from a point where the puck is 5 yards from one goal post and 6 yards from the other goal post. Within what angle must Jill make her shot to hit the net? Draw the diagram first. (3 feet per yard)



<u>Problem 4</u>: The longest slide in the world is in Vermont, U.S.A. It drops 213 m in a horizontal distance of 1200 m. How long is the slide? What is its angle of inclination? Draw a diagram first.



Solutions: 1. $\angle A = 64^{\circ}$, $a = 25.0 \, m$, $c = 26.6 \, m$ 2. 68 ft 3. about 14° 4. 1219 m, 10 degrees

Practice Test: Trigonometry

This is the communication rubric that I will be using to evaluate communication on every test. Please look at it and see what you have to do to receive 10/10

	incomplete 0	Unacceptable 3.0 4.0	Poor 5.2 5.5 5.8	Acceptable 6.2 6.5 6.8	Good 7.2 7.5 7.8	Outstanding 8.4 8.9 9.5 10
Technical correctness of solutions	All or most solutions are blank	No solutions are correct or many left blank	Few solutions are technically correct	Some solutions are technically correct	Most solutions are technically correct	All or almost all solutions are technically correct
Presentation of solutions	All or most solutions are blank	No evidence of presentation or many solutions left blank	Solutions to few problems stand alone	Solutions to <u>some</u> problems can stand alone	Solutions to most problems can stand alone	Solutions to <u>all</u> or <u>almost all</u> problems can stand alone

Read each question carefully and follow the instructions.

Write your answers in the spaces provided.

Show all of your work for maximum marks.

Useful Formulae:

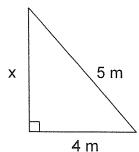
SOH CAH TOA
$$c^2 = a^2 + b^2 - \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
 $c^2 = a^2 + b^2 - 2ab \cos C$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Knowledge & Understanding: Please show your work. (8 marks)

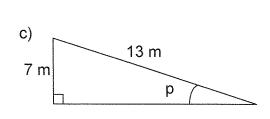
1. Find the missing value on each triangle using the proper trigonometry law or theory. (8 marks)

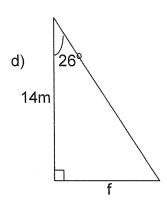
a)



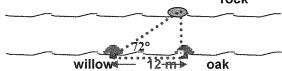
b) 2 cm 45°

Χ





Application: Please include labelled diagrams for marks. (15 marks)

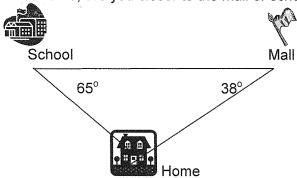


2. A ladder 3.5 m long is leaning against a wall. The foot of the ladder is 1.0 m from the wall. What angle does the ladder make with the ground, to the nearest degree.



Practice Test: Trigonometry

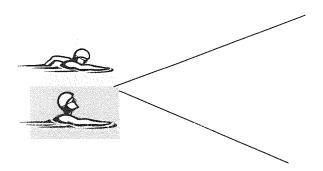
3. The distance between school and the mall is 2.5 km. The angle between the school and home with respect to the mall is 65°, and the angle between the mall and home is 38°. If you are at home, are you closer to the mall or school? (diagram is not to scale) (4 marks)



4. Sam and Jenn are shopping when they notice a really nice purse. Sam is 50 feet from the purse and Jenn is 72 feet away. The purse is between them at an angle of 32°. How far is Sam from Jenn? (3 marks)



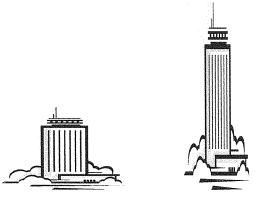
5. Two swimmers part at a beach. One swims at 8 km/hr and the other at 6km/hr. After one hour the swimmers are 4.5 km apart. What was the original angle at which the swimmers left each other? (3 marks)



Practice Test: Trigonometry

TIPS: (8 marks)

1. Two buildings are 38 m apart. From the top of the shorter building, the angle of elevation to the top of the taller building is 27° and the angle of depression of the base is 35° . Determine the height of each building.



2. Two cars head off down roads that are 57° apart. One car is traveling at 60 km/h and the other car is traveling at 75 km/h. How far apart are the cars after 3 hours? Include a diagram. (3 marks)

MBF3C Unit 2 - Quadratics

Section	Date	Date Lesson Title		Homework Assigned
1		Graphs of Quadratic Relations	U02A01	U02A06
			U02A02	
			U02A03	
			U02A04	
			U02A05	
2		The Parabola	U02A07	U02A10
			U02A08	U02A11
			U02A09	
3	-	Exploring Vertex Form	U02A12	U02A13
4	.: .:	Graphing Parabolas	U02A14	U02A17
			U02A15	
			U02A16	
5		Factored Form of a Quadratic Relation	U02A18	U02A20
			U02A19	
6		Quadratics Consolidation	U02A21	
7		Formative Assignment Work Period		
8		Binomial Multiplication using Algebra Tiles	U02A22	
9		Binomial Multiplication 2	U02A23	Supplement with Worksheet
				(not provided)
10		Standard Form of a Quadratic Relation	U02A24	U02A26
			U02A25	
11		Expanding to Standard Form from Vertex	U02A27	U02A29
		and Factored Forms	U02A28	
12		Factoring Quadratic Expressions using Algebra Tiles	U02A30	Supplement with Worksheet
	SECTION AND ADMINISTRATION AND A			(not provided)
13		Factoring Quadratic Expressions 2	U02A31	
14		Factoring and Solving Quadratic Expressions	U02A32	U02A33
15		Problem Solving with Quadratics	U02A34	U02A36
			U02A35	
16		More Problem Solving with Quadratics	U02A37	
17		Review Day	U02A38	
18		Practice Test	U02A39	U02A40
19		Test Day		

Welcome to Fasool's Fantastic Funland

Where FUN is all that matters...

Today's Special Group Rates:

- > A group of 20 costs \$40 per person.
- For every extra person, you save 50¢ per person. (Example... a group of 21 costs \$39.50 each)
- For groups below 20, it costs 50¢ more for each person below 20 (Example... a group of 17 costs \$41.50 each)

Row	# in group	Total \$

Quadratics Warm-Up:

Congratulations! You have made it to the math cheerleading team. Just imagine: a group of dedicated <u>mathletes</u> spreading the cheer of math throughout the school! The best part about being on the math cheerleading team is that you get paid... per cheer! Of course, since the team is a MATH team, it takes a bit of calculating to figure out how much you get paid.

Here's what the coach told you:

If you do 10 cheers, you get paid \$2 per cheer (NOT BAD!) You will get 10¢ less per cheer for every cheer over 10 cheers, but you will get 10¢ more per cheer for every cheer under 10 cheers.

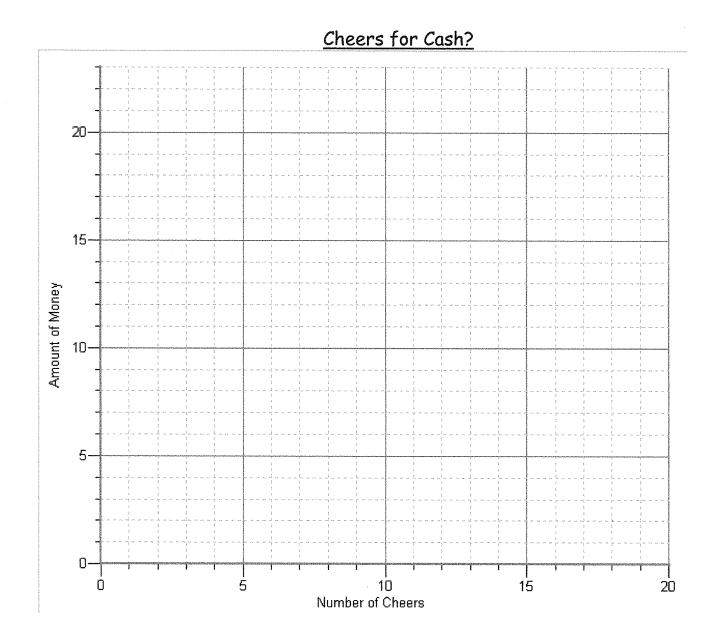
The question going around the team is "How many cheers do we need to do in order to get the most money possible?"

Fill in the table below to find out (start at 10 cheers and work up and down)

Number of Cheers	Price per Cheer	Total Money Paid (1 st × 2 nd columns)
7	\$2.30	
8	\$2.10 + 10¢ = \$2.20	
9	\$2.00 + 10¢ = \$2.10	
10	\$2.00	10 × \$2.00 = \$20.00
11	\$2.00 - 10¢ = \$1.90	
12		
13		
14		
15		
16		
17		
18		

Conclusion: The maximum money of	is paid when you do	_ math cheers.
----------------------------------	---------------------	----------------

Plot the # of Cheers vs. Amount of Money on the grid below:



Quadratics Warm-Up:

You have been given 20 sections of chain-link fence to reserve an area in a new park which will be used as a wading pool in the future. The only instruction from the construction foreman was to reserve the "biggest rectangular area possible."

The 20 toothpicks you have will represent the sections of the fence. Use the table below to design 9 different "pool areas". On the graph paper provided draw all 9 rectangles (one grid space = one section of fence) and label them with the correct rectangle label (A, B, C, etc)

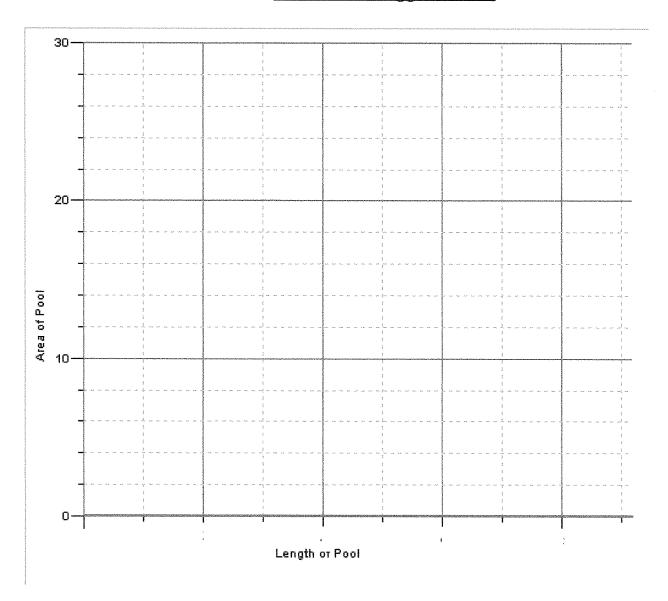
Remember, area of a rectangle is length \times width! (Or count the # of squares in the rectangle on your graph paper!)

Rectangle Label	If the length	Diagram	Then the width is	And the area
Ladei	of the pool	(not drawn	15	(units are
	13	to scale)		sections ²)
Α	1 section	S Scale)	9 sections	1 × 9 = 9
В	2 sections			
С	3 sections			
D	4 sections			
Е	5 sections			
F	6 sections			
G	7 sections			
Н	8 sections			
I	9 sections			

<u>Conclusion</u>	•	aximum area of ns long and	sections w	ctions ² occurs when vide	n the area is _	

Plot the Length of the Pool vs. Area of Pool on the grid below:

What's The Biggest Pool?

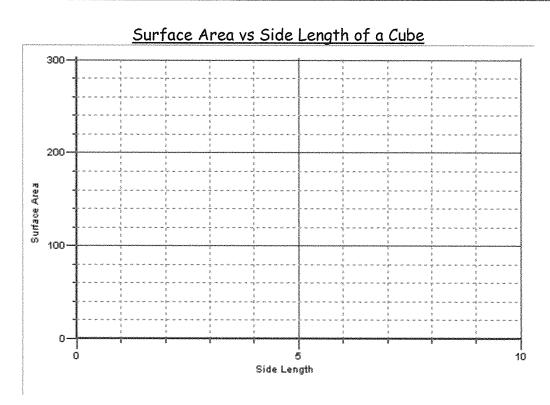


Quadratics Warm-Up:

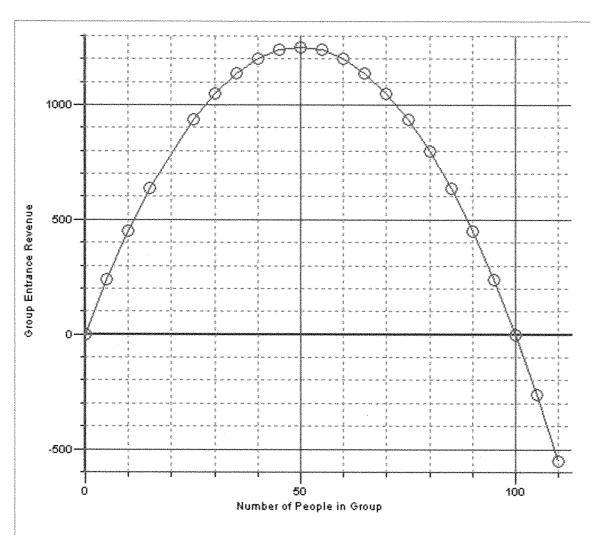
In this activity you will determine the relationship between the side-length of a cube and its surface area.

You can use hexalink cubes for the first few examples of this activity, but you will have to mentally calculate the surface area when the cubes become too big for you to build. Fill in the side-length and surface area in the table below and then plot the data in the grid provided (as much data as can fit on the plot). The first one has been done for you. This is basically a single cube. It has a side length of one (it's made of only 1 cube!) and it has 6 squares showing on all its faces (that's why the surface area is 6). A cube with a side length of 2 would be a $2 \times 2 \times 2$ cube. The surface area is the area of all the faces (count the number of squares on all the faces!)

Side Length	Surface Area (Side Area x # of sides)	Side Length	Surface Area (Side Area x # of sides)
1	6	5	
2		6	
3		7	
4		8	



Remember Fasool's Fantastic Funland?



Questions

- 1. In order to get the most money from a group, how many members should the group have?
- 2. What is the maximum revenue?
- 3. What is the revenue from a group of 20?
- 4. What happens at 0 people and 100 people?
- 5. What happens after 100 people? Does this make sense? Explain.

Quadratics Warm-Up: Homework

Question 1:

Complete the table below and then graph the data on graph paper.

A rectangular display is to be surrounded by neon string lights. The area of the display is to be as large as possible and it must be completely surrounded by the string lights which have a total length of 120 cm.

If the length of one side	Diagram	Then the length of the other side	
is	(not drawn to	is	(units are
	scale)		cm ²)
5 cm	55		
15 cm			
25 cm			
35 cm			
45 cm			
55 cm			

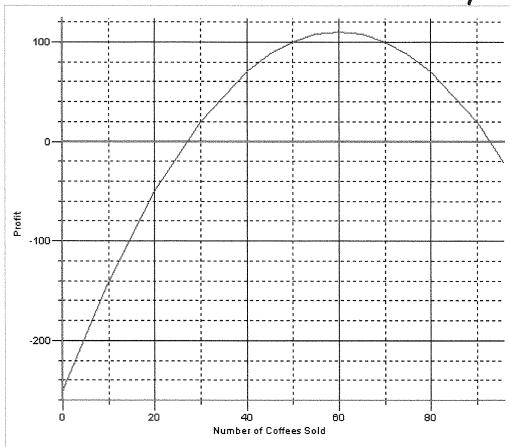
Quadratics Warm-Up: Homework

Question 2:

The promotions manager of a new band is deciding how much to charge for concert tickets. She has calculated that if the tickets are \$30 each, then 200 people will come to the concert. For every \$1 increase in the price, 10 less people will come. Create a table to calculate how much should be charged to MAXMIZE the revenue from the ticket sales.

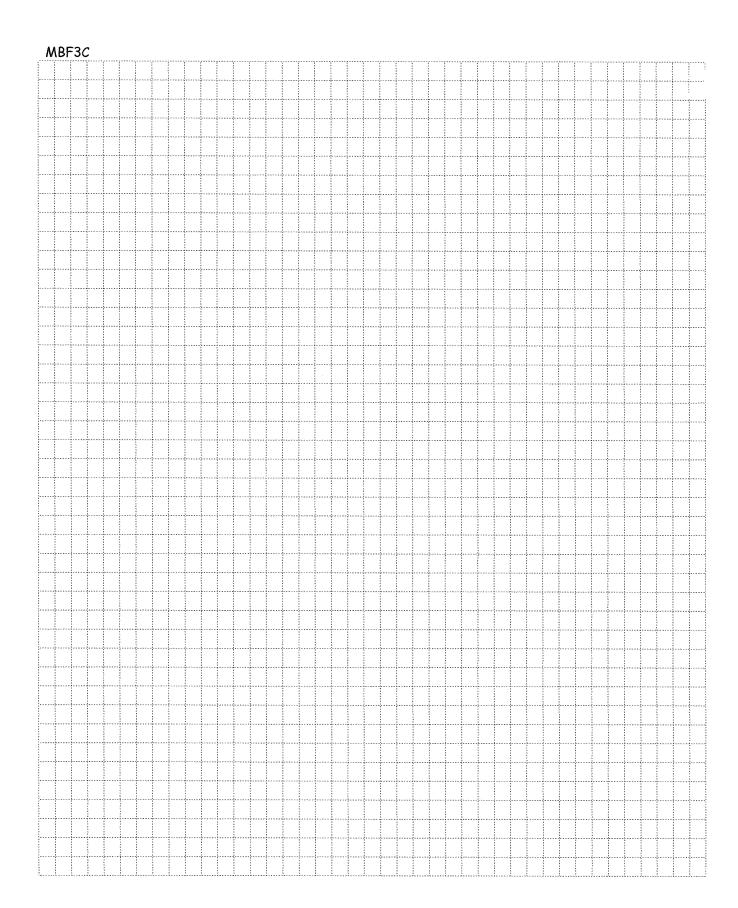
Ticket Price	Number of People	Total Money From Tickets
	117.74.74.74.74.74.74.74.74.74.74.74.74.74	
\$29		
\$30	200	\$30 × 200 = \$6000
ΨΟΟ	200	\$30 × 200 - \$0000





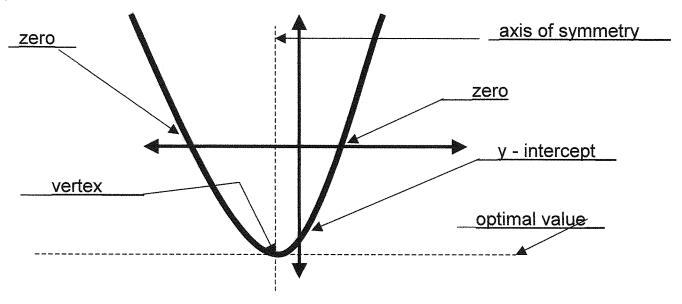
The graph above shows the profit each day for Jim Norton's roadside coffee stand.

- A. Approximate how many coffees that Jim needs to sell in order to "break even"?
- B. How many coffees does Jim need to sell to make the maximum possible profit?
- C. If he sells no coffees in a day, how much money does he make (or lose)?
- D. How many coffees does he need to sell to make \$100?



Introducing... The Parabola!

The graph of a quadratic relation is called a <u>parabola</u>. The parabola has some important features:



Everything you ever wanted to know about parabolas...

- > Parabolas can open up or down
- \triangleright The zero of a parabola is where the graph crosses the x axis
- "Zeroes" can also be called "x intercepts" or "roots"
- > The axis of symmetry divides the parabola into two equal halves
- > The vertex of a parabola is the point where the axis of symmetry and the parabola meet. It is the point where the parabola is at its maximum or minimum value.
- > The optimal value is the value of the y co-ordinate of the vertex
- The y-intercept of a parabola is where the graph crosses the y axis

Introducing... The Parabola! (Continued)

For the following parabolas, fill in the table which follows.

Lot. the Lotton	ing parabolas, till in the to	udie which follows.	
Parabola Graph	2 2 4 6	4 -2 2	-6/-4-2
Vertex			
Optimal Value			
Axis of			
Symmetry			
Zeroes			
Direction of			
Opening			
Y - intercept			

<u>True or False</u>... (use the above for answers)

The axis of symmetry goes through the y - intercept.

The vertex is always located halfway between the zeroes.

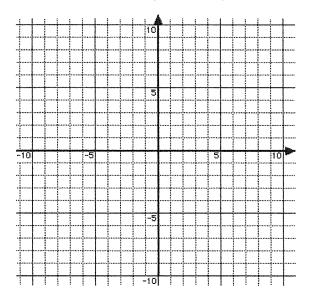
The y - coordinate of the vertex is always the same as the optimal value.

The x - coordinate of the vertex is always the same as the axis of symmetry.

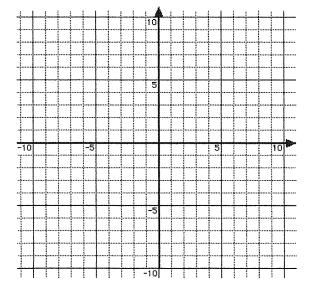
A parabola must open up.

The y - intercept is always positive.

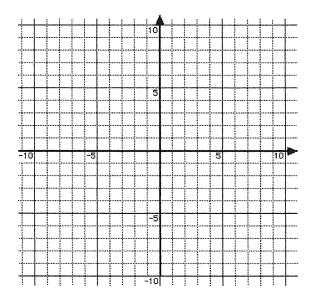
1) Sketch a parabola that has a vertex at (-3,1), has an optimal value which is a maximum and a y-intercept at -8.



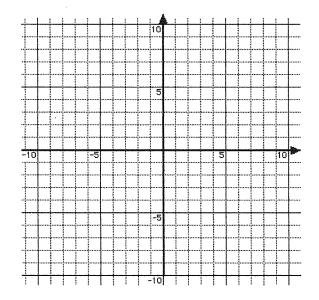
3) Draw the parabola with vertex of (3, 4) and zeros at 1 and 5.



Sketch a parabola that has an axis of symmetry at x = 4 a minimum of -7.

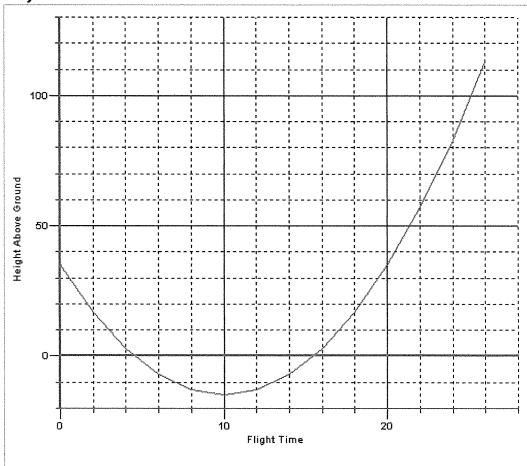


4) Draw the parabola with axis of symmetry of x = -2, optimal value of y = -3. This parabola has no zeros.

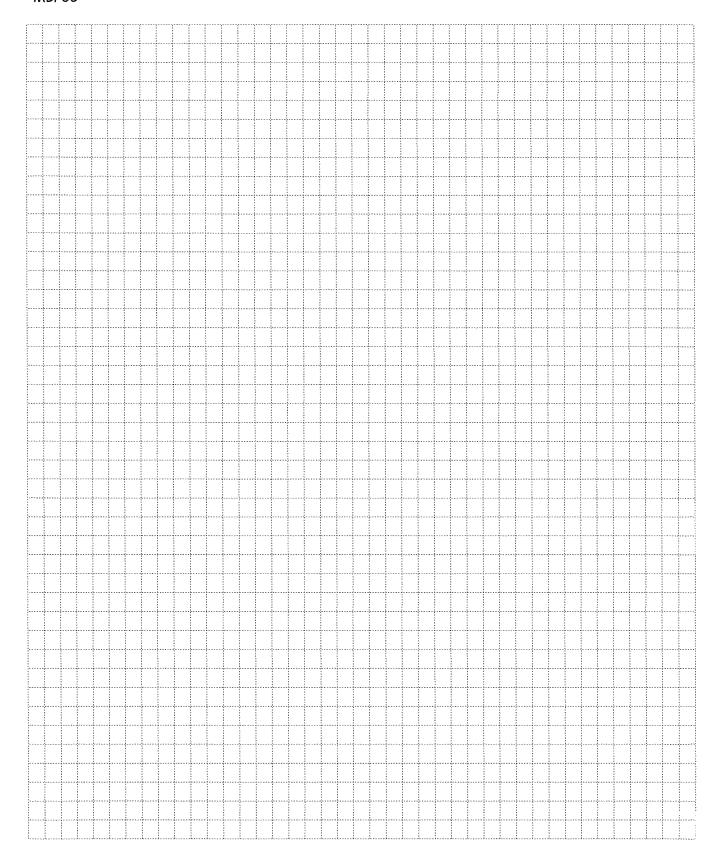


A Paper Airplane Ride

Tatiana has walked to a water tower beside a nearby gorge in order to launch her newly designed paper airplane. The graph below shows the flight of the paper airplane. A negative height means the airplane is below the level of the ground. (Height in feet and time in seconds)



- 1. Estimate the height of the water tower _____
- 2. How long does it take for the paper airplane to reach its minimum height? _____
- 3. How high is the minimum height?
- 4. When has the paper airplane reached ground level?
- 5. Write the vertex of this parabola:
- 6. Will the airplane continue in a parabolic path? Explain why or why not.



Vertex Form of a Parabola

In this investigation you will graph different parabolas and compare them to what is known as the "Basic Parabola".

TECHNOLOGY OPTION

To help you graph and plot the parabolas, enter the equation in the

Y =

screen on your TI - 83 graphing calculator, press

graph

to see the graph and

press 2nd

graph

to see a table of values for the parabola

THE BASIC PARABOLA

Equation	$y = x^2$				
Table o	of Values				
X	y				
-3					
-2					
-1					
0					
1					
2					
3					
Fill in the follow	ving information				
about the parabo	ola:	<u> </u>			
What is the	What is the	What's the "step pattern" of	Over 1		
vertex?	Direction of	the parabola? (how do you			
	Opening?	move from point to point,	Over 1		
		starting from the vertex? –			
		and it doesn't matter if you	Over 1		
		go to the right or left)			

Since all parabolas have their "over" steps the same, we usually refer to these three numbers as the Step Pattern of the parabola

So, the Step Pattern of this parabola is

PARABOLA INVESTIGATION #1

Equation	$y = x^2 + 2$								_	
									Ļ	
Table o	of Values								-	
X	y						++		Г	
-3									_	
-2							1		_ 	
-1										
0									L	
1									<u> </u>	
2										
3									2	
									F	
Fill in the follow	ving information						+++		-	
about the parabo										
What is the	Direction of	What's th	e Step	(Over 1					
vertex?	Opening?		•		(Over 1				
					(Over 1				
	PARABOLA IN	VESTIGATIO	N #2					***************************************	-	
Equation	$y = x^2 - 3$								I	
									1	
Table of	of Values								+	
X	y					+			+	
-3	***************************************								+	
-2	and resource and the second se								+	
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X	y
-3 -2	
-2	
-1	
0	
1	
2	
3	

Fill in the following information about the parabola:

What is the	Direction of	What's the Step Pattern?	Over 1	
vertex?	Opening?		Over 1	
			Over 1	

What is the effect when a number is added or subtracted to the equation of the Basic Parabola?

PARABOLA INVESTIGATION #3

Equation	$y = (x - 3)^2$
Table o	f Values
X	y
0	
1	
2	
3	
4	
5	
. 6	
Fill in the follow	ing information
about the parabo	
What is the	Direction of
vertex?	Opening?

PARABOLA INVESTIGATION #4

Equation $y = (x + 4)^2$		
TN . I. I C X7 . I		
Table of Values		
x y		
-7		
-6		
-5		
-4		
-3		
-2		
-1		
Fill in the following information		
about the parabola:		
What is the Direction of	What's the Step Pattern?	Over 1
vertex? Opening?		Over 1

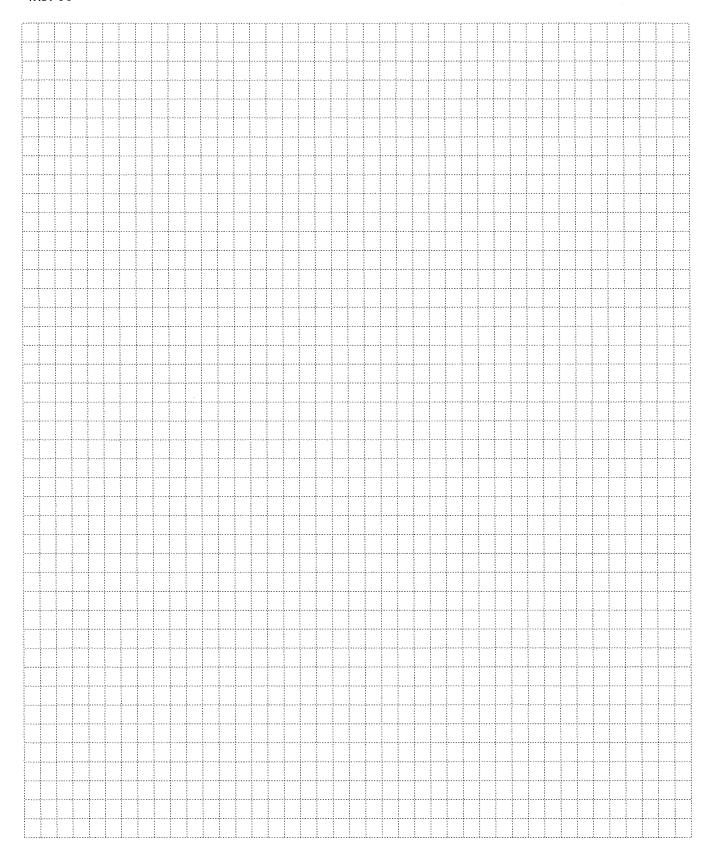
What is the effect when a number is added or subtracted to the x (in brackets) of the equation of the Basic Parabola?

MBF3	30	•		
UO2A	1	2	_	P4

PARABOLA INVESTIGATION #5

Equation	$y = 2x^2$												
	· · · · · · · · · · · · · · · · · · ·												
Table of	f Values												
X	y												
-3	#.·												
-2													
-1													
0													
1													
2													
3													
Fill in the following													
about the parabol	a:												
What is the	Direction of												
vertex?	Direction of What's the Step Pattern Opening?												
	PARABOLA INV												
	<u> </u>												
Equation	$y = -3x^2$												
Table of	f Values												
X	y												
-3													
-3													
-3 -2													
-2													
-2 -1													
-2 -1													
-2 -1 0 1													
-2 -1 0 1 2													
-2 -1 0 1 2 3	ing information												
-2 -1 0 1 2 3 Fill in the follow													
-2 -1 0 1 2 3													
-2 -1 0 1 2 3 Fill in the follow about the parabol	la:												

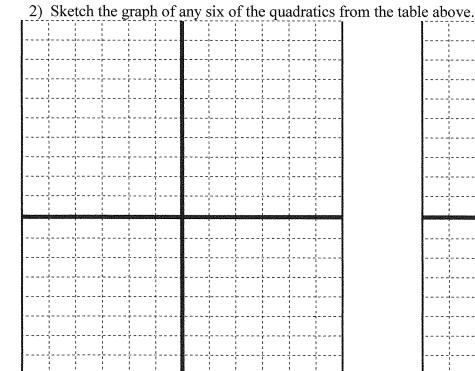
What is the effect when a positive or negative number is multiplied to the equation of the Basic Parabola?

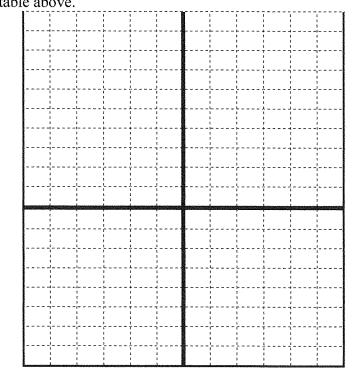


Properties of a Parabola

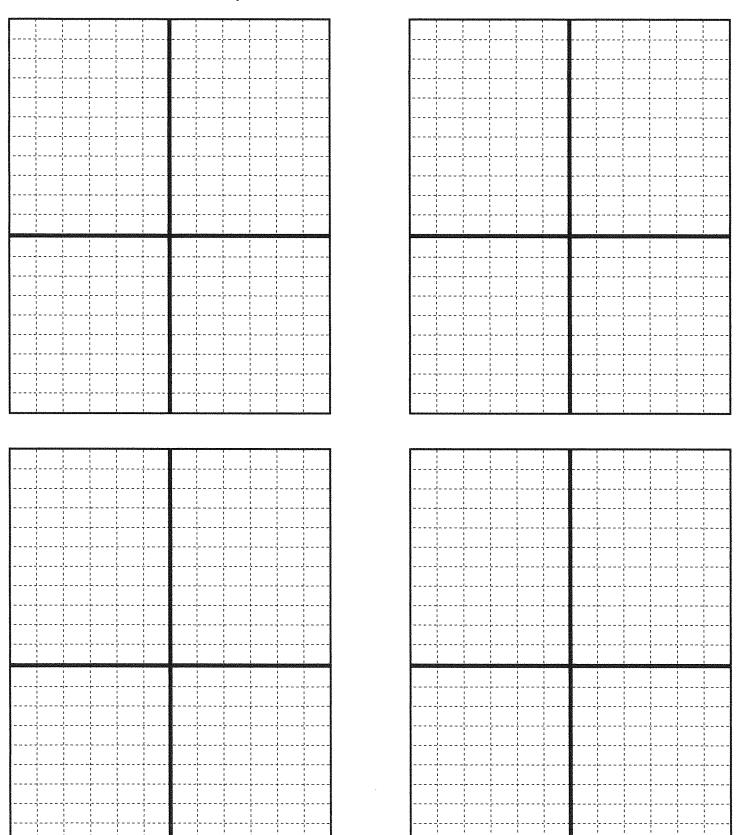
1) Complete the following table.

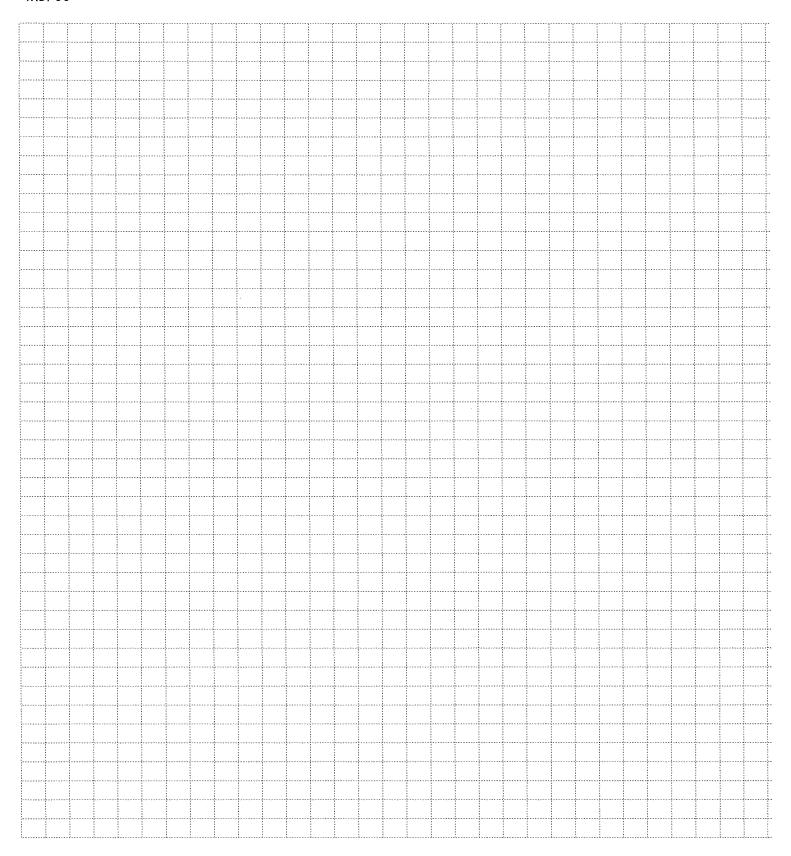
Equation	Vertex	Direction of Opening	Value of "a"	Step Pattern from Vertex
$y = (x - 2)^2 + 1$	(2,1)	Up	+ 1	1,3,5
$y = -(x + 4)^2 + 6$				
$y = 4(x - 4)^2 - 1$				
$y = 3(x + 7)^2 - 4$				
$y = -2(x - 10)^2 + 100$				
$y = (x - 4)^2 + 15$				
$y = -2(x + 2)^2 + 64$				
$y = 5(x - 10)^2 - 11$				
y =	(-3, -3)	Up	2	2,6,10
y =	(20, -10)	Down	- 1	-1 , -3 , -5





Properties of a Parabola – Continued.....





Graphing Using Vertex Form

Parabola 1

Quadratic Relation is...

$$y = 3(x + 3)^2 - 12$$

From the equation it can be seen...

The vertex is _____

The parabola opens _____

The step pattern is ____, ___,

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Parabola 2

Quadratic Relation is...

$$y = -2(x - 3)^2 + 8$$

From the equation it can be seen...

The vertex is _____

The parabola opens _____

The step pattern is ____, ___,

Graph the parabola

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The zeros are _____ and ____

The y – intercept is _____

The parabola has a max. / min. _____

The optimal value is _____

The axis of symmetry is _____

The zeros are _____ and ____

The y – intercept is _____

The parabola has a max. / min. _____

The optimal value is _____

The axis of symmetry is _____

U02A15

Money, Money, Money

A study of the finances of Dominion Motors has shown that the profit of the company can be described by the equation

$$P = -2(n - 200)^2 + 450000$$

Where P represents profit and n represents number of cars sold

- (a) What is the maximum possible profit possible?
- (b) How many cars need to be sold to achieve this profit?
- (c) What other information could the graph of this function provide?

Sub's Way

The Canadian Armed Forces are testing their new aerial-entry rescue yacht. This craft is dropped into the water from an air transport; it then follows a parabolic underwater path while resurfacing. The specifications indicate that the parabolic path is 6m in width, and reaches a depth of 18m.

(a) On the graph, the yacht enters the water at (0, 0). Sketch its path.

(b) Write the parabola's equation

(c) If properly "aimed" could the yacht be sent under a rectangular boat that was 4m wide, but floated 3m deep?

U02A16

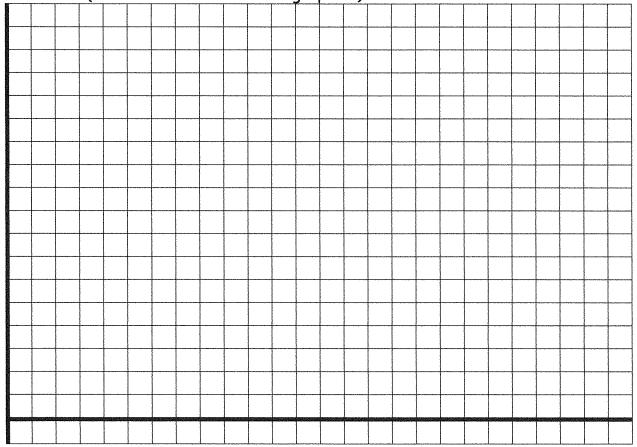
The Golden Arch

A decorative arch is to be built over a fountain at the Mathematician's Hall of Fame. The arch will be in the shape of a parabola and be 4m tall at the centre. Inscribed on a plaque located near the fountain will be the equation of the arch:

$$y = -0.25(x - 4)^2 + 4$$

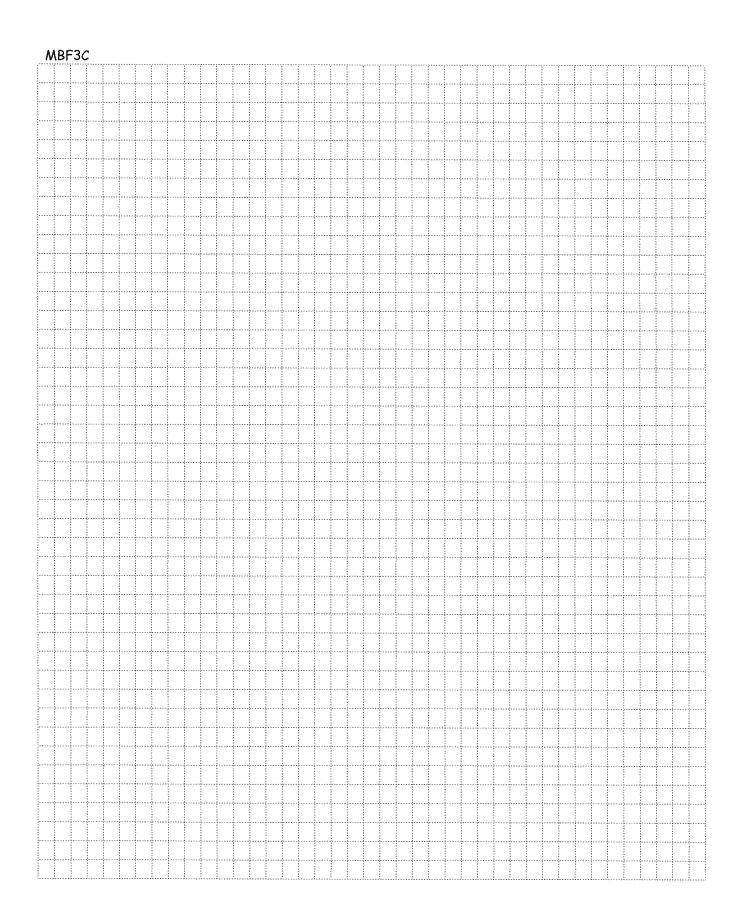
(a) graph the shape of the arch, using the grid below where one square represents 0.25m.

(the entire arch will not be graphed!)



- (b) How far apart are the "feet" of the arch?
- (c) A stunt pilot wants to hang-glide through the arch during the opening ceremonies.

 She can modify her hang-glider to have a wingspan of 2m and will fly at a height of 1.5 m above the ground. Will she fit through the arch? Explain your reasoning.
- (d) At what points is the arch 2m high? Find a graphical and algebraic way of answering this question.

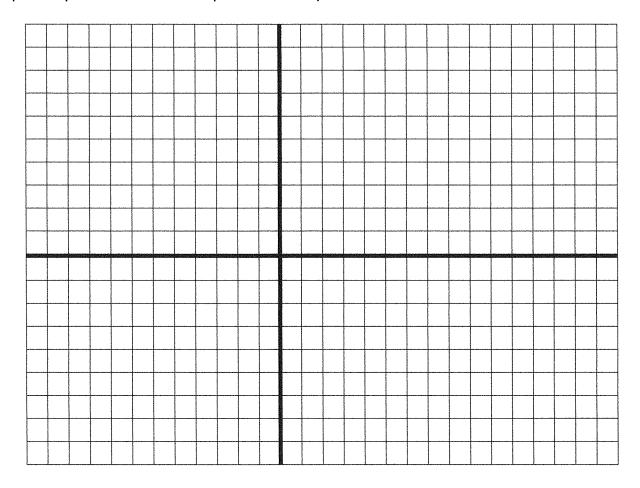


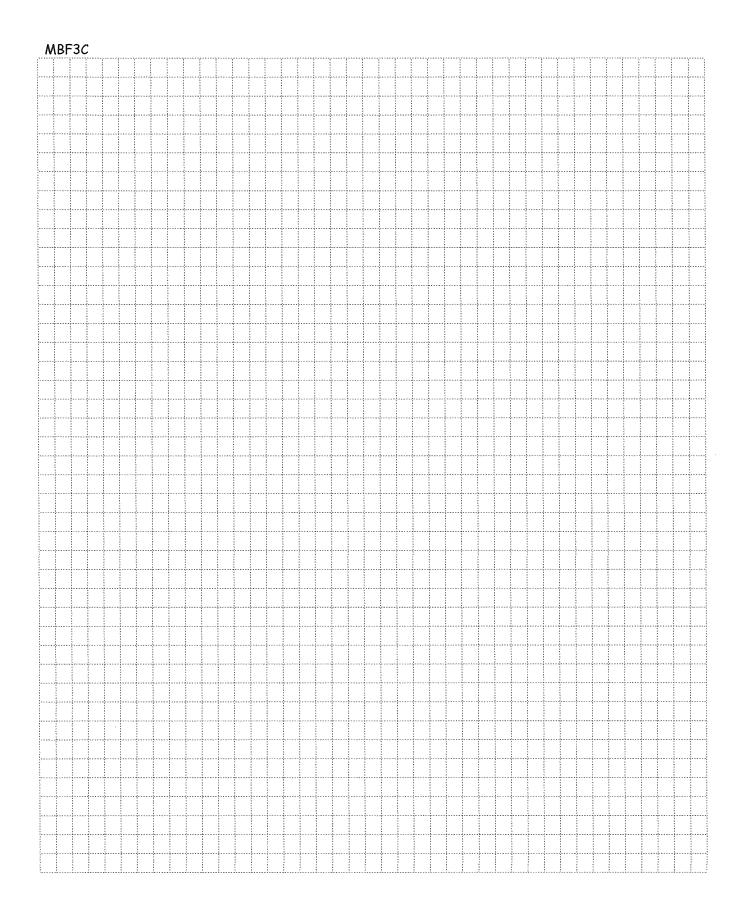
Graphing Parabolas Worksheet

1. For the following quadratic relations, fill in the table:

Equation	$y = 3(x-4)^2 - 8$	$y = -2(x + 1)^2$	$y = -(x + 2)^2 + 10$
Vertex			
Axis of Symmetry			
Value of "a"			
Direction of			
Opening			
(Max or Min)			
Step Pattern from			
Vertex			
Optimal Value			
	<u> </u>	<u> </u>	

2. Graph the parabolas defined by the above equations.





U02A18 Exploring the Factored Form of a Parabola

In this investigation you will graph different parabolas and determine the link between the equation in "factored form" and the zeroes of the parabola.

You will need to be able to determine the following about a parabola:

The zeroes

The direction of opening

The axis of symmetry

The step pattern

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To help you graph and plot the parabolas, enter the equation in the Y = Y

screen on your TI - 83 graphing calculator, press graph to see the graph and

press 2nd graph to see a table of values for the parabola

Parabola Investigation # 1

Equation	y = (x - 1)(x + 1)										
Table of Values											
X	y										
-3											
-2											
-1											
0											
1											
2											
3											
h	······································										

			SECURIO		A 50 45 45 45 45 45 45 45 45 45 45 45 45 45		SION SENS				
-		 	-	 		-				 	
-	ļ	 		 	 		ļ	ļ	 	 	

Fill in the following information about the parabola:

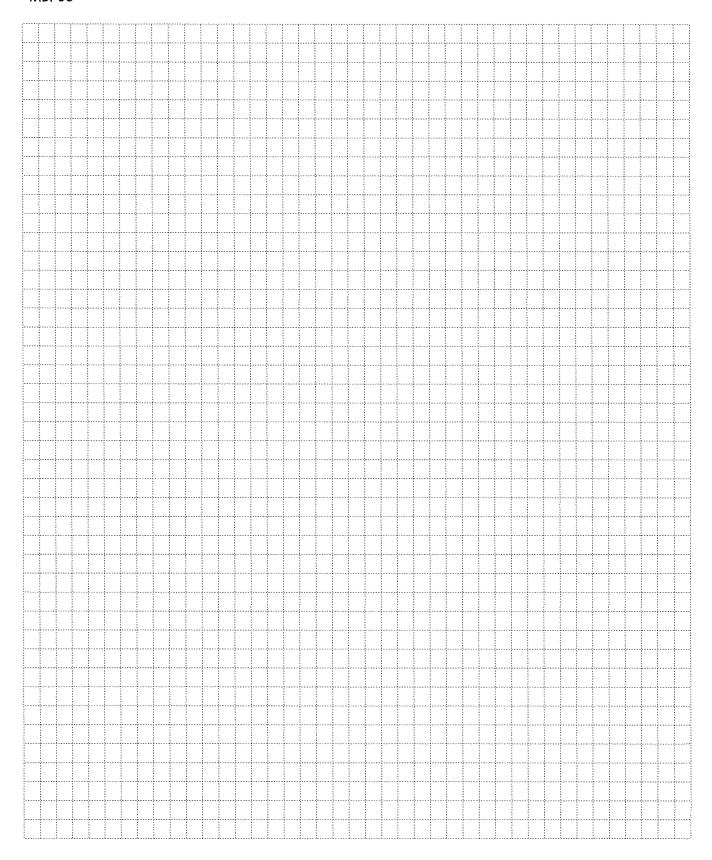
What is the	What are the zeroes?	What is the axis of	What is the step
Direction of		symmetry?	pattern?
Opening?	and		-

Parabola Investigation # 2

	L															
Equation	y = (x - 3)(x + 1)															
1.1 de manuel de la constant de la c		」		-						-	_	-				
Table	e of Values			+			-			+	-	-		\vdash		
X	y			+	-		-				-	-	$\left \cdot \cdot \right $			
-2		1		+	-			+			-	+				
-1		1 -		+			-				+					
0							\top			1						
1																
2		1	<u> </u>								_					
3		1						+			_	_				
4							isotoperans					ena estación				
		1		+	-		-	+				-				
Fill in the follo	owing information	1		-	 		_	+	\vdash		+	-				
about the para	•			,					,		•	·			•	
Direction of		roes?	Wh	at is	s th	ne ax	cis c	f		St	ten	pat	teri	1?		
Opening?						etry					- F	F				
-1 - 3	and										•		_, _		_	
	Parabola	Inves	tiga	tio	n #	£ 3								• • • • •		
Equation	y = -2(x + 1)(x + 5)	5)				Π	<u> </u>	T								
LIGHTION	J - 1/21 - 1/21 - 1			1	T			+		\neg	\dashv	\top	T			
Tab	le of Values				T											_

Equation	y = -2(x + 1)(x + 5)																
		-															
Tab	le of Values	-	-		\perp	-	_										
X	v	-	-			+		-									
-6		-	-		-	\perp	-	-									
-5		-			+	-	-		\vdash							 	
-4		-	 		_	+											
-3		-			1	\dagger	\top										
-2					\top	1											
-1																	
0		ESSENSE															
Fill in the follo	owing information																
about the para	bola:																
Direction of	What are the zeroes	?	What is the axis of			f	Step pattern?										
Opening?		symm			mmetry?												
and		•						_		_		,			•	 	

What is the relationship between factored form and the zeroes of the parabola?



U02A19

Factored Form of a Parabola

Factored Form of a Quadratic Relation:

y = a(x - s)(x - t)

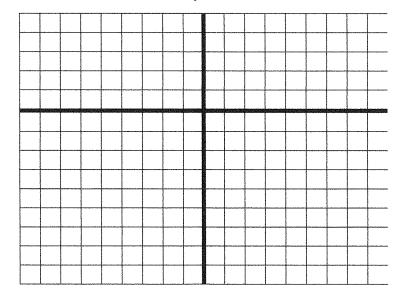
This controls the direction and opening as well as the step pattern (same as in vertex form!)

The opposites of these numbers are the zeroes of the parabola. In this case, the parabola would have zeroes of s and t. (or officially, (s, 0) and (t, 0)

Practice: Fill in the table for each parabola equation.

riuciice. I iii iii ii	ue lable for each		
Equation	$y = 3(x - 4)^2 - 8$	$y = -2(x + 1)^2$	$y = -(x + 2)^2 + 10$
Vertex			
Axis of Symmetry			
Value of "a"			
Direction of			
Opening			
(Max or Min)			
Step Pattern from			
Vertex			
Optimal Value			
Lance to the second sec	I	<u> </u>	

Practice: Find the vertex of the middle parabola, and then sketch it.



U02A20

More about the parabola!

1. Fill in the table for each parabola equation. BE CAREFUL! Some information is not given by certain equations!

Equation	y = 2(x - 5)(x + 9)	$y = -(x + 2)^2 + 6$	y = 4(x + 2)(x + 8)
Zeros			
Direction of Opening			
Axis of Symmetry			Au de la constant de la constant de la constant de la constant de la constant de la constant de la constant de
Step Pattern			
Vertex			

- 2. A cannonball is shot into the air. Its height can be described by the equation h = -3(t-1)(t-9) where h is height in feet and t is time in seconds.
- (a) What are the zeroes of this relation? ____ and ____
- (b) What do the zeroes mean in this situation?
- (c) What is the axis of symmetry and what does it represent?
- (d) Use the axis of symmetry to find the vertex and explain what the vertex means for the cannonball.
- 3. The equation P = -0.5(n 500)(n 10) describes a company's profit P, based on how many units are sold, n. What are the break even points of the company, and how many units must be sold to make a maximum profit?

Question 1	/	Question 2						
	/2 marks		/2 marks					
What are the zer	oes of	What are the zer	oes of					
y = (x - 4))(x + 8) ?	y = -2(x - 5)	5)(x + 17) ?					
Question 3		Question 4						
	2 marks	_	1 mark					
Show you	r work!!!							
What is the axis	of symmetry of	What is the axis	of symmetry of					
y = (x - 5)(x + 13)?	$y = 3(x - 4)^2 + 8$?						
	,	y 3(x 4) / 3 :						
Question 5		Question 6						
	5 marks		5 marks					
Show you	r work!!!	Show you	r work!!!					
Find the zeroes a	algebraically not	Find the vertex a	Idebraically for					
graphically for	a.g.o.o. a.o.a.ry, 1101	Find the vertex algebraically for						
	0)2 0 0	y = (x - 4)(x + 8) ?						
y = 2(x +	$(3)^2 - 8$?							
GRANDES CLICATION OF THE PROPERTY OF THE PROPE	•							

Que	st	II.	or	1 7

1 marks

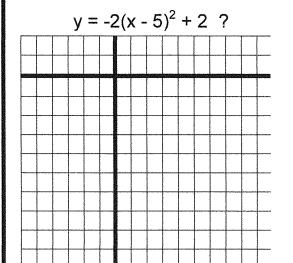
Question 8

/2 marks

What is the vertex of

$$y = -3(x + 8)^2 - 6$$
 ?

Sketch the graph of



Question 9

/2 marks

Question 10

/3 marks

Show your work!!!

Algebraically calculate the y-intercept of

$$y = 3(x - 5)(x + 1)$$
?

Algebraically calculate the y –intercept of

$$y = 3(x-4)^2 - 3$$
 ?

Question 11	Question 12	/5 marks		
	Show your work !!!			

A company's profit is described by

$$P = -2(n - 3000)^2 + 80000$$

What is the company's maximum profit and how many units must be sold to obtain this maximum?

Answer in a complete sentence form.

A company's profit is described by

$$P = -(n - 400)(n - 8000)$$

What is the company's maximum profit and how many units must be sold to obtain this maximum? Answer in a complete sentence form.

Question	,	Question	
13	1 marks	14	/2 marks

A company's profit is described by

$$P = -(n - 400)(n - 8000)$$

How many units must the company's sell in order to break even?

A football's flight is described by the equation:

$$h = -3(t-5)^2 + 45$$

If h is measured in feet and t is measured in seconds, how high does the football reach and how long does it take to get that high? 15

Question

/3 marks

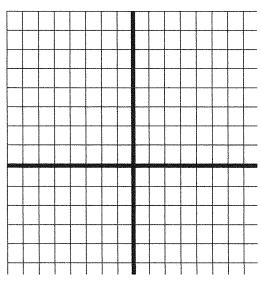
Question

16

2 marks

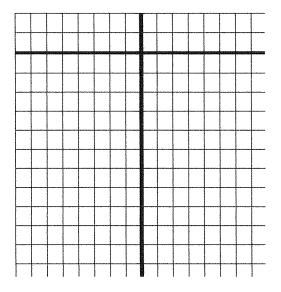
Sketch the graph of

$$y = (x - 3)(x + 1)$$



Sketch the graph of

$$y = 3(x-4)^2 - 8$$
?



Question

17

3 marks

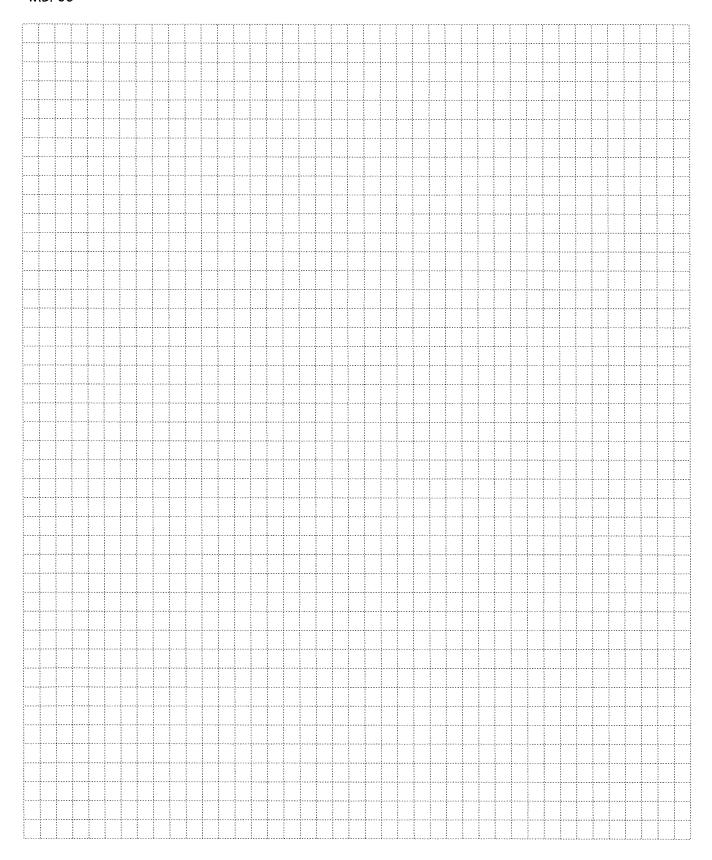
Question

18

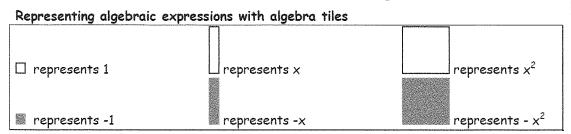
/3 marks

Write the equation of a parabola that opens down, has a step pattern of -6, -18, -30 and has a vertex located at (-3, -4)

Write the equation of a parabola that opens up, has a step pattern of 2.5, 7.5, 12.5 and has zeros of x = -2 and x = 5



Introduction to Algebra Tiles



- 1) Represent the following expressions using algebra tiles.
- a) x + 2

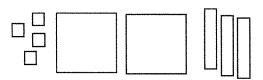
b) 3 - 2x

c) $x^2 + 2x + 4$

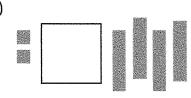
d) $x^2 - 3x - 4$

2) Write an algebraic expression that represents the following algebra tiles:

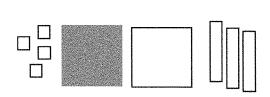
a)



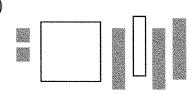
b)



c)

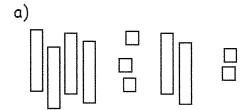


d)

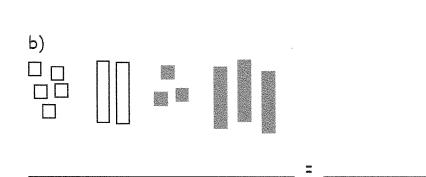


U02A22-P2

3) Simplify the following expressions using algebra tiles



=



4) Represent the following expressions using algebra tiles and then group like terms

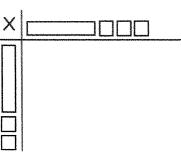
a)
$$3x - 2 - 2x + 4 =$$

b)
$$2x + 3 - 4x - 5 =$$

5) Multiply the following binomials using algebra tiles, the diagram and then algebraically

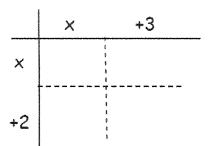
Algebra tiles

a)

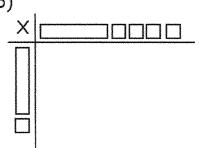


Diagram

Algebraically (x + 3)(x + 2)

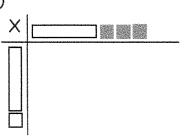


b)



1 1 2 5 5

c)



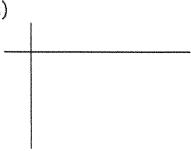
d)

	×	+3
×		1
-1		1 1 1 1 1 1 1

6) Multiply the following binomials using algebra tiles, the diagram and then algebraically

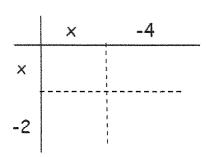
Algebra tiles

a)

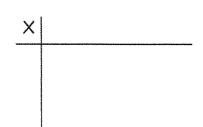


Diagram

Algebraically



b)



2x +3 x

c) _X

	f f
	1
	£ 1.
	1 1

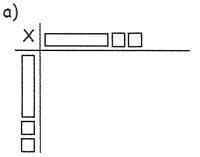
(x - 2)(2x - 1)

d)	
Х	

	5x	-8
2x		1
+3		

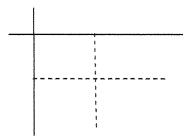
7) Multiply the following binomials using algebra tiles, the diagram and then algebraically

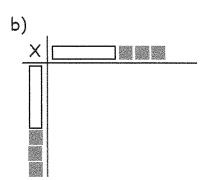
Algebra tiles

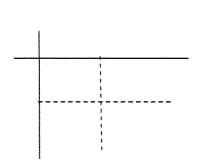


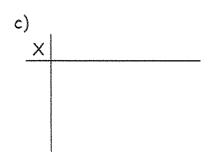
Diagram

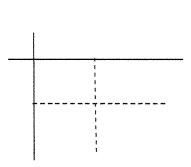
Algebraically



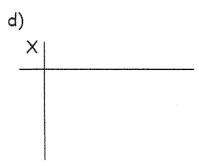


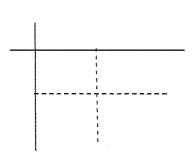






$$(2x+1)(2x+1)$$





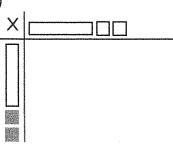
$$(x - 3)(x - 3)$$

All the above rectangles are _____

8) Multiply the following binomials using algebra tiles, the diagram et then algebraically

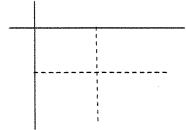
Algebra tiles

a)

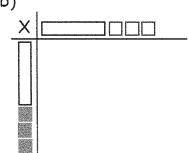


Diagram

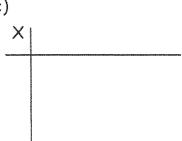
Algebraically



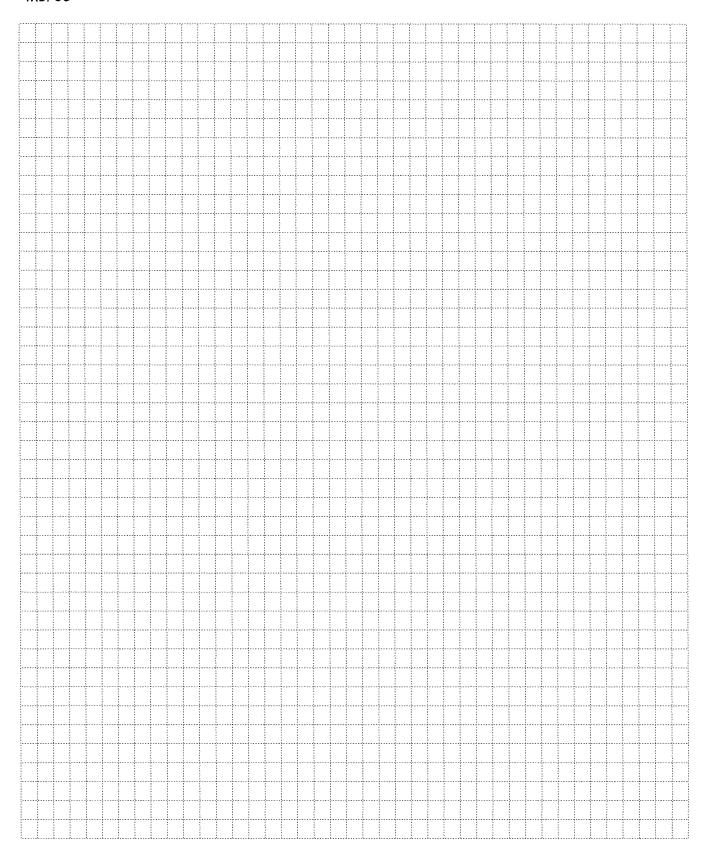
b)



c)



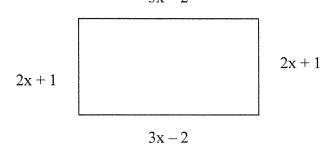
(2x - 3)(2x + 3)



A. Expand the following.

7. CAPARA INO TOROWI	W	I
1. (x + 3)(x + 4)	2. (x - 3)(x + 4)	3. (x - 2)(x - 3)
$4. \left(x+4\right)\left(x+\frac{1}{4}\right)$	5. (x - 3) ²	6. (2x + 3)(3x - 1)
7. 4(x - 2) ²	8. 5(3x - 1) ²	9. $\left(\frac{1}{2}x+1\right)(3x-2)$

B. Using the formulas for area and perimeter of a rectangle, find the area and perimeter of the rectangle shown. 3x-2

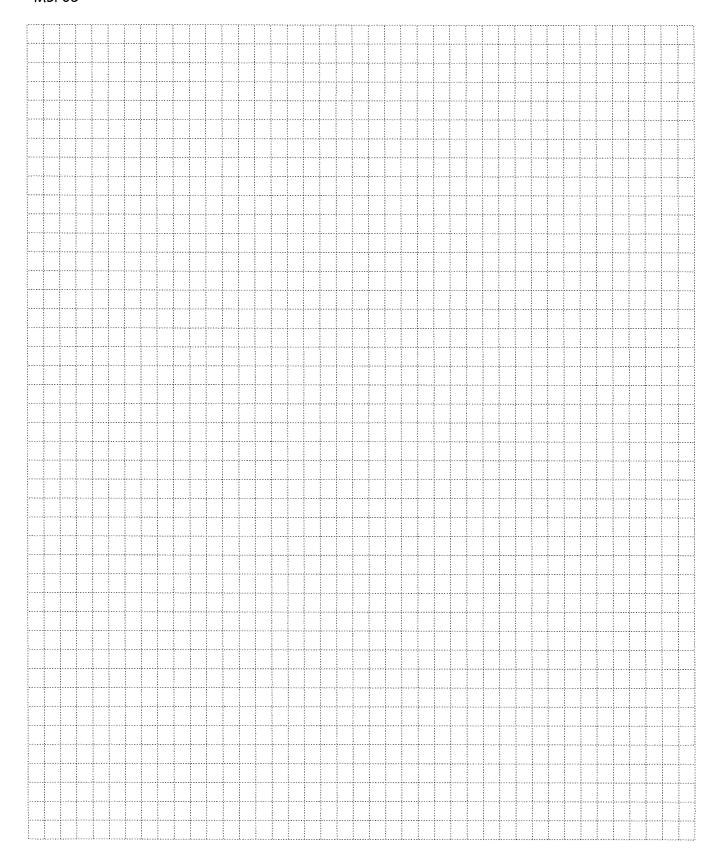


C. Using your skills of binomial multiplication expand the following.

1.
$$y = (x - 3)(x + 5)$$

2.
$$y = 2(x - 3)^2 + 5$$

3.
$$y = -3(x + 1)^2 + \frac{1}{2}$$



MBF3*C* **UO2***A***2**4

Exploring the Standard Form of a Parabola

In this investigation you will graph different parabolas and determine the information about the equation of a quadratic relation in "standard form".

You will need to be able to determine the following about a parabola:

The y - intercept

The direction of opening

The step pattern

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	U 1	\cdots		U : :	

To help you graph and plot the parabolas, enter the equation in the Y = Y

screen on your TI – 83 graphing calculator, press graph to see the graph and

press 2nd graph to see a table of values for the parabola

Parabola Investigation #1

Equation	$y = x^2 - 2x -$	- 3
Table	of Values	
X	y	
-2		
-1		
0		
1		
2		
3		
4		
	·	
	owing informat	ion
out the paral		
What is the I	1	Wha
Open	ing?	

What do you notice about the y-intercept and the equation?

Parabola Investigation # 2

Table of Values x y 0 1 1 2 3 4 5 6 Fill in the following information about the parabola: What is the Direction of Opening? How does the y – intercept relate to the equation? Parabola Investigation # 3 Equation y = 3x² + 12x + 9 Table of Values	
x y 0 0 1 1 2 2 3 3 4 5 5 6 6 Fill in the following information about the parabola: What is the Direction of Opening? What is the step pattern? What is the y-int Opening? What is the equation? Parabola Investigation # 3 Equation y = 3x² + 12x + 9	
x y 0 1 2 2 3 3 4 5 5 6 6 Fill in the following information about the parabola: What is the Direction of Opening? Clow does the y – intercept relate to the equation? Parabola Investigation # 3 Equation y = 3x² + 12x + 9	
0 1 2 3 4 5 6 Fill in the following information about the parabola: What is the Direction of Opening? Dopening? Parabola Investigation # 3 Equation y = 3x² + 12x + 9 Equation y = 3x² + 12x + 9	
1 2 3 4 5 6 Fill in the following information about the parabola: What is the Direction of Opening? Opening? Parabola Investigation # 3 Equation y = 3x² + 12x + 9 Equation y = 3x² + 12x + 9	
2 3 4 5 6 Fill in the following information about the parabola: What is the Direction of Opening? Opening? Parabola Investigation # 3 Equation y = 3x² + 12x + 9 Equation y = 3x² + 12x + 9	
3 4 5 6 Fill in the following information about the parabola: What is the Direction of Opening? Opening? Parabola Investigation # 3 Equation $y = 3x^2 + 12x + 9$	
Fill in the following information about the parabola: What is the Direction of Opening? Opening? Parabola Investigation # 3 Equation $y = 3x^2 + 12x + 9$	
Fill in the following information about the parabola: What is the Direction of Opening? Ow does the y – intercept relate to the equation? Parabola Investigation # 3 Equation y = 3x² + 12x + 9	
Fill in the following information about the parabola: What is the Direction of Opening? Tow does the y – intercept relate to the equation? Parabola Investigation # 3 Equation y = 3x² + 12x + 9	
Fill in the following information about the parabola: What is the Direction of Opening? Ow does the y – intercept relate to the equation? Parabola Investigation # 3 Equation y = 3x² + 12x + 9	_
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what is the Direction of Opening? Opening? ow does the y – intercept relate to the equation? Parabola Investigation # 3 Equation y = 3x² + 12x + 9	
What is the Direction of Opening? What is the step pattern? What is the y-int What is the y-int What is the y-int Opening? Parabola Investigation # 3 Equation y = 3x² + 12x + 9	
Opening? ow does the y – intercept relate to the equation? Parabola Investigation # 3 Equation y = 3x² + 12x + 9	1
ow does the y – intercept relate to the equation? Parabola Investigation # 3 Equation y = 3x ² + 12x + 9	ercept?
ow does the y – intercept relate to the equation? Parabola Investigation # 3 Equation y = 3x ² + 12x + 9	
Parabola Investigation # 3 Equation y = 3x ² + 12x + 9	
Equation $y = 3x^2 + 12x + 9$	
	
Table of Values	
Table of Values	
<u>x</u> <u>y</u> <u>5</u>	

Values
У

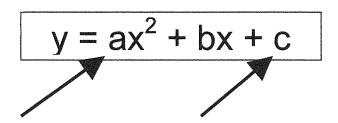
Fill in the following information about the parabola:

What is the Direction of Opening?	What is the step pattern?	What is the y-intercept?

U02A25

Standard Form of a Parabola

Standard Form of a Quadratic Relation:



This controls the direction and opening as well as the step pattern (same as in vertex form and factored form!)

This number is the y – intercept! In this case, the y – intercept would be (0, c)

Getting to Standard Form...

In order to get to standard form, some algebra is required. Let's review what we know about polynomial multiplication:

Examples: Expand & Simplify

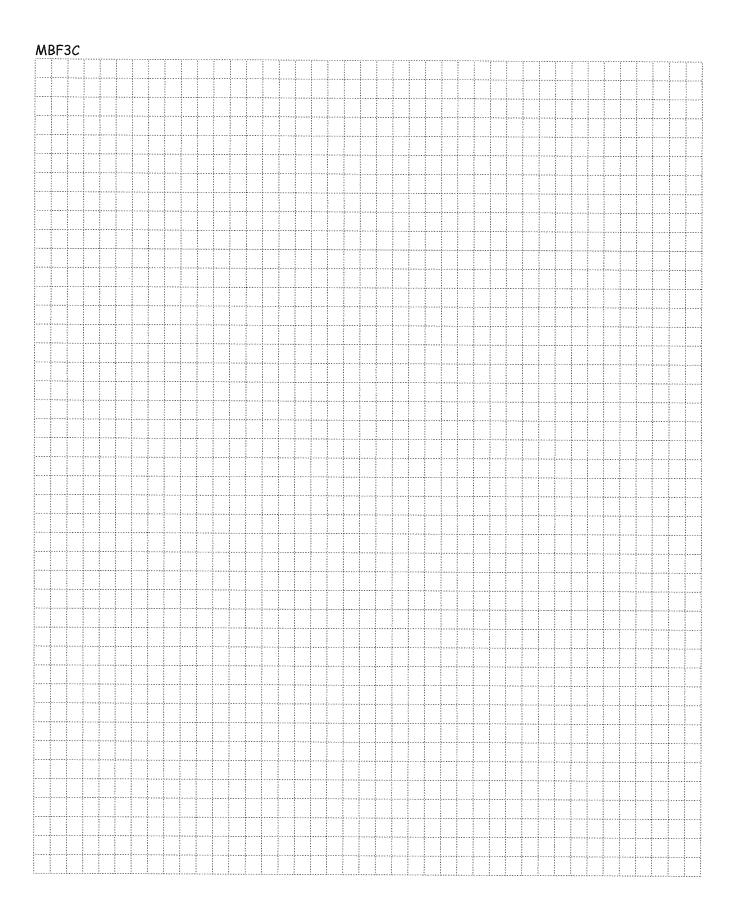
$$(x + 3)^2$$

$$3(x-4)^2$$

$$3(x-4)(x+2)$$

Practice: Expand the following expressions.

1.	(x – 5)(x + 2)	2.	$2(x-1)^2 + 3$	3.	2(x - 3)(x + 1)
4.	2(x + 4) ² - 3	5.	3(x + 2)(x – 1)	6.	- (x - 3) ² - 2



1. Match each expression in the left column with one in the right column.

(Hint: simplify)

	(HINT: SIMPLITY)		
1.	2x(x – 3)	A.	2x + 8
2.	4x - 2(x - 4)	B.	$x^2 + 3 - (3 + 6x - x^2)$
3.	$3(x^2 - 4x + 2)$	<i>C</i> .	$x(x-6) + 2(x^2 - 3x + 1)$

- 2. a) Expand to express $y = 2(x-3)^2 2$ in standard form.
 - b) Expand each of the following and compare with the equation found in a). y = -(x-2)(x-4) y = 3(x-4)(x+2)

$$y = 2(x-4)(x-2)$$
 $y = 2(x-3)(x+1)$

- c) By comparing the expanded form of the equations in 2a and 2b find the two quadratics that represent the same parabola.
- 3. List all the information you can about the parabola $y = 2x^2 4x 6$ and then find its match (in another form, of course) in question # 2

Same Parabola, Different Equation

For each of the following parabolas,

- 1. Expand the equation to standard form.
- 2. Using a graphing calculator graph both the original equation and the Standard Form equation in the same window. (Use Y1 for the Original and Y2 for the Standard Form, remember to change the curve thickness so you can see the differences)
- 3. If you have expanded the equation properly you should see the second parabola being graphed on top of the first parabola.
- 4. If you see two parabolas... go back and check your algebra!

	Original Equation	Standard Form (show your work)	Same parabola? (yes/no)
A	y = 2(x – 3)(x + 4)		
В	y = -3(x + 1)(x + 2)		
С	y = - (x - 6) ² + 12		
D	y = (x + 5) ² – 1		

Therefore, when vertex or factored form is expanded to standard form it still represents the SAME PARABOLA!

Why Standard Form?

The information about a parabola given by standard form is not as useful as the information given by vertex or factored form. So why do we use standard form?

1. Finding y-Intercepts

You can always find a y-intercept by substituting in a value of 0 for x, but if you forget this you can use standard form as well.

Example: Find the y-intercepts of $y = 2(x - 3)^2 - 14$ Solution:

Method 1: Substitute zero for x	Method 2: Expand and use equation
	·
$y = 2(x - 3)^2 - 14$	$y = 2(x - 3)^2 - 14$
$y = 2(0 - 3)^2 - 14$ *Sub $x = 0$ *	y = 2(x - 3)(x - 3) - 14
$y = 2(-3)^2 - 14$	² - 3x - 3x + 9) - 14
y = 2(9) - 14	$y = 2(x^2 - 6x + 9) - 14$
y = 18 - 14	$y = 2x^2 - 12x + 18 - 14$
y = 4	$y = 2x^2 - 12x + 4$
The y - intercept is (0, 4)	The y - intercept is (0, 4)

Practice:

Use both methods above to find the y-intercept of $y = (x + 2)^2 - 9$

2. Finding Points

If you know the x-value of a point on the parabola you can use the parabola's equation to find the y-value simply by substituting in the value for x and evaluating. BUT... if you know the y-value and want the x-value the algebra takes more work and standard form is simpler to use in this case. ** We will see more of this in a later lesson!**

3. Easier Algebra

Standard form is a "simplified" parabola equation since there are no brackets, or exponents, to be expanded. This makes certain algebraic tasks easier to accomplish. For example, a computer or graphing calculator can calculate points on a parabola more efficiently using standard form than using vertex or factored form.

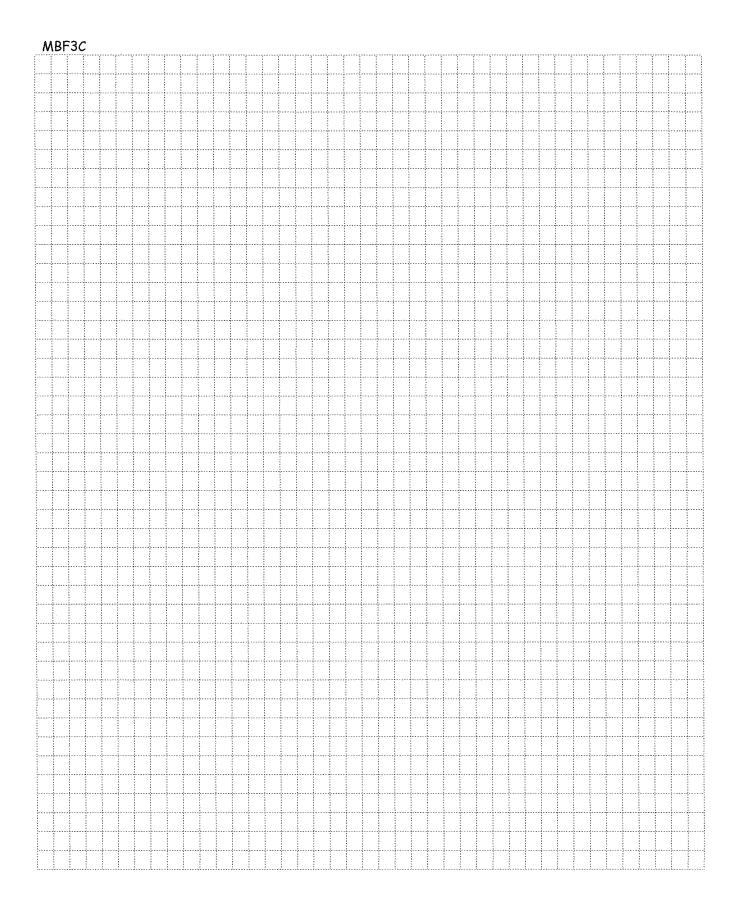
U02A29

Standard Form of a Quadratic Relation

1. Write the following quadratic relations in standard form.

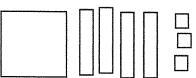
(a) $y = (x + 2)(x + 1)$	(b) $y = (x - 3)^2 - 3$	(c) $y = 2(x - 3)(x + 4)$
(d) $y = 2(x - 3)^2 - 1$	(e) $y = -3(x-2)(x+3)$	(f) $y = -(x + 2)^2 - 7$

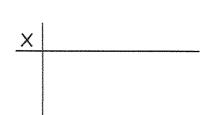
2. "When a parabola has its vertex on the y - axis the equation looks the same in vertex form and in standard form." Is this true? Provide an example as proof.



1) Create a rectangle using the following algebra tiles

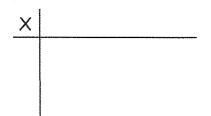






b)

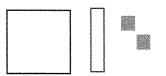




_____=

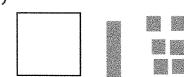
2) Create a rectangle using the following algebra tiles

a)





b)



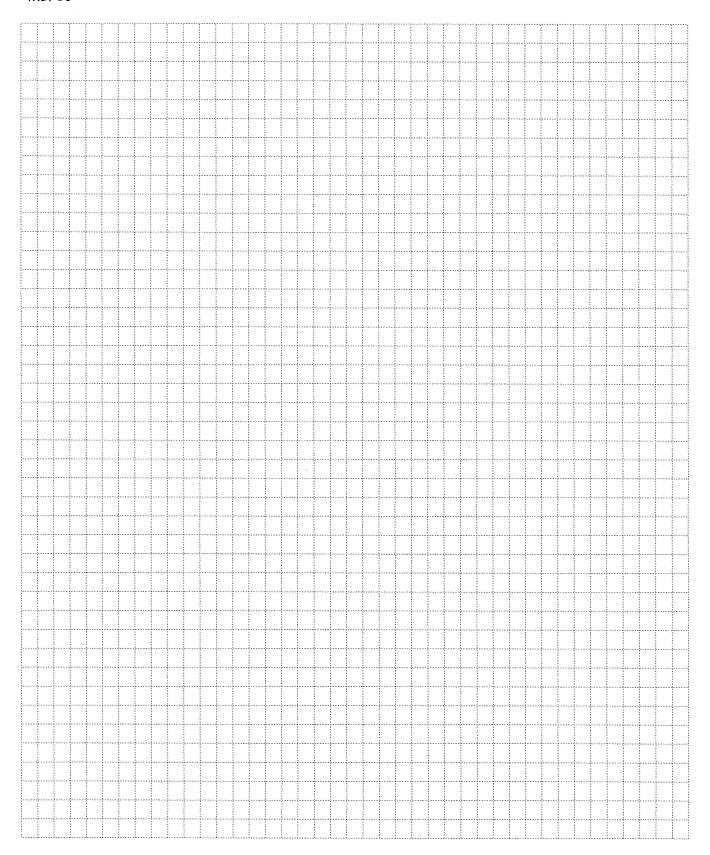
X	

_____=

U02A30-P2

3) Create a rectangle using the following algebra tile

X	X	



U02A31

Factoring Quadratic Expressions

1. Fill in the blanks with the missing numbers. (Be sure to put the plus "+" or "-" sign)

(a)
$$(x-3)(x+4) = x^2 + x$$

(b)
$$(x-6)(x+2) = x^2 ___x$$

(c)
$$(x - 1)(x + 2) = x^2 + 5x + 6$$

(c)
$$(x - 1)(x + 2) = x^2 + 5x + 6$$
 (d) $(x + 3)(x - 1) = x^2 - 6x - 27$

(e)
$$(x ___)(x ___) = x^2 + 9x + 14$$
 (f) $(x + 3)(x ___) = x^2 - 5x - 24$

(f)
$$(x + 3)(x _) = x^2 - 5x - 24$$

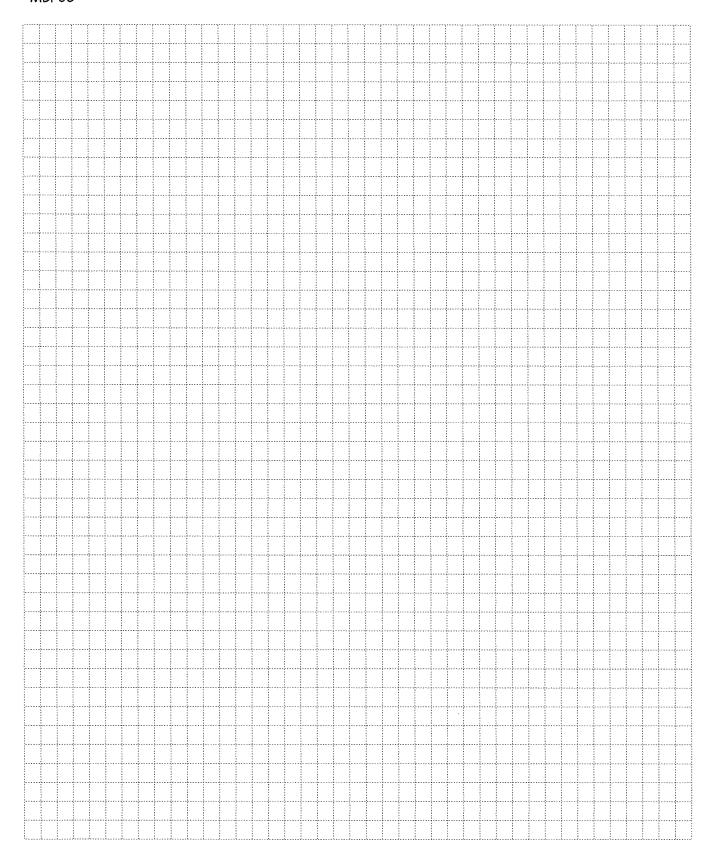
2. Factor each expression.

(a) $x^2 - 3x - 4$	(b) x ² - 11x + 28	(c) $x^2 + 7x + 12$
(d) $x^2 - 4x - 32$	(e) x ² – 13x + 42	(f) $x^2 - 4x + 4$

3. Connecting to prior lessons, by factoring standard form, we can change a parabola's equation into factored form!

Given the equation: $y = x^2 + 8x + 15$

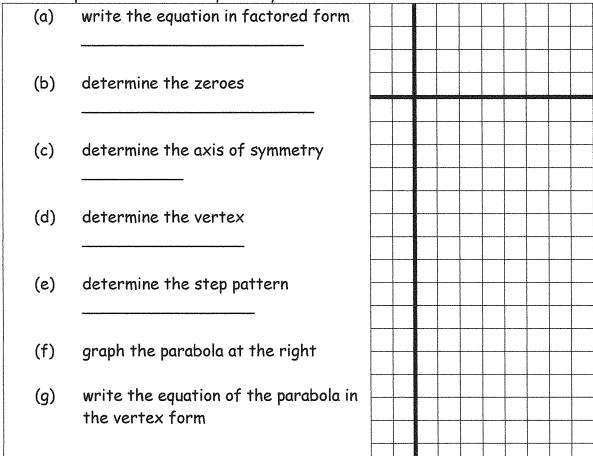
- (a) state the y - intercept _____
- (b) write the expression in factored form y =_____
- the zeros of the parabola are _____ and ____ (c)
- the vertex of the parabola is _____ (d) (hint: the vertex is located halfway between the zeros)
- the axis of symmetry of the parabola is _____ (e)



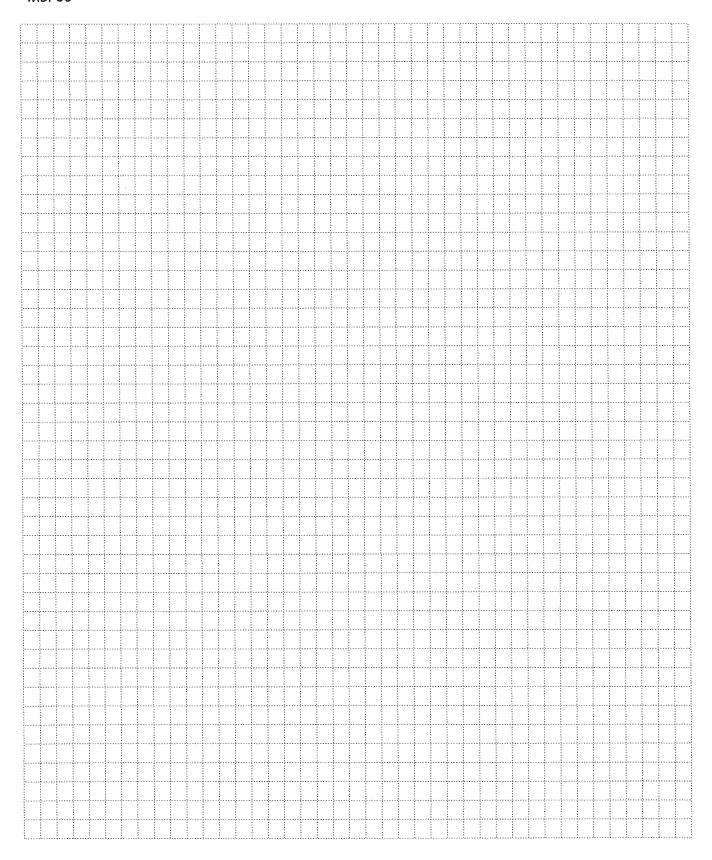
Finding Factored Form!

We can use trinomial factoring to change standard form to factored form to answer many different types of problems.

1. A parabola has the equation $y = 2x^2 - 4x - 6$

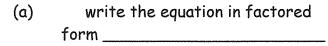


- 2. Imagine you had a quadratic relation in standard form. What steps would you take to make an accurate graph of the parabola?
- 3. A ball is thrown upwards. Its height is described by the equation $h = -5t^2 + 20t$, where h is measured in meters and t is measured in seconds.
- (a) Common factor $h = -5t^2 + 20t$,
- (b) how high is the ball at 0, 1, and 2 seconds?
- (c) Using the factored expression find the time when the ball hits the ground (hint: when h = 0)
- (d) use your answers from (c) to find the maximum height of the ball and when it occurs.

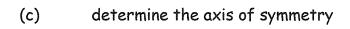


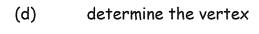
Quadratics Homework

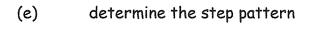
1. A parabola has the equation $y = -2x^2 + 12x - 10$



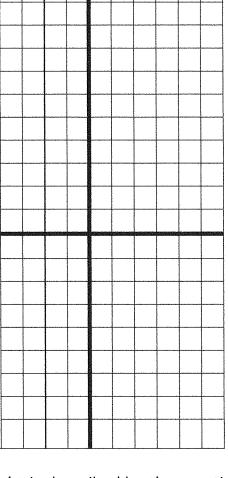
(b) determine the zeroes







- (f) graph the parabola at the right
- (g) write the equation of the above parabola in vertex form



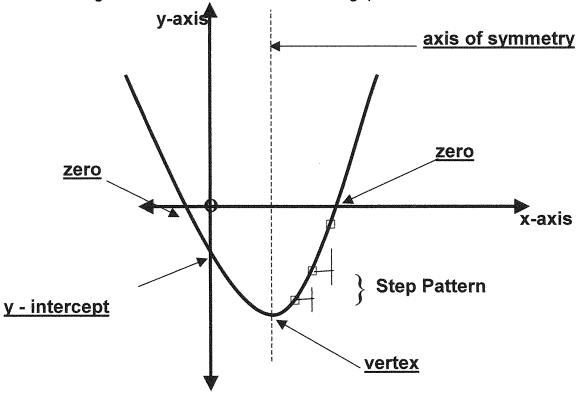
2. A cannonball is launched upwards. Its height is described by the equation $h = -5t^2 + 40t + 45$, where h is measured in yards and t is measured in seconds.

- a) how high is the cannonball at 0, 1, and 2 seconds?
- b) from what height was the cannonball launched?
- c) factor the expression to find when the cannonball hits the ground
- d) use your answers from (c) to find the maximum height of the cannonball and when it occurs.

U02A34

Summarizing Parabolas

Use the diagram below to answer the following questions.



What are the important parts of the parabola?

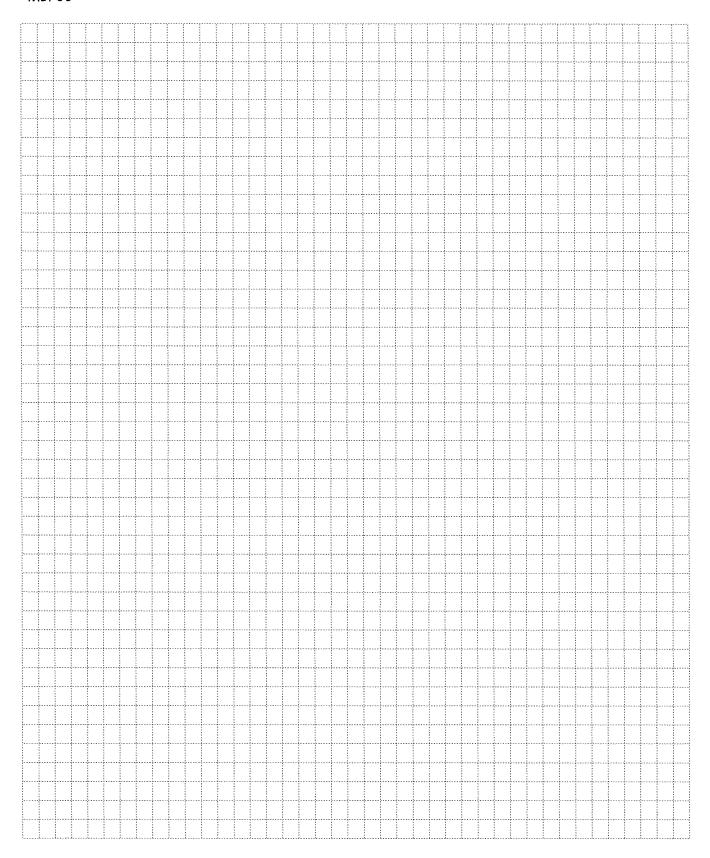
What information does each of the following algebraic representations of the parabola give us?

- i) Factored Form
- ii) Standard Form iii) Vertex Form

What information do you need to graph a parabola?

How do you factor a quadratic expression?

How do you multiply two binomials?



Problem Solving with Quadratics

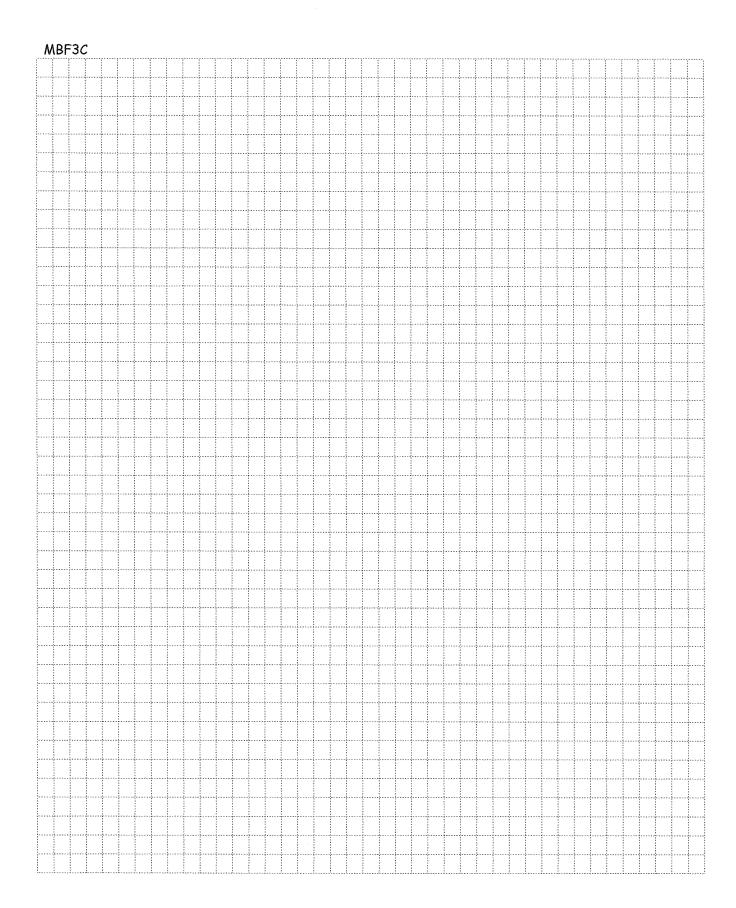
On Planet X, the height, h (in metres), of an object fired upward from the ground at 48m/s is described by the equation $h = 48t - 16t^2$, where t (in seconds), is the time since the object was fired upward.

(a) Determine the maximum height reached by the object during its flight?

(b) Determine the time(s) at which the object is 32m above the ground?

(c) Determine the time at which the object hits the ground?

(d) Determine the equation for the flight of the object in vertex form? Hint: $y = a(x - h)^2 + k$



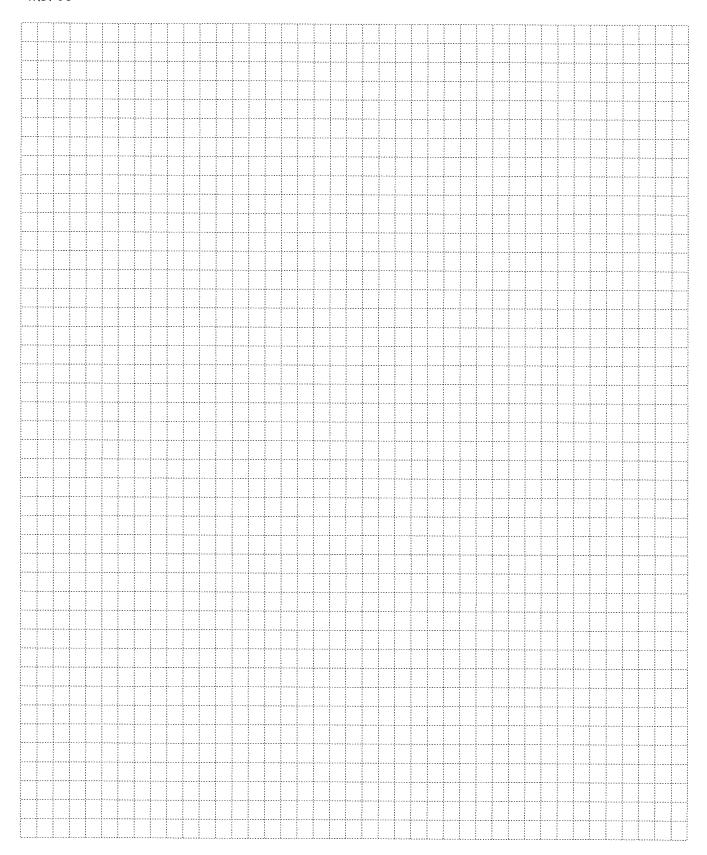
Quadratic Quandry!

The "Quadratics Cup" is a new coffee shop that sells various coffee-related items, but with a quadratics twist (like an equation written on your cup, or your caramel drizzle in the shape of parabolas, etc)

- 1) The profit of this new coffee shop can be described by the equation, $P = -4n^2 + 64n 112$, where P represents profit (in thousands of dollars) and n represents the number of customers (in ten thousands).
 - (a) What is the profit if no customers visit the store?
 - (b) What are the break-even points of the store?

- (c) What is the maximum possible profit of the store?
- (d) If a company makes a \$80 000 profit they receive a municipal Silver Business Award. How many customers are needed in order for the Quadratics Cup to receive the award?
- 2) Across the road another coffee shop has opened (It's called the "Mathees Muffins"). It's profit equation is $P = -5t^2 + 60t 100$

Answer (a) to (c) from above about the new coffee shop.



Problem Solving With Quadratics

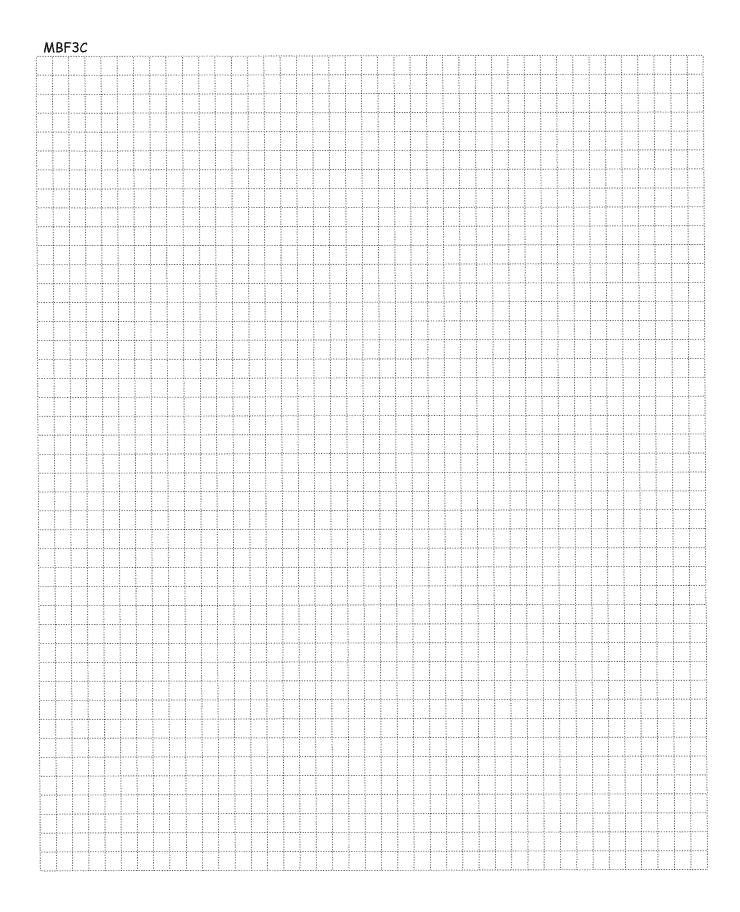
- 1. A parabola has the equation $y = 2x^2 + 4x 6$. Determine:
 - (a) the zeros of the parabola

(b) the vertex of the parabola

(c) the points where the parabola reaches a "height" of y = 10

(d) the equation of the parabola in vertex form and factored form

(e) sketch the parabola on a piece of graph paper



Problem Solving With Quadratics

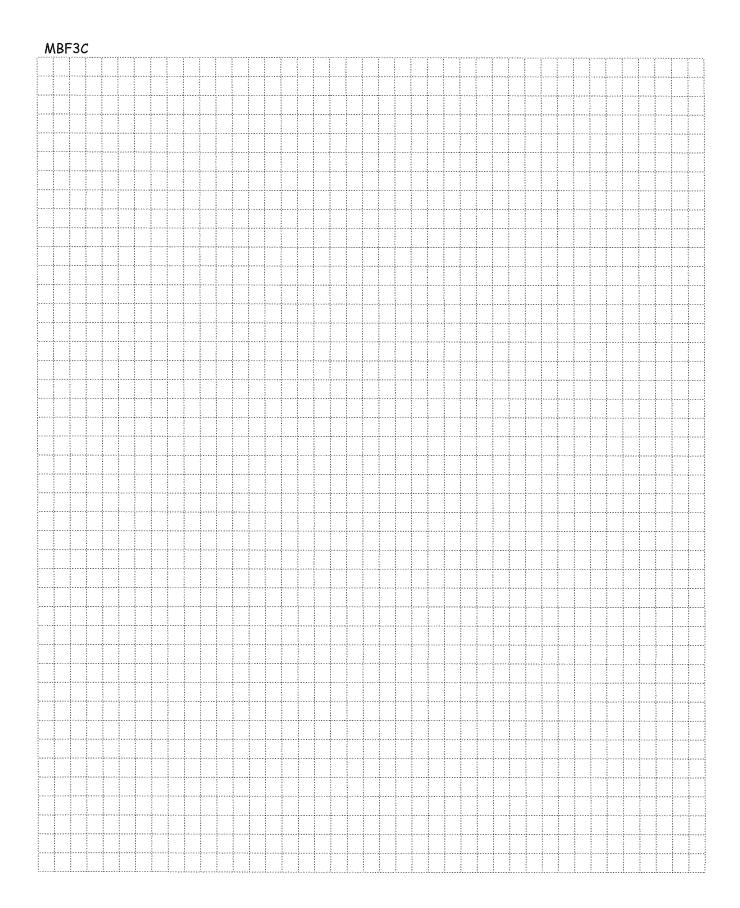
2. The effectiveness of an internet pop-up is based on how many viewers it receives per day. There is an optimal number of hits and beyond this the effectiveness drops off. The effectiveness can be described by the equation $E = -2n^2 + 18n + 44$ where E represents the percent effectiveness and n represents the number of viewers in millions of people.

Determine:

(a) the maximum effectiveness and how many viewers are needed to achieve this

(b) the number of viewers needed to make the pop-up 80% effective

(c) when the pop-up is 0% effective



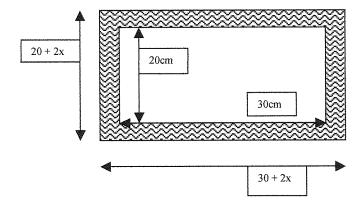
Problem Solving With Quadratics

3. EXTENSION: Yasmeen is trying to calculate the width of a matte to go around a 20cm by 30cm photograph she wants to display. She has decided that the matte will have the same area as the photo (the Home Depot book suggested this). She just isn't sure how thick the matte should be.

Using the diagram shown determine:

- (a) an expression for the area of the photo's matte (remember, the area would be the big rectangle subtract the small rectangle)
- (b) the area of the photograph
- (c) an equation for the area of the matte

- (d) simplify the equation in (c) (put it in "standard" form)
- (e) solve the equation in (d) and decide how thick the matte should be



U02A38

Quadratic Equations & Parabola Graphs - Review

			will we know	if it is QUA	ADRATIC?	(Don't forget	we know
n	nore than on	e form!)					
		· · · · · · · · · · · · · · · · · · ·		***************************************	· · · · · · · · · · · · · · · · · · ·		
		· · · · · · · · · · · · · · · · · · ·					
					····		

- 2. Given a <u>Table of Values</u>
 - *Calculate the first differences
 - *Calculate the second differences

$y = x^2 - 5$						
x	у	First Differences	Second Differences			
3						
2						
1						
0						
-1						
-2						
-3						

$y = 2x^2 - 4x - 5$							
х	y	First Differences	Second Differences				
4	11						
3	1						
2	-5						
1	-7						
0	-5						
-1	1						
-2	11						

Is there a pattern in the first differences?

Is there a pattern in the second differences?

If the first differences are the same, the relation is linear. If the second differences are the same, what can you say about the relation??

3. Given these equations, state i) The direction of the opening; ii) The y-intercept.

a)
$$y = 3x^2 + 9$$

b)
$$y = -x^2 + 4x - 3$$

i)_____

c)
$$y = -2x^2 + 6x - 1$$

$$d) y = x^2 + 5x$$

i)_____

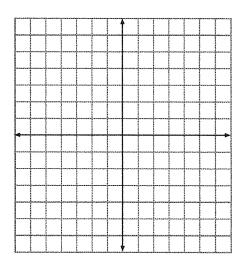
- 4. Given these equations, state i) The direction of the opening;
 - ii) The zeroes
- a) y = -(x+3)(x+6)
- b) y = (x + 2)(x 3)

- c) y = -(x-2)(x-8)
- i) _____ i) ____ ii) ____ iii) _____
- 5. Given the equation, state: i) The direction of the opening
 - ii) The zeroes
 - iii) The y-intercept

Then GRAPH.

$$y = (x + 1)(x - 3)$$

- ii)
- iii)



Where should the line of symmetry be? (Remember, it's halfway between the zeroes.) Mark this on the graph as well.

6. MULTIPLE CHOICE

Which equation is most likely to match with the graph shown? (Hint: Eliminate the ones it cannot be.)

a)
$$y = -(x+4)(x-2)$$

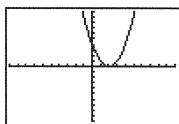
OR b)
$$y = (x + 4)(x - 2)$$

OR c)
$$y = x^2 + 2x - 2$$

OR d)
$$y = (x-2)(x-2)$$

OR e) $y = -x^2 + 3x + 3$

OR e)
$$y = -x^2 + 3x + 3$$



7. Expand the following equations, from Factored form into Standard form:

a)
$$y = (x-2)(x-5)$$

b)
$$y = (x-3)(x+3)$$

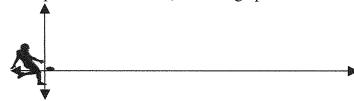
c)
$$y = -(x+5)(x+4)$$

d)
$$y = (x-1)(x-1)$$

8. A football player kicks a ball into the air; it's flight can be modelled by the equation: y = -x(x-6) (y is the height of the ball in metres;

x is the time taken in seconds)

a) Given that the zeroes of this equation are +6 and 0, sketch a graph:



- b) What is the maximum height reached by the ball? (Hint: You need the vertex.)
- c) At x = 5, there is a player 1.5 m tall who wants to make the catch. Determine y (the height) at this x-value. Will the player be able to catch the ball?

BONUS: On the back, sketch a parabola which has no x-intercepts.

U02A38-P4

I Binomial Multiplication:

1.
$$(x + 3)(x + 2) =$$

2.
$$(x-5)(x+3)$$
 =

3.
$$(x-4)(x-\frac{1}{2})$$
 =

4.
$$(2x + 1)(x + 2)$$
 =

5.
$$(x + 4)^2$$
 =

6.:
$$2(x+1)^2$$
 =

7.
$$2(x-3)^2+5$$
 =

8.
$$-(x-3)^2-2$$
 =

II Quadratic Equation in Standard Form:

9. Write the following equations in standard form and identify the y-intercepts:

a)
$$y = 2(x - 3)^2 - 1$$

b)
$$y = -3(x - 2)(x + 3)$$

c)
$$y = -(x + 2)^2 - 7$$

10. Use two methods to find the y-intercept of $y = (x - 3)^2 + 4$

11. Are $y = (x + 5)^2 - 1$ and $y = x^2 + 10x - 24$ the same parabola? Show work!

III Factoring:

12. Factor the following expressions:

a)
$$x^2 + 5x + 6 =$$

b)
$$x^2 + 3x - 18 =$$

c)
$$x^2 - 9x + 20 =$$

d)
$$x^2 - 10 x + 16 =$$

e)
$$x^2 + 8x + 16 =$$

f)
$$x^2 - 17x + 72 =$$

$$g) x^2 + 2x - 48 =$$

U02A38-P6

IV Info from Standard Form:

13. Given the equation: $y = x^2 - 2x - 3$

(a) state the y - intercept _____

(b) write the expression in factored form y =

(c) the zeros of the parabola are _____ and ____

(d) symmetry of the parabola is _____

(e) the vertex of the parabola is _____ the axis of

14. A parabola has the equation $y = 2x^2 + 4x - 6$. Determine:

(a) the zeros of the parabola

(b) the vertex of the parabola

(c) the points where the parabola reaches a "height" of y = 10

(d) the equation of the parabola in vertex form and factored form

(e) sketch the parabola

Answers: 1) $x^2 + 5x + 6$ 2) $x^2 - 2x - 15$ 3) $x^2 - \frac{9}{2}x + 2$ 4) $2x^2 + 5x + 2$ 5) $x^2 + 8x + 16$ 6) $2x^2 + 4x + 2$ 7) $2x^2 - 12x + 23$ 8) $-x^2 + 6x - 11$ 9) (a) $y = 2x^2 - 12x + 17$ 8, (b) $y = -3x^2 - 3x + 18$, (c) $y = -x^2 - 4x - 11$ 10) y-int=13 11) no 12) a) (x + 2)(x + 3) b) (x + 6)(x - 3) c) (x - 4)(x - 5) d) (x - 2)(x - 8) (e) (x + 4)(x + 4) f) (x - 8)(x - 9) g) (x + 8)(x - 6) 13 (a) 0,-3 (b) (x - 3)(x + 1) (c) 3 and 1 (d) 1 (e) 1,-4 14 (a) -3 and 1, (b) (-1, -8), (c) (2, 10) and (-4, 10) (d) $y = 2(x + 1)^2 - 8$ and y = 2(x + 3)(x - 1)

QUADRATICS - Practice Test Part I

	Incomplete 0	Unacceptable 3.0 4.0	Poor 5.2 5.5 5.8	Acceptable 6.2 6.5 6.8	Good 7.2 7.5 7.8	Outstanding 8.4 8.9 9.5 10
Technical correctness of solutions	All or most solutions are blank	No solutions are correct or many left blank	Few solutions are technically correct	Some solutions are technically correct	Most solutions are technically correct	All or almost all solutions are technically correct
Presentation of solutions	All or most solutions are blank	No evidence of presentation or many solutions left blank	Solutions to few problems stand alone	Solutions to some problems can stand alone	Solutions to <u>most</u> problems can stand alone	Solutions to <u>all</u> or <u>almost all</u> problems can stand alone

Part A - The following will be marked for Knowledge/Understanding. (23 marks)

1. Circle the equation of the function that is **not** quadratic:

a)
$$y = -x^2 - 4x + 2$$

b)
$$y = \frac{1}{x} + 1$$

c)
$$y = (2 - x)^2$$
 d) $y = 9x^2 - 8$

d)
$$y = 9x^2 - 8$$

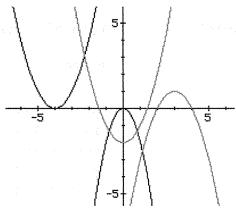
- 2. True or False: the parabola $y = -3x^2 + 2$ opens up.
- 3. T or F: The vertex is always located halfway between the zeroes.
- 4. The point where the graph cuts the y-axis is called
- 5. The vertical line which divides a parabola in half is called
- 6. The maximum or minimum point on a parabola is called
- 7. Use the vertex to match each graph with its defining equation:

a)
$$y = (x + 4)^2$$

b)
$$y = -2x^2$$

c)
$$y = x^2 - 2$$

d)
$$y = -(x-3)^2 + 1$$



8. Which of the following are quadratic? Explain how you know. (3 marks)

a)
$$y = 3x - 15$$

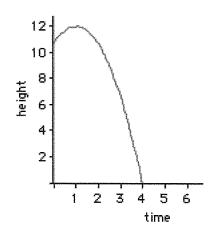
b)
$$y = 4x^2 - 2x + 8$$



Х	У
0	-32
1	-18
2 3	-8
3	-2
4	0
5	-2

Part B – The following will be marked for Application: (11 marks)

1. The following graph shows the height, in metres, of a platform diver, over time, in seconds. (3 marks)



a) What is the maximum height of the diver?

b) When did he reach maximum height?

c) What happened at the 4 second mark?

(4 marks)

2. A football player kicks a football and the path can be modeled by the relation $H = -0.091(d - 18)^2 + 30$ where H represents the height above the ground and d represents horizontal distance from the player, both in metres. (2 marks)

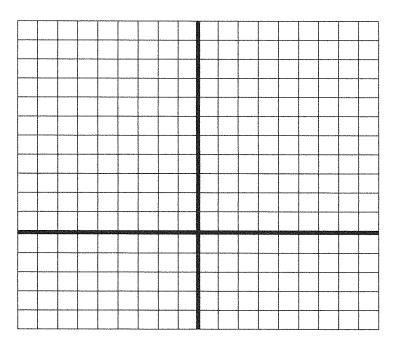
a) What is the football's maximum height?

b) How far away from the player is the ball when it is at that height?

3. One of the largest solar furnaces in the world is in Odeillo, France. The parabolic mirror is 54 m wide and 10 m deep. Write a relation that **might** model the parabolic shape of the mirror. Explain your model. (2 marks)

4. a) Sketch the graph of the parabola that opens upward, has a vertex of (2,-4), and passes through point (4,8).

b) Determine its equation.



Part C - The following will be marked for TIPS: (5 marks)

A decorative arch is to be built over a fountain at the Mathematician's Hall of Fame. The arch will be in the shape of a parabola and be 4m tall at the centre. Inscribed on a plaque located near the fountain will be the equation of the arch: $y = -0.25(x - 4)^2 + 4$

(a) graph the shape of the arch, using the grid below where **one square represents 0.25m**. (the entire arch will not be graphed!)

- (b) How far apart are the "feet" of the arch?
- (c) At what points is the arch 2m high? Find a graphical **and** algebraic way of answering this question.

$QUADRATICS\ Practice\ Test\ Part\ II$

<u>Part A</u> - The following will be marked for Knowledge/Understanding.

i. irue c	or false! (5 marks)												
• The p	parabola $y = (x - 5)^2$	has been shifted left.						-			···		
• The j	parabola $y = 0.2x^2$ is	wider than $y = x^2$.											
• The p	parabola $y = (x + 4)^2$	- 1 has a vertex of (-4, 1)	١.					*****	V-1				
• The j	parabola $y = x^2 + 7 h$	as been shifted horizonta	ally.										
The	parabola $y = (x - 3)^2$	has a vertex of (-3,0)											
	e following words to than once. (9marks)	o complete the statement	ts. Not a	ıll wor	ds ar	e us	ed a	and	son	ne w	ord	s can	be u
•	actored form eros	standard form y-intercept		x form pattern					is of rtex	sym	met:	ry	
The f	factored form of a pa	arabola easily shows us t	he										
The y	y-intercept is easy to	find when using the						_of	a pa	irab	ola.		
The		can be calcul	lated by	using	the z	eros	of a	an e	qua	tion			
The	vertex is easy to find	when using the						_of	the	para	abol	a.	
The		are also known as th	ne break-	even p	oint	s.							
The		can tell us if a paral	bola ope	ns up	or do	wn.							
		at a ball reaches when too											
	-	nd level is known as the						*********		***************************************			
	_					s the							
If tha	it ball is tossed on o	f a water tower, the start	mg neig	nt wot	na be	e uie	2		**********	•			
		$quation y = 2x^2 - 4x -$	- 6			(10	mai	rks)	,	·····	,		
(a)	write the equation	on in factored form											
(b)	determine the ze	eroes											
(c)	determine the ax	ris of symmetry											
(d)	determine the ve	ertex		_									
(e)	determine the st												
` ,		ep pattern							1				
(f)		ep pattern											
	graph the parabo	ep pattern ola on the grid above											
(g)			vertex										
(g)		ola on the grid above	vertex										

<u>Part B</u> - The following will be marked for Application.

1. Change the equations as directed and fill in the blank. (12 marks)

 $y = 3(x - 2)^2 + 1$ a)

identify the vertex then change to standard form

 $y = x^2 - 2x - 15$ b)

change to factored form and identify the zeros

c) y = (x - 2)(x - 3) identify the zeros then

change to standard form

y = -2(x - 1)(x - 5) change to standard form and d)

identify the step pattern

2. a) Write an expression, in simplified form, for the area of the rectangle. (3 marks)

3x - 2

2x + 1

b) Find the area of the rectangle when x = 5 cm

U02A40-P3

3. The equation $h = -2(t-1)^2 + 12$ models the dive of a platform diver where h is the height, in metres, of a platform diver, over time t, in seconds. What is the maximum height of the dive and when did it occur? (2 marks)



4. A ball is thrown upwards. Its height is described by the equation $h = -5t^2 + 20t$, where h is measured in meters and t is measured in seconds. (5 marks)



- (b) Using the factored expression find the time when the ball hits the ground
- (c) use your answers from (b) to find the maximum height of the ball and when it occurs.

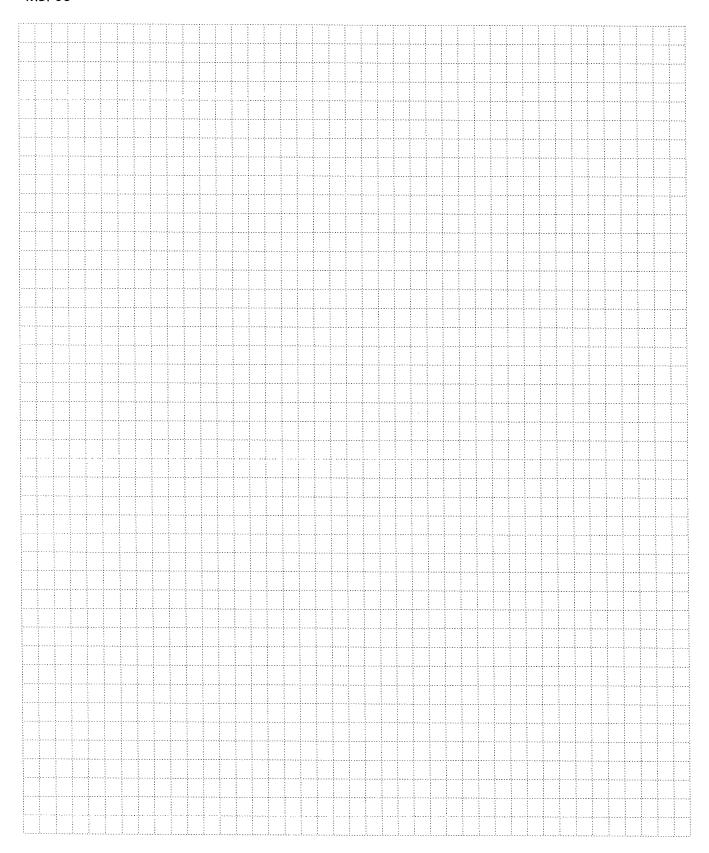
<u>Part C</u> - The following will be marked for TIPS. (5 marks) Choose one of questions A or B.

- A. Find the y-intercept of $y = (x 3)^2 + 4$ using the method of your choice. Make a sketch of the parabola.
- B. Are $y = (x + 5)^2 1$ and $y = x^2 + 10x 24$ the same parabola? Show work!

	Incomplete 0	Unacceptable 3.0 4.0	Poor 5.2 5.5 5.8	Acceptable 6.2 6.5 6.8	Good 7.2 7.5 7.8	Outstanding 8.4 8.9 9.5 10
Technical correctness of solutions	All or most solutions are blank	No solutions are correct or many left blank	Few solutions are technically correct	Some solutions are technically correct	Most solutions are technically correct	All or almost all solutions are technically correct
Presentation of solutions	All or most solutions are blank	No evidence of presentation or many solutions left blank	Solutions to few problems stand alone	Solutions to <u>some</u> problems can stand alone	Solutions to most problems can stand alone	Solutions to <u>all</u> or <u>almost all</u> problems can stand alone

MBF3C Unit 3 - Exponential Relations

Section	Date	Lesson Title	Handouts	Homework Assigned
1		Exponent Rules (Product, Quotient & Power Laws)	U03A01	U03A02
2		Negative and Zero Exponents	U03A03	U03A04 U03A05
3		More Exponents Practice	Quiz	Supplement (Handout not Provided)
4		Comparing Models of Growth	U03A06 U03A07 - (in Appendix)	U03A08
5		Properties of Exponential Relations	U03A09	
6		Applications of Exponential Relations	U03A10 U03A12	U03A11 U03A13
7		More Exp. Growth/Decay Word Problems	U03A14	1000
8		Cat's & Mice Assignment	U03A15	· · · · · · · · · · · · · · · · · · ·
9		Review	U03A16	U03A17
10		Practice Test	U03A18	
11		Unit Test		



Exponents Take I

In your groups of 3, label yourselves Partner A, Partner B, and Partner C.

- Each partner completes their respective section of the investigation.
- When your group is finished, check your answers and complete Part B.
- Each group member should be prepared to speak about their findings.

Part A

For question 1-3, Partner A completes (a), Partner B completes (b), and Partner C completes (c). Keep fractions in fraction form.

1. a) Fill in the table

a.
$$3^2 \times 3^3 =$$

$$(\frac{1}{2})^2 \times (\frac{1}{2})^3 =$$
 $(\frac{1}{2})^5 =$

$$(.1)^5 \times (.1)^3 =$$
 $(.1)^8 =$

b.
$$2^3 \times 2^5 =$$

$$(\frac{1}{3})^2 \times (\frac{1}{3})^1 = \frac{1}{3}$$

$$(.2)^3 \times (.2)^4 =$$
 $(.2)^7 =$

c.
$$5^2 \times 5^4 =$$

$$\left(\frac{1}{4}\right)^3 \times \left(\frac{1}{4}\right)^2 =$$
 $\left(\frac{1}{4}\right)^5 =$

- b) Describe what you notice.
- 2. a) Fill in the table

a.
$$2^8 \div 2^5 =$$

$$(\frac{1}{4})^3 \div (\frac{1}{4})^1 =$$

$$(.2)^7 \div (.2)^3 =$$

$$(\frac{1}{4})^2 =$$

b.
$$5^6 \div 5^2 =$$

$$(\frac{1}{2})^5 \div (\frac{1}{2})^3 =$$

$$(.3)^6 \div (.3)^5 =$$

$$(\frac{1}{2})^2 =$$

c.
$$3^5 \div 3^2 =$$

$$(\frac{1}{3})^5 \div (\frac{1}{3})^3 = \frac{1}{3}^2 = \frac{1}{3}$$

$$(.1)^8 \div (.1)^5 =$$
 $(.1)^3 =$

- b) Describe what you notice.
- 3. a) Fill in the table

a.
$$(2^3)^2 =$$

$$(.3^2)^2 =$$

b.
$$(3^2)^3 =$$

$$(5^3)^2 = 5^6 =$$

$$(\frac{1}{3}^4)^2 =$$

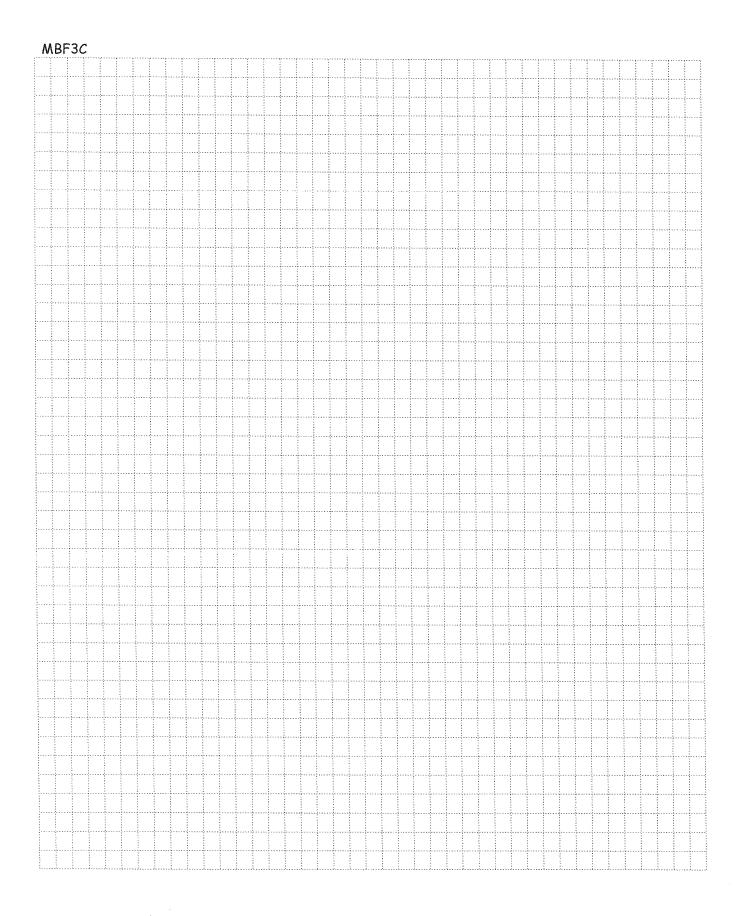
c.
$$(.1^3)^2 =$$

$$(2^2)^4 =$$

b) Describe what you notice.

<u>Part B</u>

- 1. As a group, summarize your findings:
- 2. Other important points:



Exponents Take I

Evaluate. Rewrite negative exponents and evaluate as fractions.

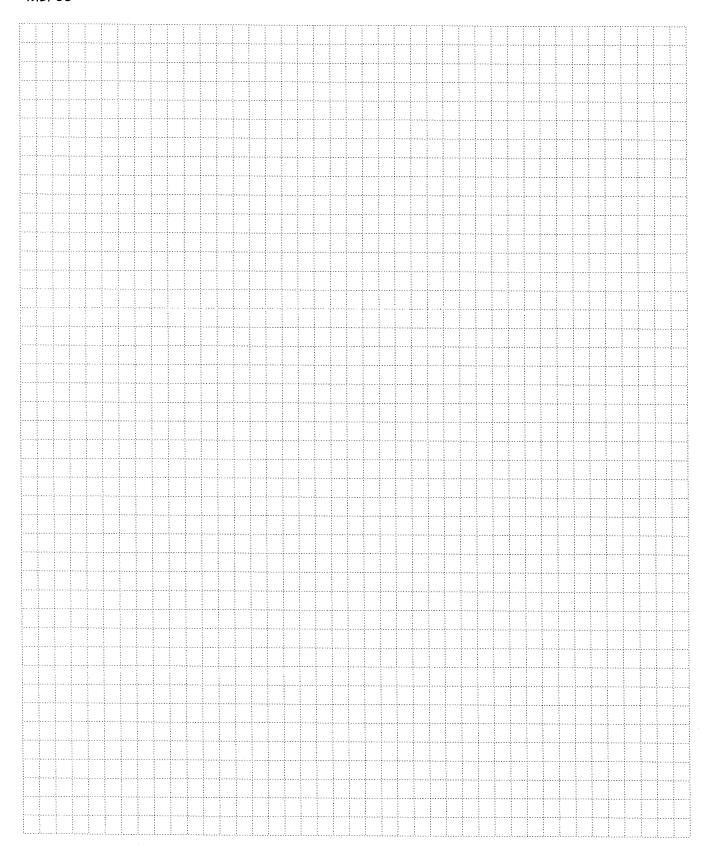
	re exponents and evaluate a	s tractions.	
1) 5 ⁴ x 5 ²	2) 2 ⁴ x 2 ⁻²	3) 3 ⁻³ x 2 ⁻²	4) 8 ⁻³ x 8 ⁵
5) 4 ³ × 4 ⁻²	6) 6 ² × 6 ⁴	7) 7 ⁻³ x 7 ⁶	8) 5 ⁴ x 5 ⁻²
9) 4 ⁴ ÷ 4 ²	10) 3 ² ÷ 3 ⁵	11) 2 ÷ 2 ⁻²	12) 5 ⁻² ÷ 5 ⁻²
13) 7 ³ ÷ 7 ²	14) 10 ⁷ ÷10 ⁴	15) 6 ⁴ ÷ 6 ⁻³	16) 8 ⁻² ÷ 8 ⁻¹
17) (2 ³) ⁴	18) (3 ²) ³	19) (4¹) ⁵	20) (5 ⁻²) ³
21) (4 ²) ⁻³	22) (2 ⁻³) ⁻⁴	23) (3 ⁻²) ⁻³	24) (4 ⁻¹) ⁻⁵
25) (2 ⁻³) ³	26) 3° × 3° × 3° =	3° 27) 4	$a+b \times 4^{a-b} = 4^{?}$

Exponents Take 1

PART A: , 1.a. $9 \times 27 = 243$; $\frac{1}{4} \times 1/8 = 1/32$; 0.00001 \times 0.001 = 0.00000001, b. $8 \times 32 = 256$; $1/9 \times 1/3 = 1/27$; 0.008 \times 0.0016 = 0.0000128, c. $25 \times 625 = 15625$; $1/64 \times 1/16 = 1/1024$; 0.3 \times 0.00243 = 0.000729, (answers will vary)Each pair of answers is the same. It looks like the exponents get added., 2.a. $256 \div 32 = 8$; $1/64 \div 1/4 = 1/16$; 0.0000128 ÷ 0.008 = 0.0016 b. $15625 \div 25 = 625$; $1/32 \div 1/8 = 1/4$; 0.000729 ÷ 0.00243 = 0.3, c. $243 \div 9 = 27$; $1/243 \div 1/27 = 1/9$; .00000001 ÷ 0.00001 = 0.001, (answers will vary)Each pair of answers is the same. It looks like the exponents get subtracted., 3.a. $(2^3)^2 = (8)^2 = 64$; $(3^2)^2 = (0.09)^2 = 0.0081$; $(\frac{1}{2}^4)^1 = (1/16)^1 = 1/16$, b. $(3^2)^3 = (9)^3 = 729$; $(5^3)^2 = (125)^2 = 15625$; $(\frac{1}{3}^4)^2 = (1/81)^2 = 1/6561$, c. $(1^3)^2 = (0.001)^2 = 0.000001$; $(2^2)^4 = (4)^4 = 256$; $(\frac{1}{4}^2)^3 = (1/16)^3 = 1/4096$, (answers will vary) Each pair of answers is the same. It looks like the exponents get multiplied. PART B: (ANSWERS WILL VARY), students ought to develop the laws of exponents; $m^2 \times m^6 = m^{a+b}$; $m^a \div m^b = m^{a-b}$; $(m^a)^b = m^{ab}$

Solutions

1. 15625 2. 4 3. 1/243 4. 64 5. 4 6. 46656 7. 343 8. 25 9. 16 10. 1/27, 11. 8 12. 1 13. 7 14. 1000 15. 279 936 16. 1/8 17. 4096 18. 729 19. 1024 , 20. 1/15625 21. 1/4096 22. 4096 23. 729 24. 1024 25. 1/512 26. $3^{\text{a-b-c}}$ 27. 4^{2a}



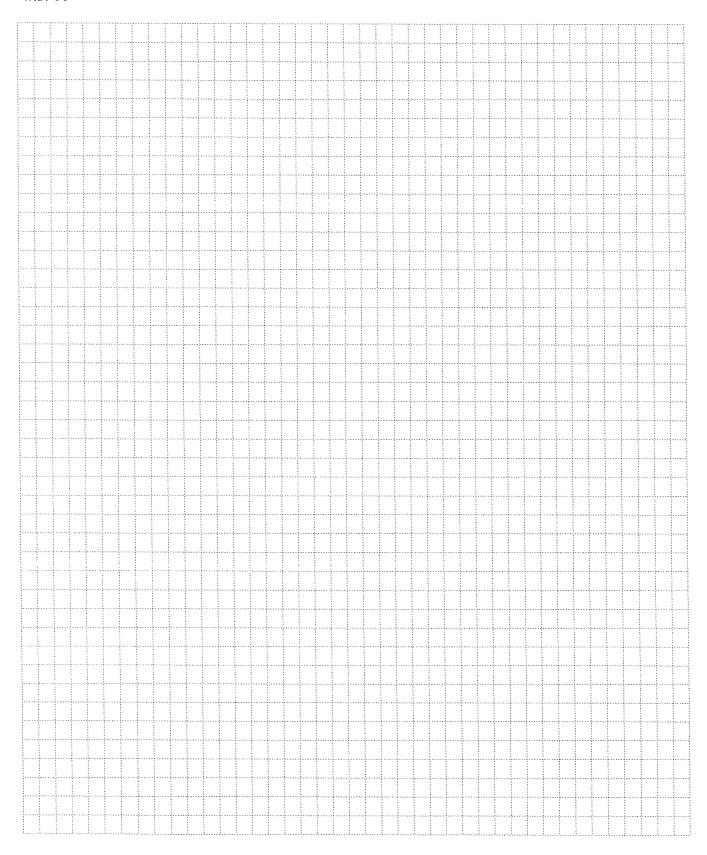
1. Fill in the chart

. Fill in the char	T	
Expression to be simplified	Expanded Form	Using Exponent Laws
$\frac{2^3}{2^1}$	$\frac{2x2x2}{2}$	$2^{3-1} = 2^2$
$\frac{2^3}{2^2}$	= 4	= 4
$\frac{2^3}{2^3}$		
$\frac{2^3}{2^4}$		
$\frac{2^3}{2^5}$		
$\frac{2^3}{2^6}$		

2. What do you notice about how the exponential expression is related to the simplified expanded expression?

3. Fill in the following Table.

J. [111 111 1	116 101108	ving ruble							
Exponent Form	3 ²		10°	3 ⁻³		6°	2 ⁻ⁿ		1250000°
Simplified		1			1			1	
Form		5			$\frac{1}{3}$			5 ^m	



Exponents: Take 2

Evaluate, using a calculator. Round to 4 decimal places where necessary.

- a) $(1.005)^{12}$
- **b)** 3.76³

c) 5^6

- **d)** 2.15⁴
- e) 4^6

f) (17)°

b) 2¹²

- **b)** (1.045)⁶
- $(2.12)^1$

h) 25^3

i) 1.036⁻³

i) 1.0075⁻¹²

- k) 2.16⁻⁴
- 1) 3-3

 $m) 2^{-5}$

- $n) (7.065)^{\circ}$
- o) 11⁻¹

2. Identify which of the expressions could have been completed without the use of a calculator. Explain.

Rewrite the following in simplified fraction form:

a) 3^{-3}

b) 2⁻⁵

c) 11⁻¹

4. Evaluate. Rewrite negative exponents and evaluate as fractions.

- a) $7^0 =$
- **b)** 16⁰=
- c) $-(16)^{0}$ =
- d) $(-5)^0$ =
- $e) (5^{\circ}) =$

- $f) 5^4 =$
- g) $(6)^3=$
- h) $7^3 =$
- i) $2^4 =$
- $j) 3^3 =$

- **k)** $(-3)^3$ =
- 1) $(-2)^4$ =
- m) $(-5)^3$ =
- n) $(-6)^4$ =
- o) $(-11)^1 =$

- p) 3^{-2} =
- r) 5^{-2} =
- s) $2^{-5}=$
- +) $-(5^4)=$

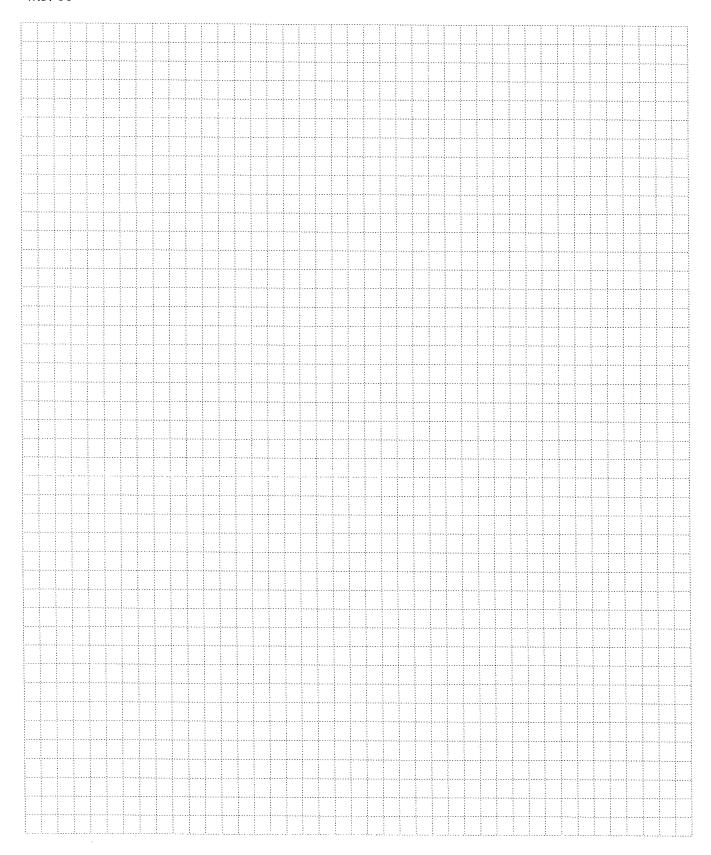
- u) $-(4)^{-3}=$ v) $-(3^{-2})=$
- **w)** $-(-3)^3$ =
- $x) -(6)^3 =$
- $y)-(-2)^5=$

Solutions

1. 1.0617; 53.1574; 15625; 21.3675; 4096; 1; 4096; 1.3023; 2.12; 15625; 0.8993; 0.9142; 0.0459; 0.0370; 0.03125; 1; 0.0909 2. (answers will vary): anything with a zero or 1 exponent could have been done without a calculator because of the exponent rules. Also, negative exponents (smaller digits) like 3^{-3} can be simplified in fraction form. 3. 1/27; 1/32; 1/11 4. a) 1 b) 1 c) -1 d) 1 e) -1 f) 625 g) 216 h) 343 i) 16 j) 27 k) -27 l) 16 o) -11 p) 1/9 or 0.11111 q) 1/64 or 0.015625 r) 1/25 or 0.04 s) 1/32 or 0.03125 m) -125 n) 1296 t) -625 u) -1/64 or -0.015625 v) -1/9 or -0.1111 w) 27 x) -216 y) 32

- 1. Evaluate WITHOUT a calculator: (decimal equivalents are not acceptable)
 - a) $2^3 =$
 - **b)** 3⁴=
 - c) $(2^3)^2$ =
 - d) $(-4)^3$ =
 - e) 2⁻³=
 - f) $-(5^3)=$
 - g) $(-5)^3$ =
 - h) $(5)^{-3}$ =
- 2. Simplify
 - a) $3^5 \times 3^2 =$
 - **b)** $2^{11} \div 2^6 =$
 - c) $4^2 \times 4^{-3} \times 4=$
 - **d)** $(5^6 \times 5^2) \div 5^5 =$
 - e) $2^6 \div 2^3 =$
 - **f)** $(3^6 \div 3^2) \div 3^4 =$
 - **g)** (-1335)⁰=
 - **h)** $8^{-2} \div 8^{-1}$
 - i) $3^2 \div 3^5$
 - **j)** (4²)⁻³

- Solutions
- 1a) 8 b. 81 c. 64 d. -64 e. 1/8 f. -125 g. -125 h. 1/125
- 2a) 3^7 b) 2^5 c) 1 d) 5^3 e) 2^3 f) 1 g) 1 h) 8^{-1} i) 3^{-3} j) 4^{-6}



Graph-fest!

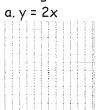
INSTRUCTIONS:

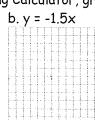
In your groups of 3, label yourselves Partner A, Partner B, and Partner C. Each partner completes their respective part of each question. (ie: Partner A completes Part A, question # 1.a., 2.a., 3a., 4.a., 5a. then Part B question # 1.a., 2.a., 3a., 4.a., 5a. then Part C question # 1.a., 2.a., 3a., 4.a., 5a.)

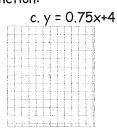
Upon completion of Parts A, B and C, prepare to present your findings to the class.

Part A

1. Using the TI-83 Graphing Calculator, graph each function:







- 2. Describe the graphs.
- 3. Complete a table of values for each:

y = 0.75x+4	

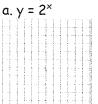
4. Find 1st and 2nd differences for each table of values. Describe any patterns you see.

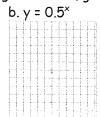
5. Prepare to explain your findings.

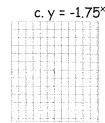
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Part B

1. Using the TI-83 Graphing Calculator, graph each function:







- 2. Describe the graphs.
- 3. Complete a table of values for each:

y = 2*					
	111000000000000000000000000000000000000				

y = 0.5 [×]	y = -1.75 ^x				
	TO THE PARTY OF TH				
and the second s					

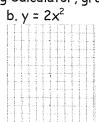
4. Find 1st and 2nd differences for each table of values. Describe any patterns you see.

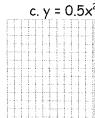
5. Prepare to explain your findings.

Part C

1. Using the TI-83 Graphing Calculator, graph each function:







2. Describe the graphs.

3. Complete a table of values for each

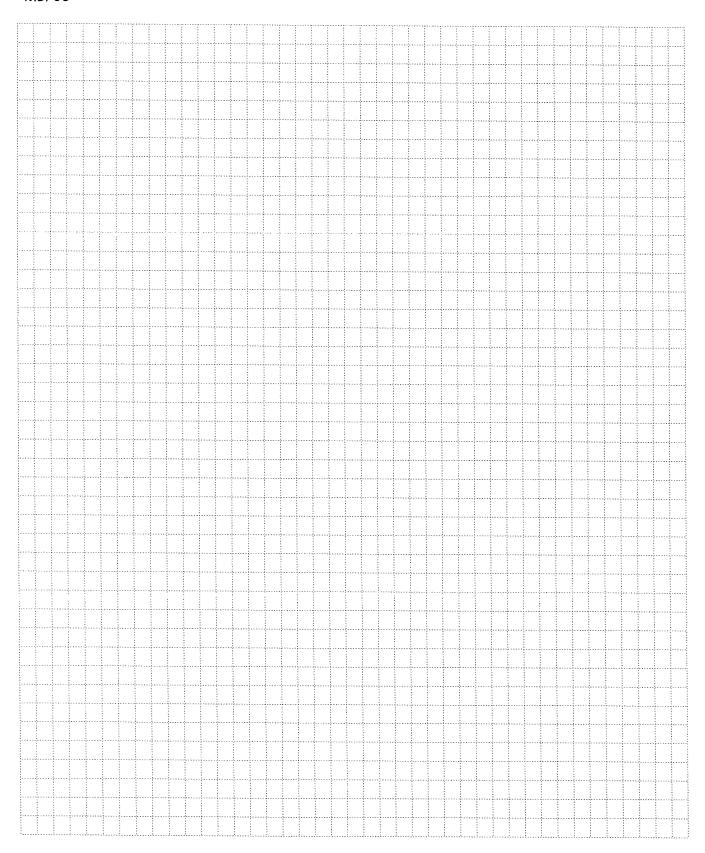
- C	00.,,,	1001000000	40,0			
	$y = x^2$					

			· · · · · · · · · · · · · · · · · · ·			
		1				

$y = 2x^2$	$y = 0.5x^2$	

4. Find 1st and 2nd differences for each table of values. Describe any patterns you see.

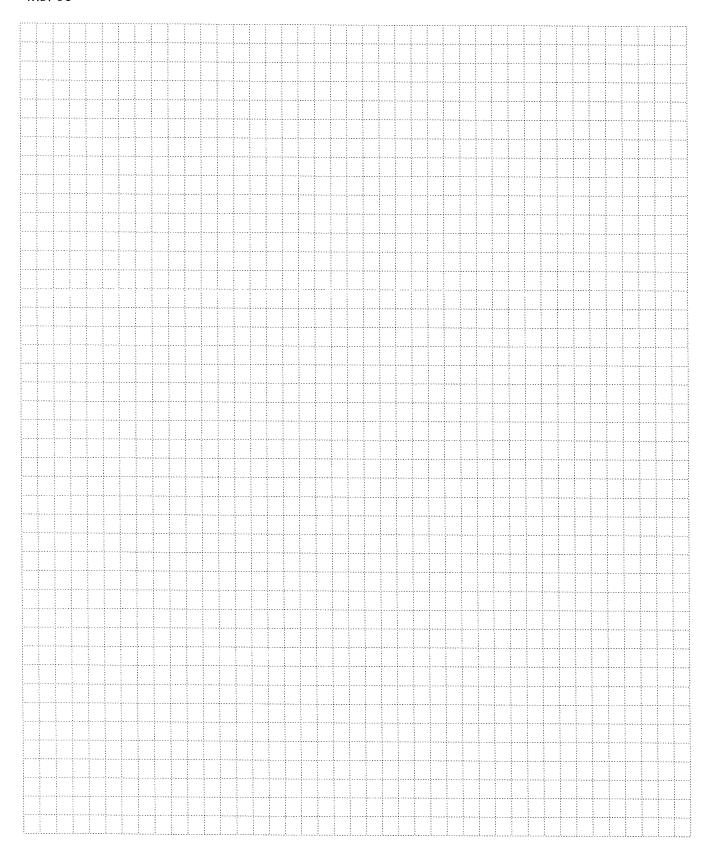
5. Prepare to explain your findings.



Relations

For each question:

- Create a table of values
- Classify as Linear/Quadratic/Exponential
- Determine a formula for Linear and Exponential relations
- 1. James stacks cans for a grocery store display. The top row has 1 can, the second row has 2 cans, the third row has 3 cans, etc. How many cans are there in the 10^{th} row?
- 2. A soccer ball is kicked. At 1 sec., its height is 20.6m. At 2 sec., its height is 30.4 m. At 3 sec., its height is again at 30.4m. At 4 sec., the height is 20.6m. At 5 sec., the height is 1m.
- 3. A vase that cost \$800 is expected to increase in value by 7% each year for 5 years.
- 4. Graham got a loan for a car for \$16 000. After the first year, he still owed \$13 171.37. After the second year, he still owed \$10 167.99. After the third year, he still owed \$6979.37. After the fourth year he still owed \$3594.08. After the fifth year, he had paid off his car so there was a zero balance remaining. Use your formula to find the amount still owing after 6 months (hint: $\frac{1}{2}$ or 0.5 of a year)
- 5. A \$600 investment is worth \$618 after 1 year, \$636 after 2 years, \$654 after 3 years and \$672 after 4 years.



Exponential Growth Investigation

- 1. In your group, complete a table of values or use the graphing calculator to graph three of the following exponential relations.
 - a) $y = 1.5^{\times}$
- **b)** $y = 2^x$ **c)** $y = 2.25^x$ **d)** $y = 3.2^x$ **e)** $y = 3.45^x$
- **f)** $y = 3.75^{x}$

- **g)** y = 4[×]
- h) $y = 4.2^{x}$ i) $y = 4.45^{x}$ j) $y = 5^{x}$
- **k)** $y = 5.3^{\times}$
- 1) $y = 6^{x}$

Display graphs and describe the characteristics in terms of y-intercept, x-intercept, increasing or decreasing.

2. For each exponential relation, describe its characteristics:

	Exponential Relation	x-intercept	y-intercept	Increasing / decreasing
a)	y = 3 [×]			
b)	y = 2.5 [×]			
c)	y = 10 [×]			
d)	y = 1.25 [×]			
e)	y = 2 [×]			

Solutions

- 2a) x-intercept: none, y-intercept: 3, increasing b)x-intercept: none, y-intercept: 2.5, increasing
- c) x-intercept: none, y-intercept: 10, increasing d) x-intercept: none, y-intercept: 1.25, increasing
- e) x-intercept: none, y-intercept: 2, increasing

Germs! Germs! Germs!

Certain bacteria, under the right conditions, multiply themselves.

You will use strips of paper, each representing a bacterium, to model its growth.

For this activity, each member of your group must choose a role:

- Recorder records data
- Counter counts pieces for recorder
- Reader reads questions for other group members
- Facilitator keeps discussion of topic going

	rip(s) of paper tal pieces to yo	into u have?	_ equal pieces
Cut each pie	ece into	equal piece	25.
	ieces do you ho		
	cut each piece		_ equal pieces to complete the table:
Cuts	Pieces		, ,
0			*.* 1 .
1		< m	itial Amount
2			
3			
4			
5			

Graph your result on the transparency provided. Identify characteristics of your graph.

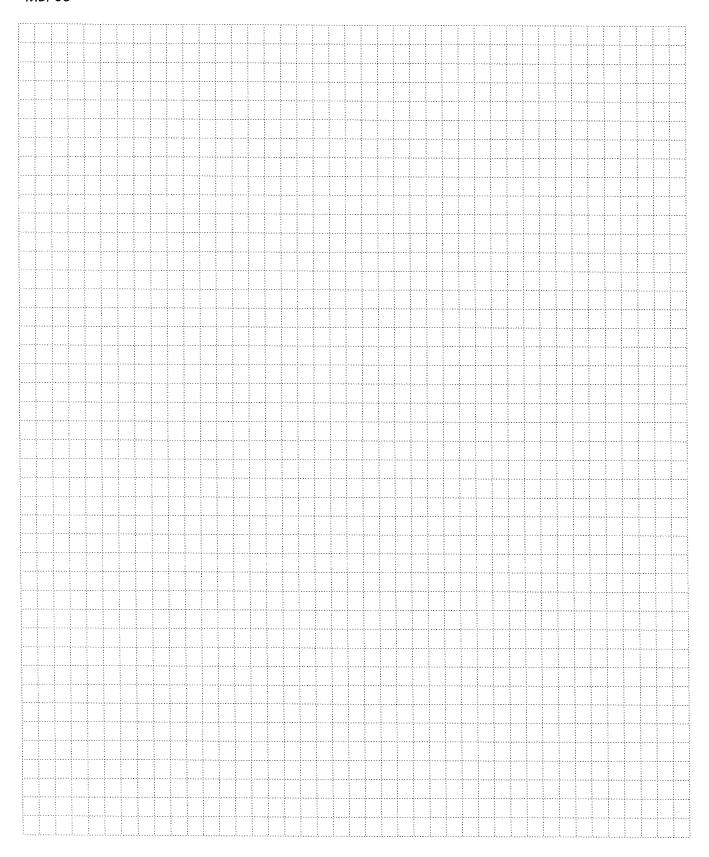
- x-intercept
 - y-intercept

Create an equation to model the data.

Prepare to present your findings.

Solutions

Answers will vary according to the number of cuts assigned to each group, as well as the number of strips assigned to each group



Exponential Relations

For each question, create a table of values to represent each situation then answer the questions that follow:

1. An antique costs \$800. Its value increases by 15% each year.

End of Year	Value
1	\$
2	\$
3	\$
4	\$

x-intercept:

y-intercept:

Equation:



2. The population of Mathville is 23 000. Each year, the population increases at a rate of 3.4%.

End of Year	Value
1	\$
2	\$
3	\$
4	\$

Solutions

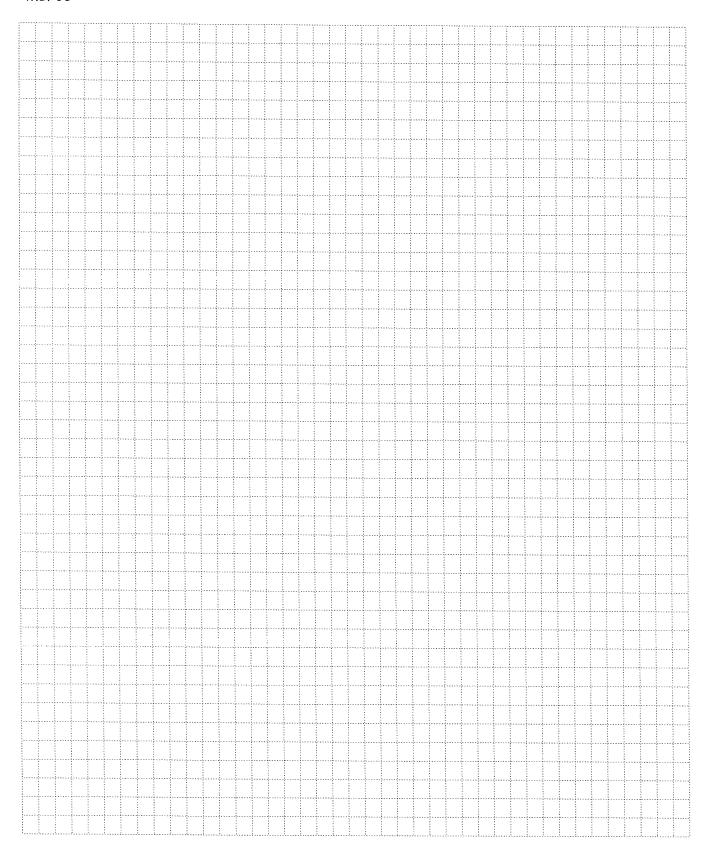
1.	
End of	Value
Year	-
1	\$920
2	\$1058
3	\$1216.70
4	\$1399.21

x-intercept: none y-intercept: 800

equation: y=800(1.15)×

2. End of Year	Value	x-intercept: none y-intercept: 23 000 – equation: y = 23 000 (1.034)
1	23782	= equation: y = 23 000 (1.034)
2	24590.588	_
3	25426.66799	_

26291.17



Exponential Decay Investigation

- 1. In your group, complete a table of values or use the graphing calculator to graph three of the following exponential relations.
 - a) $y = 0.5^{\times}$
- **b)** $y = 0.12^{x}$
- **c)** $y = 0.15^{x}$
- **d)** $y = 0.2^{\times}$
- **e)** y = 0.25[×]
- **f)** $y = 0.3^{\times}$

- **g)** $y = 0.4^{\times}$
- h) $y = 0.35^{\times}$
 - i) $y = 0.6^{\times}$
- $j) y = 0.7^{x}$
- **k)** $y = 0.8^{\times}$
- 1) $y = 0.9^{x}$

Display graphs and describe the characteristics in terms of y-intercept, x-intercept, increasing or decreasing.

2. For each exponential relation, describe its characteristics:

	Exponential Relation	x-intercept	y-intercept	Increasing / decreasing
a)	y = 0.75 [×]			
b)	y = 0.35 [×]			
c)	y = 0.2×			
d)	y = 0.15 [×]			
e)	y = 0.8*			

Solutions

- 2a) x-intercept: none, y-intercept: 0.75, decreasing b) x-intercept: none, y-intercept: 0.35, decreasing
- c) x-intercept: none, y-intercept: 0.2, decreasing d) x-intercept: none, y-intercept: 0.15, decreasing
- e) x-intercept: none, y-intercept: 0.8, decreasing

Half-Life of Radioactive Materials

Radioactive materials break down in a process known as radioactive decay.

The rate of decay varies from substance to substance.

The TIME it takes for half $(\frac{1}{2})$ of a substance to decay is known as half-life.

e.g. If the half-life of a substance is 20 minutes, then a 6kg mass will decay to 3 kg in 20 minutes.

Procedure: In this investigation, you will model the decay of radioactive iodine over several days using 100 numbered cubes to represent 100 atoms of iodine.

Part A: Data Collection

- 1. If approximately 1/6 of the atoms decay each day, then each "pour" of the atoms represents one day. Atoms with the marking of "one" facing UP will decay each day.
- 2. Repeat for several days, recording the number of atoms that remain after each "day of radioactive decay" until there are less than five (5) atoms remaining.

Part B: Data Presentation

Represent your findings by constructing a graph of Number of Days versus Number of Atoms Remaining on the graph paper provided.

Part C: Conclusions

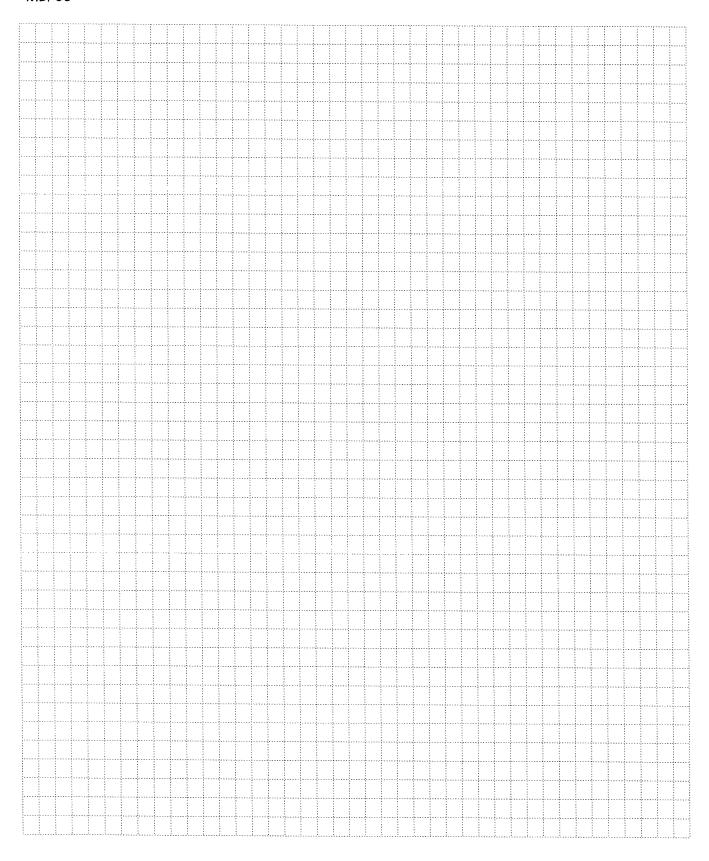
- 1. Based on your observations, numerically and graphically, predict the half-life of iodine. Explain your reasoning.
- 2. Will the sample of iodine ever completely decay to zero atoms? Explain your answer.
- 3. Write an equation to represent the number of atoms remaining as a function of time.

Part D: Extension

Create an activity to introduce and model half-life using two coloured counters. Be sure to include the solution with your activity.

Solutions

(answers will vary) PART C: It will take between 3 and 4 days for half of a sample of iodine to decay (half-life), The amount of the sample will never reach zero because there will always be 5/6 of the sample left. So even with a small sample, an even smaller sample will be left. $Y=100(5/6)^{\times}$ PART D: (answers will vary) One roll or flip is a half-life



Exponential Growth and Decay Word Problems

The general formula that can be used for all exponential growth and decay problems is;

y=A(b)^x
where **A** represents the initial value **b** represents the growth or decay rate **x** represents the length of time

Exponential Growth: Two types of problems

eg. 1 Jason decides to grow bacteria for a living. The bacteria doubles in size every 3 minutes. If there are 1000 bacteria initially present, how many will there be in 15 minutes? The question involves <i>doubling period</i> .	eg. 2 Mrs. James is accumulating money in her Swiss bank account from her oil stocks investment. The stocks give her an interest rate of 30% per year. She's looking to retire in 22 years from now. If she invests 1000 dollars today how much money will she make when she retires? This question involves growth rate.

Exponential Decay: Two types of problems

eg. 3 When Earnest Rutherford discovered radioactive decay of radium, he noticed it has a half life of 100 days. This means whatever mass of radium he has initially it will be half as much in 100 days. He starts with 320 mg of radium and stores it for 40 days. How much will be left after that time? This question involves half-life.	eg. 4 Carly buys a new car for \$20,000. It is a nice one, but the value depreciates at a rate of 6% per year. If she wants to sell her car in 8 years from now how much will it be worth? This question involves deprication.

U03A14-P3

- 1. Which of these functions describe exponential decay? Explain

- (a) $y = -4(3)^x$ (b) $y = 0.8(1.2)^x$ (c) $y = 3(0.8)^{2x}$ (d) $y = \frac{1}{3}(0.9)^{\frac{x}{2}}$

- 2. In 1990, a sum of \$100 is invested at a rate of 6% per year for 15 years.
 - (a) What is the growth rate (i.e. the value of "b")
 - (b) What is the initial amount?
 - (c) Write an equation that models the growth of the investment, and use it to determine the value of the investment after 15 years?

- 3. A species of bacteria has a population of 500 at noon. It doubles every 10 hours. The equation that models the gorwht of the population, P, at any hour, t, is $P = 500(2)^{\frac{r}{10}}$
 - (a) Why is the exponent $\frac{t}{40}$?
 - (b) Why is the base 2?
 - (c) Why is the multiplier 500?
 - (d) Determine the population at midnight
 - (e) Determine the population at noon the next day



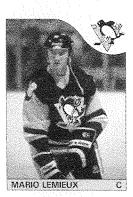
U03A14-P4

- 4. A town with a population of 12000 has been growing at an average rate of 2.5% for the last 10 years. Suppose the growth rate will be maintained in the future. The equation that models the town's growth is $P = 12\ 000(1.025)^n$, where P represents the population and n is the number of years from now.
 - (a) Determine the population of the town in 10 years.
 - (b) Determine the number of years until the population doubles.

- 5. In each case, write an equation that models the situation described. Explain what each part of the equation represents.
 - (a) The percent colour remaining if your blue jeans lose 1% of their original colour everytime they are washed
 - (b) The population if a town had 2500 residents in 1990 and grew at a rate of 0.5% each year after that for t years.
 - (c) The population of a colony if a single bacterium doubles every day, what is the population P after t days.

U03A14-P6

- 6. A population of yeast cells doubles every hour. Assume an initial population of 80 cells.
 - (a) Write an equation that can be used to determine the population after t hours.
 - (b) Use your equation to determine the population after 6 hrs.
 - (c) Use your equation to determine the population after 90 minutes.
 - (d) Approximately how many hours would it take for the population of cells to reach 1 million.
- 7. A collector's hockey card is purchased in 1990 for \$5. The value increases by 6% every year.
 - (a) Write an equation that models the value of the card, given the number of years since 1990.
 - (b) Determine the value of the card in the 4th year after purchase.
 - (c) Determine the value of the card in the year 2010.



- 8. Light intensity in a lake falls by 9% per meter of a depth relative to the surface.
 - (a) Write an equation that models the intensity of light per meter of depth. Assume that the intensity is 100% at the surface.
 - (b) Determine the intensity of light at a depth of 7.5 meters

Cats and Mice!

There is an isolated island off the West coast of Canada. The island has become overrun with mice, so the Wildlife Federation of Canada released a cat population on the island to stabilize the mouse population. In 1999, the population of the mice was 23576 and began to decrease at a rate of 2.5% per year. In the same year, the population of cats was at 15786 and was increasing at a rate of 1.8% per year. Assume that there is no outside factors, and that these rates continue in order to answer the following questions.

- 1. Create a well labelled table of values for <u>both the mice and cat</u> populations from 1999 to 2009 (<u>You need to carry a minimum of 3 decimal places for the population</u>). Find the first-differences, second-differences and the constant ratio for both populations (Show your work). Describe each of the populations as either Linear, Quadratic, Exponential-growth or Exponential –decay and justify your reason.
- 2. Using your table of values as your knowledge of what a linear or quadratic or exponential equation looks like, create an equation to describe the population of the mice AND create an equation to describe the population of the cats. Explain how you come up with these equations?
- 3. Create a graph (use the full page of graph paper) that illustrates the population vs. the years of the mice AND on the same graph the population vs. years of the cats. Label each of your graphs with the appropriate equation that you created in question # 2. Your y-axis (population) will need to go between 15 000 and 24 000 and the x-axis (years) will need to go between 1999 and 2009.
- **4.** Describe how the populations are different? Describe how they are related?
- 5. When would the population of the cats be greater than the population of the mice?
- 6. When would the populations be the same (approximately)? Explain how you can tell when they are the same or how you came to your conclusion.
- 7. What will happen to both the mice and cats populations if this trend continues?



U03A16

Exponential Relations

1. Simplify first, then evaluate:

a)
$$4^6 \div 4^3$$

b)
$$4^5 \times 4^{-2}$$

$$f) (3^2)^3$$

$$j) 2^5$$

c)
$$(2^4)^2$$

g)
$$5^8 \div 5^4$$

k)
$$5^2 \times 5^2$$

d)
$$3^8 \div 3^5$$

h)
$$5^{-3}$$

$$(4^3)^2$$

Identify each of the following equations as linear, exponential or quadratic.

2.
$$y = 3x$$
 3. $y = 3^{x}$ **4.** $y = 3x^{2}$ **5.** $y = -0.75x$ **6.** $y = -0.75^{x}$ **7.** $y = -0.75x^{2} + 2$ **8.** $y = x^{2} + 5$ **9.** $y = 16^{x}$

$$0.75x^2 + 2$$
 8. $y = x^2 + 5$ **9**. $y = 16^x$

For each exponential situation, identify its characteristics:

10. A club uses email to contact its members. The chain starts with 3 members who each contact three more members. Then those members (9) each contact 3 members, and so the contacts continue.

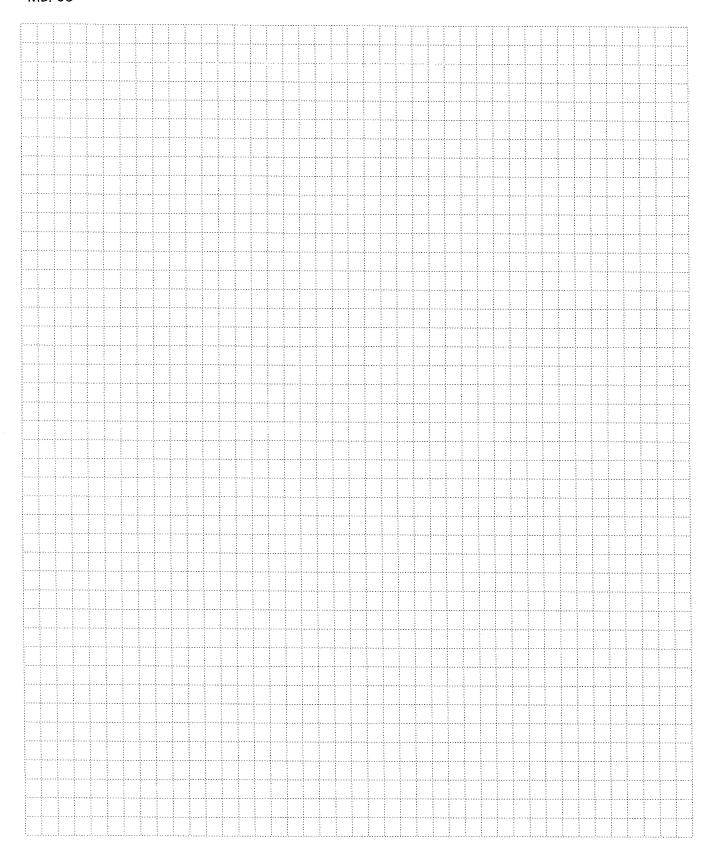
11. A bouncing ball rebounds to 0.75 of its height on each bounce. The ball was dropped from a height of 30 metres.

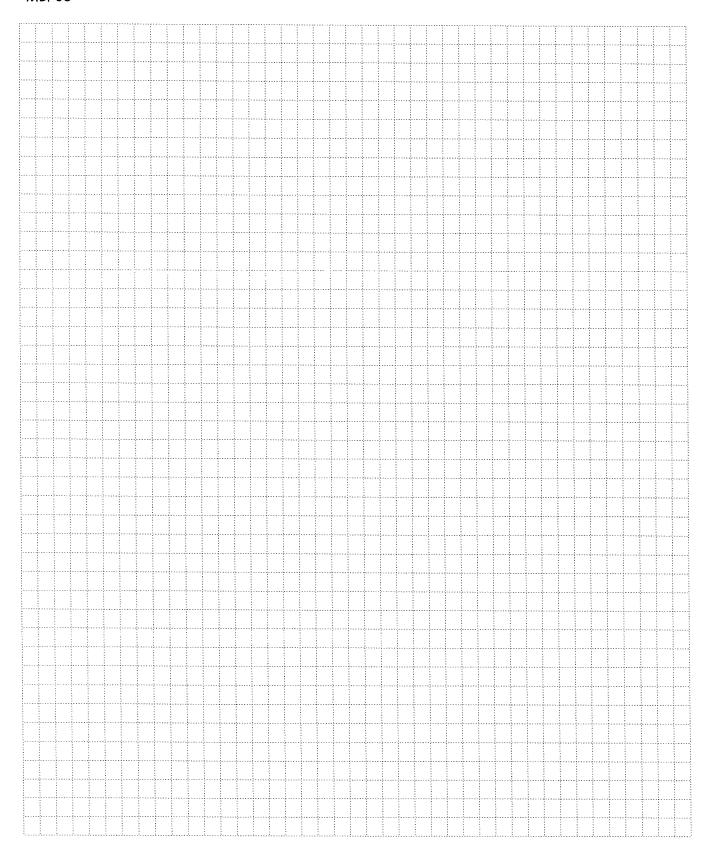
12. A painting was bought for \$475. Each year, its value increases by 8%.

Solutions

$$1a) 4^3=64$$
 b) $4^3=64$ c) $2^8=256$ e) 1 f) $3^6=729$ g) $5^4=625$ h) $1/125$ i) $1/11$ j) 32 k) $5^3=125$ l) $.4^6=4096$ d) $3^3=27$

2. Linear 3. Exponential 4. Quadratic 5. Linear 6. Exponential, 7. Quadratic 8. Quadratic 9. Exponential 10. xintercept: none, y-intercept: 11, increasing 12. x-intercept: none, y-intercept: 30, decreasing 23. x-intercept: none, yintercept: 475, increasing





Exponential Relations Review

1. Identify each of the following equations as either linear, quadratic or exponential.

a.
$$y = 3x$$

b.
$$y = 3^x$$

c.
$$y = 3x^2$$

d.
$$y = -0.75^x$$

e.
$$y = -0.75x$$

f.
$$y = -0.75x^2 + 2$$

g.
$$y = x^2 + 5$$

h.
$$y = 16^x$$

2. Simplify each of the following expressions and express your final answers using only positive exponents. Decimal equivalents will not be accepted.

a.
$$(3x^5)(2x^{-2})$$

b.
$$\frac{x^7 y^3 z}{x^9 y^3 z^4}$$

c.
$$\frac{65a^7b^6c^{-4}}{-5a^2b^{-3}c}$$

d.
$$(2a^{-3}b^2)^3$$

e.
$$(-2a^{-3}b^3c)^{-5}$$

- 3. For each of the following situations, identify the relation as linear, quadratic or exponential.
 - a. Larry is paid \$10 per hour, he receives a \$1 per hour raise each year.
 - b. A painting was bought for \$475. Each year its value increases by 8%.
 - c. A principal of \$500 is invested at 5% simple interest for 20 years.
 - d. A ball is kicked from the ground. At 1 second it is 3 m off the ground. At 2 seconds it is 12 m off the ground, at 3 seconds it is 27 m off the ground.
 - e. Actinium has a half life of 5 years.
 - f. The population of Estonia is 3.75 million. It increases at a rate of 1.3% each year.
 - g. The temperature of the Sahara desert is 35°C always.
 - h. A bouncing ball rebounds to ¾ of it's height on each bounce of the ball. The ball was dropped from a height of 45 meters.
 - i. A colony of bacteria triples every 10 minutes. Initially there were 20 cells in the colony.
- 4. For each exponential relation in question 3, write an equation for the situation and state whether it represents exponential growth or exponential decay.
- 5. How much will the painting be worth after 7 years?
- 6. After 1 hour, how many bacteria will be present in the colony?

MBF3C U03A17-P2

- 7. After 3 bounces, how high is the ball?
- 8. What percentage of the Actinium remains after 50 years?
- 9. What will the population of Estonia be in 15 years?
- 10. The value of a car can be represented by the equation $V = 6200(0.915)^{x}$, where x is in years.
 - a. What was the initial value of the car?
 - b. What is the rate of depreciation per year (as a percentage)?
 - c. Does this model exponential growth or exponential decay?
- 11. The value of an investment with compound interest can be represented by the equation,

$$A = 500(1.075)^n$$
, where n is in years.

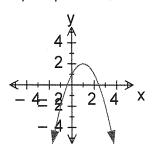
- a. What was the initial value of the investment?
- b. What is the rate of return on the investment per year as a percent?
- c. Does this model exponential growth or exponential decay?
- 12. Convert the following growth rates to a growth factor (b).
 - a. 7%
 - b. 80 %
 - c. 0.58 %
 - d. 3.14 %
- 13. Convert the following growth factors to growth rates.
 - a. 1.03
 - b. 1.72
 - c. 1.0075
 - d. 1.006
- 14. Convert the following decay rates to a decay factor (b).
 - a. 5%
 - b. 50%
 - c. 85%
 - d. 7.5%
- 15. Convert the following decay factors to a decay rate
 - a. 0.75
 - b. 0.085
 - c. 0.005
 - d. 0.03

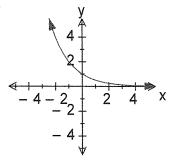
Exponential Relations Practice Test

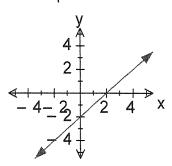
Simplify using exponent laws, then evaluate.

a) $3^5 \div 3^2$ b) $(2^4)^2$ c) $\frac{6^5 \times 6^3}{6^6}$ d) $4^3 \times 4^{-6}$ e) $\frac{(10^3)^5}{10^{15}}$ f) $\frac{(2^3)^3 \div 2^7}{2} \times 3$

In the space provided, identify the following relations as linear, quadratic or exponential. 2.







d)
$$y = (1.02)^n$$

e)
$$y = 2x^2 - 3$$

f)
$$v = -4x + 8$$

In the space provided, identify the following relations as an increasing or decreasing exponential.

a) $v = (1.02)^n$

b) $v = 3^x$

$$\mathbf{c)} \quad y = \left(\frac{3}{2}\right)^x$$

Arrange the following functions in order of increasing steepness.

 $v = 6^x$

$$y = 1.5^{x}$$

$$v = 10^{x}$$

$$v = 5^x$$

Arrange the following functions in order of increasing steepness. 5.

1
$$y = \left(\frac{1}{3}\right)^x$$
 2 $y = \left(\frac{1}{10}\right)^x$ **3** $y = 0.5^x$ **4** $y = 0.25^x$

$$y = 0.25^x$$

Given the equations below, complete a table of values and graph the relation on a clearly labeled grid.

a)
$$y=2^x$$

×	у
-2	
-1	
0	
1	
2	

$$b) \quad y = \left(\frac{1}{2}\right)^x$$

×	У
-2	
-1	
0	
1	
2	

MBF3C Unit 4 - Personal Finance

Section	Date	Lesson Title	Handouts	Homework Assigned
1		Introduction to Simple Interest	U04A01	U04A02
2		Simple Interest Assignment		U04A03
3		Compound Interest From Simple Interest	U04A04	U04A05
4		Finance on a Spreadsheet	U04A06	
		Book a Computer Lab	U04A07	
5		Introduction to Compound Interest	U04A08	Supplemental Worksheet
W-FARMWAY.				(not provided)
6		Compound Interest	U04A09	Supplemental Worksheet
				(not provided
7		Interest Calculations with TVM Solver	U04A10	
8		Interest and Savings Alternatives	U04A11	
		Book a Computer Lab		
9		Introduction To Credit Cards	U04A12	
:		Book a Computer Lab		
10		Vehicles: Costs Associated With Owning	U04A13	
		Book a Computer Lab	U04A15	
11		Vehicles: Buying or Leasing	U04A14	
		Book a Computer Lab	U04A15-	
			continued	
12		Vehicles: Buying Old or New	U04A15-	
		Book a Computer Lab	continued	
13		Review		Supplementa Worksheet
				(not provided
14		Practice Test	U04A16	
15		Unit Test		

U04A01

Simple Interest

Simple Interest: Interest that is calculated only on the original principle, using the simple interest formula I = Prt ... where:

P = Principal (the original amt.)

r = interest rate (expressed as a decimal)

t = length of time (expressed in terms of years)

Examples

- Show the interest rates as they would appear in the formula as r. (Divide by 100, or move decimal 2 spaces to the right)
 - a) 13%
- b) 2.5%
- c) 0.5%
- d) 217%

- 2. Express the following lengths of time in terms of years. (t in the formula)
 - a) 24 months
- b) 8 months c) 14 weeks d) 82 days

=

- Given the following scenario,
 - a) Calculate how much interest is earned if \$2000 is invested at 4% simple interest for 26 weeks.

Solution

The interest earned on this investment is _____

b) How much is the investment worth?

Solution:

$$A = I + P$$
, where A represents total amount.

$$A = $$$

The total amount of the investment is ______

What principle is needed to have \$500 in interest in 2 years invested at 2.5% simple interest?

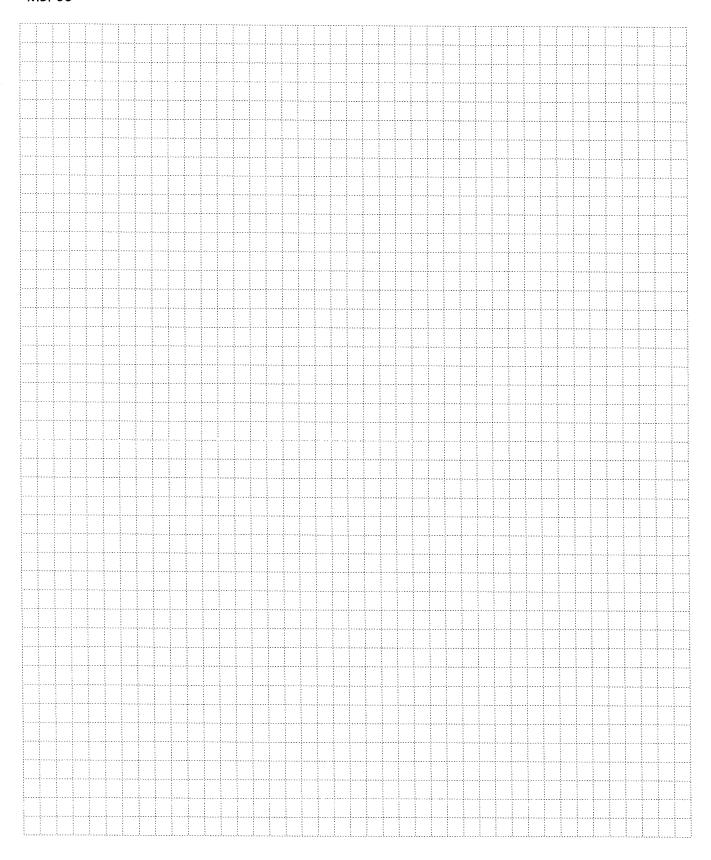
Solution:
$$P = \frac{I}{rt} =$$

The principal that you need to invest is _____.

5. What rate of simple interest is needed to get \$7000 to grow to \$10000 in 5 years?

6. How long would it take \$1500 to grow to \$2000 at a simple interest rate of 3%?

Solution: $t = \frac{I}{Pr}$ t = t = t = Therefore it would take approximately _____ years.



Simple Interest Homework

1. Express the following interest rates as (r) in the simple interest formula.

a) 6% b

b) 4.5%

c) 1.25%

d) 0.85%

e) 32%

2. Express the following lengths of time a (t) in the simple interest formula.

a) 18 months

b) 16 weeks

c) 88 days

d) 4 years

e) 52 weeks

3. Complete the following chart.

Principle (\$)	Interest rate %	Time	Interest Earned (\$)	Total Amount (\$)
2000	4.5	3 months		
550	0.5	36 months		
1500	1.5		320	
	7.2	16 weeks	100	
2500		18 months	275	
	6.75	240 days	55	
10000		6 weeks	125	
780	1.3		58	

- 4. \$300 is invested for 2.5 years at 6% simple interest. How much interest is earned?
- 5. Joe borrowed \$500 from his parents to buy an ipod. They charged him 2.5% simple interest. He paid them back in 14 months. How much interest did he pay them? How much did he pay them in total?
- 6. Peter invested in a GIC that paid 3.25% simple interest. In 36 months, he earned \$485. How much did he invest originally?
- 7. What rate of simple interest is needed for \$700 to double, in 3 years?
- 8. Kadeem's investment matured from \$1300 to \$1750. It was invested at a simple interest rate of 4.25%. How long was it invested for?
- 9. \$4500 was invested at a $5\frac{3}{8}$ % simple interest for 300 days. How much interest was earned? What was the total amount of the investment?
- 10. \$600 is invested at 4% simple interest for 2 years.
 - a) How much interest is earned?
 - b) If the interest rate is doubled to 8% is the interest earned doubled?
 - c) If the time was doubled to 4 years, would the interest earned be doubled?

Unit 4 - Personal Finance

Name:____

Instructions:

This assignment should be done on a separate sheet of lined paper and submitted at the beginning of the period on ______ for assessment. You may use your notes, work in groups but each student must submit their own work. Make sure it is organized, neat, well labeled and include all formulas and appropriate units.

Recall:

Simple Interest Formula:

I = Prt where;

I is the interest earned in dollars

P is the principal invested in dollars (the original amount)

R is the annual interest rate, expressed as a decimal

T is the time in years

A = P + I where;

A is the total amount of the investment

P is the principal amount

I is the interest earned

1. Write each of the following percents as a decimal.

a) 29 %

b) 16 %

c) 4°

1) 7.2 %

e) 0.49 %

f) 2.49 %

g) $5\frac{2}{3}\%$

h) $\frac{3}{5}\%$

i) $2\frac{3}{6}\%$

j) $14\frac{5}{8}\%$

2. Write each of the following decimals as a percent.

a) 0.02

b) 0.079

c) 0.265

d) 4.6

3. Calculate the interest earned on each investment. Remember to change the interest rate to a decimal and make sure the time is in years or portion of years.

	Principal	Rate	Time
a)	\$3450	4.9 %	7.5 years
b)	\$250	$3\frac{2}{8}\%$	760 days
c)	\$920	$5\frac{2}{8}\%$	16 months
d)	\$624	8.69 %	19 weeks

4. Suzanne had an outstanding balance of \$2625.10 on her credit card for 59 days. The annual interest rate is 17.8 %. How much interest did Suzanne pay?

Recall: The simple interest formula can be rearranged for any variable that you want to solve for.

$$I = Prt P = \frac{I}{rt} r = \frac{I}{Pt} t = \frac{I}{Pr}$$

5. Calculate each missing item from the chart below. Show your work for each calculation.

	Principal	Rate	Time	Interest
a)	\$489	5.24%		\$86.23
b)		$6\frac{2}{3}\%$	74 days	\$18.25
c)	\$1098		15.5 years	\$732.65
d)	\$2698	$4\frac{3}{5}\%$	27 months	
e)		9.8%	260 days	\$160.25
f)	\$778	13.6%		\$689.25
g)	\$9526		9.85 years	\$8965.26
h)	\$2005	4.98%		\$56.98
i)		9.4%	250 days	\$46.29
j)	\$8964		6.5 years	\$965.26

- 6. Emma's retirement account grew from \$320 to \$562.36 in 9 months. What annual rate of interest does her account pay?
- 7. Cecille cashed a \$2563 CSB that earns 2.6% simple interest per year. She received \$265 interest. How long did Cecille hold the bond?
- 8. Joan cashed a CSB after 4 years and received \$1056 interest. The annual rate on the bond was 4.6%. What was the original value of the bond?

U04A04

Compound Interest using Simple Interest Formula

- 1. \$500 is invested at 2.4% interest compounded monthly for 3 months.
 - a) Use the simple interest formula to calculate the total amount after 3 months.

- b) If the principle was not compounded, how would the final amount be different?
- 2. Complete the charts and graphs for the following information.
 - a) \$700 is invested at 9% simple interest for 10 years.

Year	Principle	Interest	Total amount
1	700	63	763
2	700	63	826
3	700		
4			
5			
6			
7			
8			
9			
10			

- b) What do you notice about the amount of interest earned each year?
- c) What is this type of growth referred to as?
- d) On graph paper, complete a graph of total amount of investment over time.

U04A04-P2

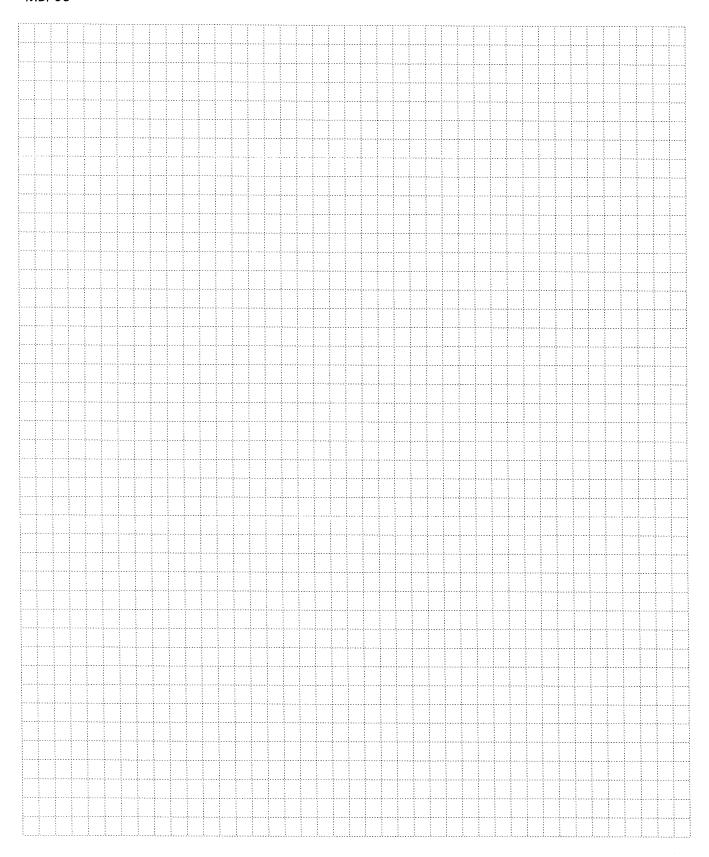
- 3. \$700 is invested at 9% interest compounded annually for 10 years.
 - a) Complete the table below.

Year	Principle	Interest	Total amount
1	700	63	763
2	763	68.67	831.67
3	831.67		
4			
5			
6			
7			
8			
9			
10			

b) What do you notice about the amount of interest earned each year?

c) What is this type of growth referred to as?

- d) On graph paper, complete a graph of total amount of investment over time.
- e) Describe a situation where compound interest would not be the better choice.



Comparing Growth Rates of Simple and Compound Interest Homework

Compound Interest: Interest that is calculated at regular compounding periods, and then, added to the principle for the next compounding period.

- 1. Complete the charts and graphs for the following information.
 - a) \$2000 is invested at 7.5% simple interest for 10 years.

Year	Principle	Interest	Total amount
1			
2			
3			
4			
5			
6		***	
7			
8			
9			
10			

Calculations:		

b) \$2000 is invested at 7.5% interest compounded annually for 10 years.

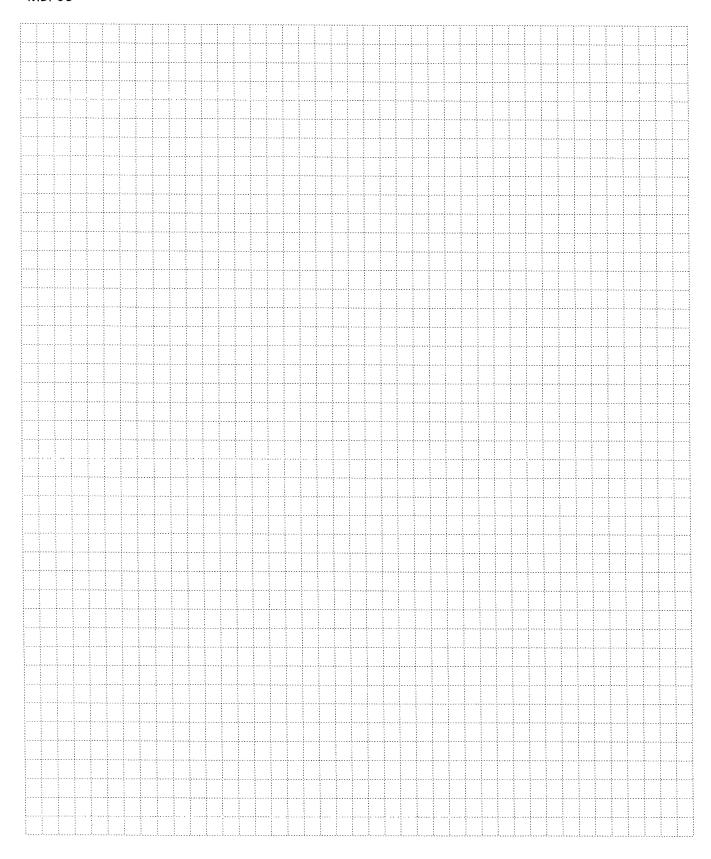
Year	Principle	Interest	Total amount
1	2000	150	2150
2	2150		
3			
4			
5			
6			
7			
8			
9			
10			

Calculations:		

^{*}On graph paper, construct a graph of total amount of investment over 10 years.

^{*}On graph paper, construct a graph of total amount of investment over 10 years.

Compare you two graphs. What do you notice about the way simple interest grows compared to compound interest?



Investigation: Simple Interest vs. Compound Interest

1. Use a spreadsheet program to create the spreadsheet page shown below, note the formulae used in the grey boxes in cells B8 through C12.

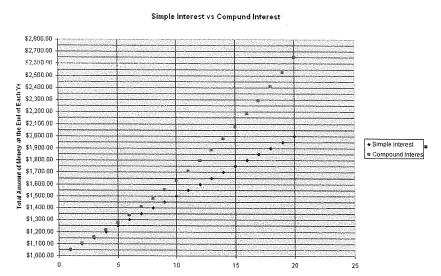
	A	В	C
1		Principal	\$1,000.00
2		Interest Rate	5.00%
3			
4		I = P * r * t	
5		A = P + I	
6		A = P + P * r * t	$A = P(1 + r)^n$
7	Year	Simple Interest	Compound Interest
8	1	\$1,050.00	\$1,050.00
9	2	\$1,100.00	\$1,102.50
	· -	Ψ1,100.00	Ψ1,102,00
10	3	\$1,150.00	\$1,157.63
	_		
10	3	\$1,150,00	\$1,157.63

The values in the table are actually the formulae shown below:

Year	Simple Interest	Compound Interest
1	=\$C\$1+\$C\$1*\$C\$2*A8	=\$C\$1*(1+\$C\$2)^A8
2	=\$C\$1+\$C\$1*\$C\$2*A9	=\$C\$1*(1+\$C\$2)^A9
3	=\$C\$1+\$C\$1*\$C\$2*A10	=\$C\$1*(1+\$C\$2)^A10
4	=\$C\$1+\$C\$1*\$C\$2*A11	=\$C\$1*(1+\$C\$2)^A11
5	=\$C\$1+\$C\$1*\$C\$2*A12	=\$C\$1*(1+\$C\$2)^A12

Also add more years to your Year column so that you have values down to 20 years. You should copy the formula cells down rather than type the formula over and over again. Use the fill handle on the bottom right of the cell or highlight from B12 to B27 and with the cells highlighted select EDIT, FILL, DOWN. Repeat for the formulas in column C.

Once you've completed the table down to 20 years you should create a chart by selecting everything from cell A7 down to the bottom of your compound interest column. (ie: A7:C27). The chart type you should select is an XY Scatter plot. It should look something like the chart shown to the right:



Investigation (continued)

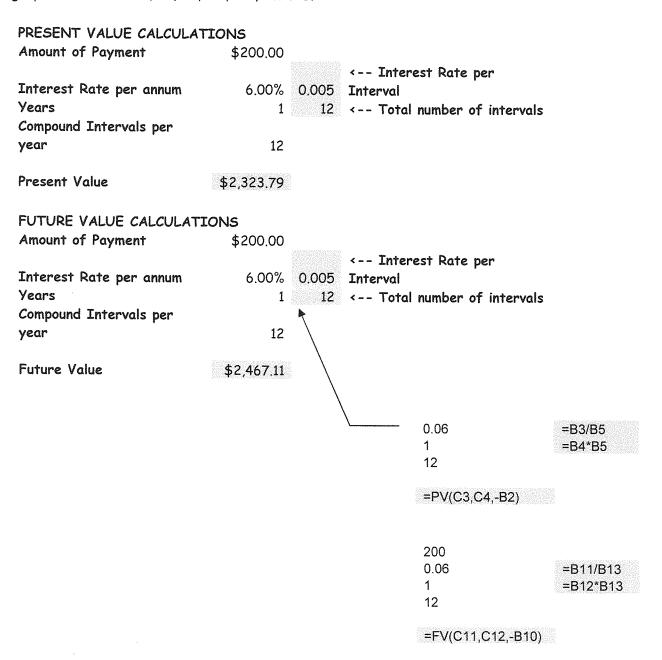
OUTAU	0-72	Investigation (continued)	
	Describe any patterns you see Simple Interest	in the two lines.	
ь)	Compound Interest		
,			
c)	Explain why you think they are	e occurring:	
You sh should	ould be able to change the Pr automatically update – try th	rincipal ($cell\ C1$) and Interest Rate ($cell\ C2$) values and the following and describe the results.	ne graph
3.	Try changing the Principal amo	ount and describe how the graph changes:	

4. Try changing the Interest rate and describe how the graph changes:

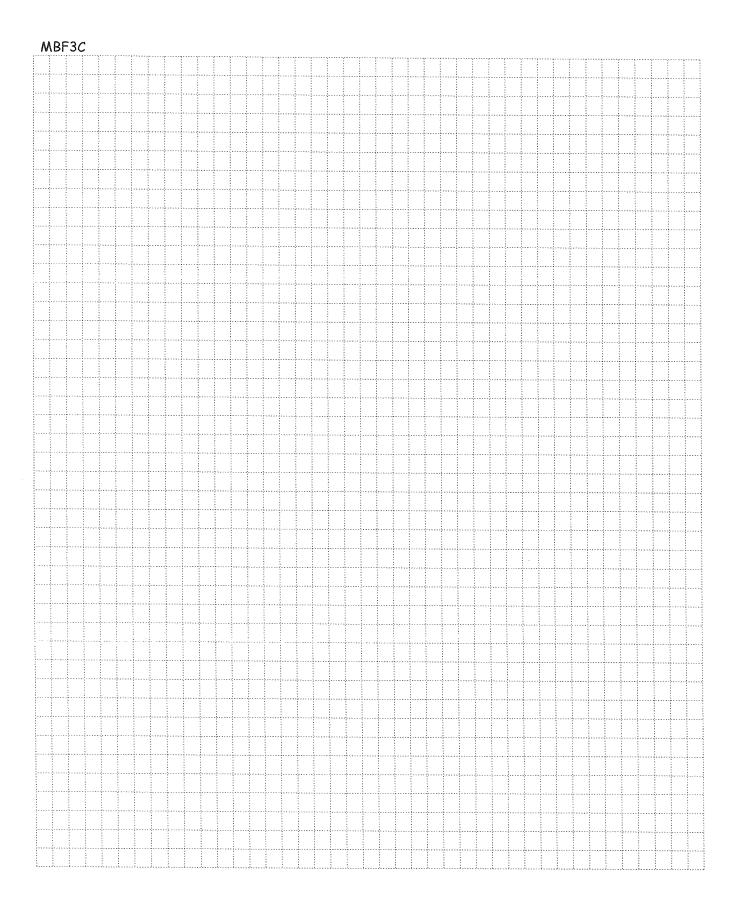
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Investigation: Present Value and Future Value

Use a spreadsheet program to create the spreadsheet page shown below, note the formulae used in the grey boxes in cells C3, C4, B7, C11, C12, and B15.



By changing the appropriate values in the spreadsheet you created, you can correct/check your homework from prior lessons.



Introduction to Compound Interest Formula

- Evaluate. Round answers to 2 decimal places
 - a) 1000(1.0097)¹²

- **b)** 575(1 + 0.0234)²⁶
- c) 900(1+ $\frac{0.3}{12}$)²⁴
- Calculate the adjusted interest rate (i) as it would appear in the formula for the given below.
 - a) 5% quarterly
- b) 0.3% semi-annually
- c) 1.25% monthly

- d) 4.2% bi-weekly
- e) 0.05% daily

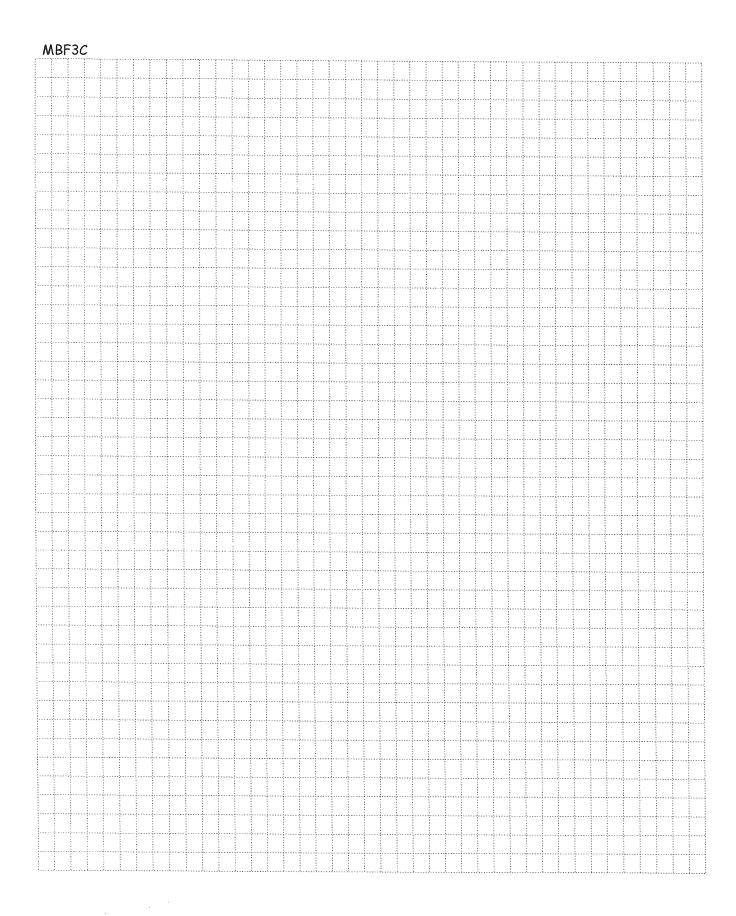
- f) 12% annually
- Calculate the adjusted compounding period (n) as it would appear in the formula for the given below. 3.
 - a) monthly for 2 years
- b) weekly for 3 years
- c) annually for 36 months
- d) semi-annually for 30 months e) bi-weekly for 6 months
- f) daily for 3 weeks

4. Complete the chart.

	Principle	Interest Rate	Time Invested	Compounding Frequency	Amount	Interest Earned
a)	\$300	2.3%	18 months	Semi- annually		
b)	\$1200	1.25%	2 years	Weekly		
c)	\$1575	0.75%	85 days	Daily		
d)	\$870	18%	3.5 years	Quarterly		
e)	\$14000	5.45%	9 months	Annually		

- 5. If interest is compounded quarterly, how much is earned by:
 - a) \$456.50 invested for 2 years at 8.5%
 - **b)** \$2550 invested for 4.5 years at 1.33%
 - c) \$5000 invested for 6 months at 0.4%

6.	If interest is compounded semi-annually, how much is earned by: a) \$456.50 invested for 2 years at 8.5%
	b) \$2550 invested for 4.5 years at 1.33%
	c) \$5000 invested for 6 months at 0.4%
7.	If interest is compounded weekly, how much is earned by: a) \$456.50 invested for 2 years at 8.5%
	b) \$2550 invested for 4.5 years at 1.33%
	c) \$5000 invested for 6 months at 0.4%
8.	What is the amount of an investment if \$650 is invested at 2.45% interest compounded monthly for 3 years?
9.	Mohammed spent \$800 on his credit card. His credit card company charged 18% compounded monthly. He forgot to pay it for 3 months. How much does he owe now? How much of that is interest?



U04A09

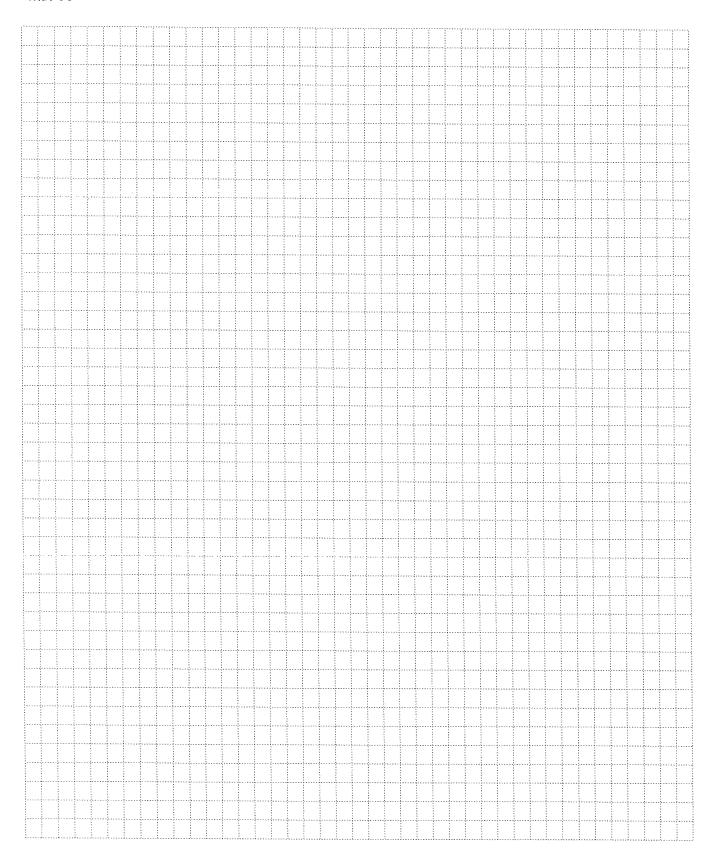
Compound Interest Part 2

- 1. Calculate the amount of compound interest paid on \$3700 at the end of 3 years for each rate.
 - a) 15% compounded semi-annually
 - b) 8% compounded quarterly
 - c) 2.6% compounded weekly
 - d) 0.67% compounded daily
- 2. Which investment is better? Explain how you determined your answer.

Investment A: 6% compounded semi-annually?

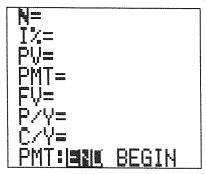
Investment B: 4.5% compounded quarterly?

- 3. Jenny deposited \$300 in an account that paid 12% interest compounded semi-annually. If she leaves it there for 6 years, how much will she have?
- 4. A loan of \$5000, at 12% compounded monthly is due to be repaid in 3 years. How much needs to be repaid?
- 5. Brett and Steven's grandparents gave them each \$10000 to use when they go to college in 6 years. Brett invested his in a GIC that paid 2.25% compounded quarterly, while Steven invested his in a GIC that paid 1.75% compounded monthly. Who made more money? How much more?
- 6. Wayne bought a computer and owes \$3000 to the computer store. They charge 7% and compound it quarterly. His payment is due in full in 3 years.
 - a) How much will he owe them in 3 years?
 - b) How much will he owe them if he pays it off in 1 year instead?
 - c) How much will he have saved?
- 7. Jamal's is getting a raise! 2.75% each year for the next 4 years. His current salary is \$35500 per year. What will his salary be in 4 years?

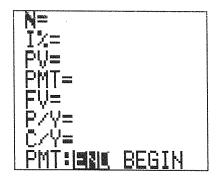


Using the TVM Solver

- 1. The following questions are based on the TVM Solver application of the graphing calculator.
 - a) State the sequence of key strokes to get to the TVM Solver
 - b) What do you press to display the value of the unknown variable?
 - c) What do you have to remember about FV and PV?
 - d) Which variables should always remain at 0.00 when dealing with compound interest?
 - e) If it is compounded quarterly, what should the value of C/Y be?
- 2. State the values that should be entered for each of the variables in the TVM solver. You do not need to solve.
 - a) Determine the amount of an investment if \$600 is invested at 3.5% interest for 7 years compounded semi-annually.

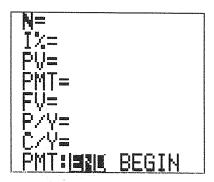


b) How long would it take \$4000 to grow to \$6000 if it is invested at 2.3% compounded monthly?

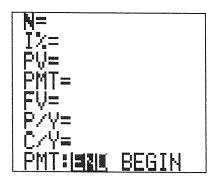


Unit 4 - Personal Finance

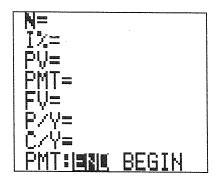
c) What interest rate is needed for \$10000 to double in 5 years if it is compounded quarterly?



d) What amount needs to be invested at 0.8% interest compounded weekly if you have \$650 after 1 year?

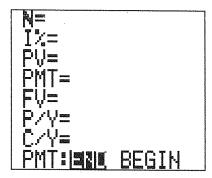


3. A deposit of \$1550 will grow to \$1680.20 in 2 years. What rate of interest is given if interest is calculated annually?

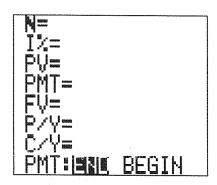


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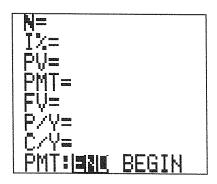
- 4. Owen invested \$3500 in a savings bond. After 2 years he cashed it in for \$4235.45.
 - a) What was the rate of interest if it was compounded quarterly?



b) What was the rate of interest if it was compounded monthly?



c) What was the rate of interest if it was compounded weekly?



- 5. Approximately, how long would it take for a \$25000 investment to double if it is compounded semi-annually at the following interest rates?
 - a) 0.5%
- b) 2%
- c) 5.5%
- d) 9%
- e) 20%
- 6. Rochelle needs to have \$3000 to buy a car. She can invest her savings in a GIC that pays 5.2% compounded monthly. How much does she need to invest now, if she waits the following lengths of time?
 - a) 6 months
- b) 1 year
- c) 2 years
- d) 5 years

Interest & Savings Alternatives

Part 1: Investment Alternatives

You will be using the website www.tdcanadatrust.com to complete this part of the assignment.

- 1. Choose "accounts" from the menu at the left. Information about chequing and savings accounts is displayed. Click on each of them, and briefly describe the differences between the two types of accounts.
- 2. Under the "tools" menu on the right, select "compare savings accounts"
 - a) Which is the only account that has unlimited debit transactions at no extra cost?
 - b) Which account charges the most for additional transactions?
 - c) Name 2 additional fees that are charged on some of the accounts.

- d) What interest rate would you receive if you had a companion account with \$6000 in it?
- e) What interest rate would you receive if you had a GIA account with \$6000 in it?

f) After comparing the different accounts, which one do you think would be best for you? Why?

Interest & Savings Alternatives Continued

3. Click on "how interest is calculated" from the menu at the right. If you scroll down to the notes on the second page you will see this:

Daily interest is calculated as follows: Daily closing balance x interest rate for the appropriate tier divided by 365.

This means, that if there is a balance of \$600 in an account, for a month with 30 days the formula to calculate the interest, if the given rate was 0.05% would be,

$$A = 600 \left(1 + \frac{0.0005}{365} \right)^{30}$$

A = 600.02, which means that \$0.02 was earned in interest.

a) Calculate the amount of interest earned for the month of May, if \$750 is in an account paying 0.06% interest.

- 4. Click on "investing" then "term deposits and GIC's" then "rate information" from the menu at the right.
 - a) What does GIC stand for?
 - b) What does the name indicate about the risk factor for this investment?

c) Where can GIC's be purchased?

Interest & Savings Alternatives Continued

5.	 Read the descriptions for the different types of GIC's and decide which would be the best option for the following situations. Explain why. Exclude the US GIC's and the specialty GIC's. a) Ralph got a \$4000 bonus from work and wants to invest it somewhere, but he also wants to do some home improvements in the next few months and needs to be able to get at the money.
	b) Fran is 29 and just inherited \$100000. She doesn't need the money now, and wants to invest the whole amount for her retirement.
	c) Dave is saving for a trip to Europe next year. He has \$1000 to invest in a GIC.
6.	Determine the amount of the following investments. a) How much would an investment of \$3000 in a "wait and see GIC" be worth at the end of a 1 year term?
	b) How much would a \$10000 investment in a "long term deposit" be worth in 5 years?
7.	In general, what is the advantage of having an investment "locked in" for a longer term?
8.	. What is a major difference between stocks and bonds?

Interest & Savings Alternatives Continued

9.	Use the internet to look up the (a) Air Canada	current price for the following	stocks
	b) Sears		
	c) Microsoft		
10	If you had 500 shares in each of worth?	f the previous companies, how n	nuch would your investments be
	a)	b)	c)
11.	You bought 80 shares in Calculat \$0.6745. a) How much money have you los		4 each. Presently, they are worth
	b) Would you want to sell them n	now? What factors would you co	onsider?

Introduction to Credit Cards

Part 2: Credit Cards

Major credit cards such as VISA and MasterCard are associated with financial institutions. TD Canada Trust for example, is associated with VISA.

Not all credit cands are the same From the menu on the left choose credit cards Refer to the chart

		credit cards are the same. From the menu on the lett, choose credit—cards Refer to the cha ded in your assignment.
1.		o to the website <u>www.tdcanadatrust.com</u> to answer the following questions. What is the interest rate for all the cards, except for the low interest card?
	b)	Which cards have annual fees?
	c)	Why might someone prefer the TD Elite VISA card, to the TD GM card?
2.		pescribe the type of person who might choose each of the following cards. TD Gold Travel Visa Card
	b)	TD Emerald Visa Card

c) TD Green Visa Card

Introduction to Credit Cards Continued

Credit Card Bills

Unit 4 - Personal Finance

3.	The following terms may appear on your credit card bill. Define what they mean with regards to a credit card statement.
	a) Previous balance
	b) Credit limit
	c) Available credit
	d) New balance
	e) Minimum payment
	f) Billing date (due date)
4.	If a payment is made in full by the billing date, no interest is charged. This month, Cindy had a previous credit card balance of \$45, bought a sweater for \$25.50, paid for a dinner out which cost \$55.00, bought a book for one of her classes for \$62.25, and treated herself to 3 new CD's which came to \$47.62. She had made an early payment of \$50. How much would she owe on her due date?
	If the entire balance is not paid off by the due date, simple interest is charged from the posting date until the due date.
5.	a) What type of people benefit from a credit card with no annual fee, but a high interest rate?
	b) What type of people would be better off with a low interest credit card, even if it has other fees?

Introduction to Credit Cards Continued

- 6. Joseph bought a stereo system on his credit card on May 14^{th} for \$2759.43. His due date is May 27^{th} . The interest rate is 19%. He does not make any payments.
 - a) How much interest would he be charged until his due date?

Hint: There are 13 days between May 14th and May 27th, so $t = \frac{13}{365}$

b) What is his new balance?

After that, interest is compounded monthly until it is paid off.

- c) If Joseph keeps ignoring his bills, how much will he owe on December 27th? Use $A = P(1+i)^n$
- d) How much does he owe in interest?

- e) How much does he owe next May?
- f) He chose his credit card because it didn't have an annual fee, but he could have chosen a low interest credit card that only charged 6% interest, but had an annual fee of \$120. After 1 year, which card would he have been better off with? By how much?

Introduction to Credit Cards Continued

Credit Rating

You will use the website www.equifax.ca to complete the next part of the assignment.

- 1. Go to the link for "credit education centre"
 - a) In 100 words or less, summarize what a credit report is.
 - b) List the seven sections that are found in your credit report.
 - c) Click on "credit score basics".......What suggestions do they have to improve your credit score?
 - d) Buying on Credit: Compare 2 different banks credit card charges for a \$1500 purchase if the full amount is paid off 55 days later.

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Costs of Operating a Vehicle

The purchase or lease of a vehicle represents only part of the cost of driving. There are also many costs involved in the day-to-day operation of a vehicle. These costs vary, depending on where you live, the type of vehicle you drive, and the distance you drive. The regular expenses involved in operating a vehicle include fuel costs, maintenance costs, insurance costs, and licence costs.

Fuel Costs

The amount of fuel a vehicle uses is often a factor in the choice of the type of vehicle to buy or lease. Information about the fuel consumption of a vehicle can be obtained from the salesperson from whom you buy or lease a vehicle. It can also be obtained from the website of National Resources Canada. The website has information on how fuel consumption ratings are determined, and provides the fuel consumption ratings for many vehicles.

Fuel consumption ratings are expressed either in miles per Imperial gallon or in liters per 100 kilometers. Once you know the fuel consumption rating for the vehicle you drive, the approximate distance you drive in a year and an estimate of the average cost per liter of fuel, you can estimate the yearly cost of fuel.

Ex.1: Oliver's car has a fuel consumption rating of 5.8 L/100 km. Oliver estimates that he drives 27000 km per year and the average cost of fuel during the year is 96¢ / L. Estimate Oliver's fuel cost for the year.

Maintenance Costs

A second major expense in operating a vehicle is the cost of maintenance. Vehicle manufacturers stress the importance of regularly scheduled maintenance to keep a vehicle in good running order and to maintain its value. The manufacturer's recommended maintenance schedule for any vehicle can be obtained through the dealer or on the internet.

Here is a sample vehicle maintenance schedule.

Vehicle	e Maintenance So	chedule	
Recommended Service	Frequency (km)	Frequency (months)	Approximate Cost (\$)
Lube, oil and filter	5000	3	29.95
Tire Rotation and Brake Inspection	10000	6	41.50
Wheel Alignment Inspection	25000	12	69.75
Cooling System	50000	24	72.95
Tune-up and Emission Control	50000	30	225.00
Automatic Transmission	85000	51	145.00

Ex.2: Thelma purchases a new car. She will use the car to drive to work every day. She estimates that she will drive 35000 km in the first year. Use the vehicle maintenance schedule to estimate Thelma's maintenance costs in the first year including taxes.

Insurance Costs

Insurance is another item that contributes to the yearly operating cost of a vehicle. Canadian law requires that

all purchases and leased vehicles be insured. All drivers are required to have *personal liability coverage*. Personal liability coverage provides insurance against lawsuits in the event that someone is injured in an accident in which the driver is involved. It also covers the cost of damage to other cars and property caused by the driver's car.

If you finance a vehicle, you must also have *collision and comprehensive coverage*. Collision coverage pays for damage to your car if you are in an accident or replaces a car that is damaged beyond repair. Comprehensive coverage covers theft and damage to your car from things such as falling trees, fire, hail and flood.

The cost of car insurance varies with the type of coverage provided and with the individual driver. Some factors that influence this cost are where you live, your age and marital status, the amount you drive, your driving record and the purposes for which you use your vehicle. Car insurance payments may be made annually, semi-annually or monthly.

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Licence Costs

Licence fees include both the driver's licence and the vehicle licence plate. At the present time, a driver's licence is renewed every 5 years, while the vehicle licence plate is renewed every year or every second year. The renewals must be done on or before your birthday in the year the renewal is required.

Ex.3: In the first 3 years Sal owns his SUV, he drive 102653 km. The average cost of fuel is 91¢/L. Sal's SUV has a fuel consumption rating of 12.7 L / 100 km. He maintains his vehicle according to the suggested maintenance schedule. Sal pays \$1259 insurance per year. He renewed his driver's licence at a cost of \$50. His vehicle licence plate is renewed every year at a cost of \$74. What was Sal's average monthly operating cost in the 3 year period?

Buying a New Vehicle Handout

When you by a new vehicle, the starting point for the price you will eventually pay is the *Manufacturer's Suggested Retail Price* (MSRP). Since this is a suggested price, it is subject to negotiations. Once you and the salesperson agree on a purchase price for the vehicle, other costs are added to obtain the actual amount you will pay for the vehicle.

The additional costs vary depending on the vehicle and the dealership. Some of these costs are freight charge, tire/air tax, gas tax, administration fee, licence plate fee, gas fee, lien payout (money you still owe on a car you are trading in), GST and PST. The additional costs are added to the agreed upon purchase price to obtain the delivery price of the vehicle.

Most people pay for a vehicle by making a down payment and then finance (borrow) the rest through the car dealership or bank. The dealership provides you with a computer printout, called a *deal review*. This itemizes the costs and financial arrangements involved in purchasing a vehicle.

Ex.1: Jorge is ready to purchase a compact car with an MSRT of \$19721. He negotiates a discount of \$1045. The following additional costs need to be included in the delivery price: a \$670 freight charge, a \$100 tire/air tax, a \$75 gas tax, an \$85 administration fee, a \$74 licence plate fee, and a \$20 gas fee. Jorge makes a down payment of \$4000, and finances the rest through the dealership at 4.9% compounded monthly for 48 months. He pays a \$54 fee to arrange the financing. Complete a deal review for Jorge's car using the following table;

DEAL REVIEW				
MSRP	Licence Plate Fee			
Discount	Gas Fee			
Purchase Price	Lien Payout			
Freight	Delivery Price			
Tire/Air Tax	MFG Rebate			
Gas Tax	Down Payment			
Administration Fee	Finance Fee			
Taxable Subtotal	Amount Financed			
Trade-in Allowance	Interest Rate			
Taxable Total	Loan Term			
HST (13%)	Total Due (incl. interest) ** A = P (1 + i) ⁿ			
	Monthly Payment			

Car Project

Part 1: Costs associated with owning a vehicle

*Be sure to include all calculation	*Be	sure	to	include	all	calculation	ns
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1. Look at website of different car dealerships. Choose a vehicle that you might like to own in the near future. You should try to be realistic (don't get carried away). It should be the current model year of the car.

State the make, model, and all options that you would choose.

Insurance

2.	You may use	the website	<u>www.ibc.ca,</u>	or others,	to help	answer	the fo	llowing q	uestions.

a) What is a deductible?

b) What happens to an insurance premium when a deductible is lowered?

3. Go to the website www.statefarm.com. Follow the links to get an insurance quote. Use the information based on the car you selected. Use your own personal information as much as possible. Example: If you don't have a licence, pretend you do! You may pick whatever coverage you would like. Assume you are the only driver on the car.

a) What are some of the major factors they consider when giving an insurance quote?

b) What was your quote?

c) Based on the factors that are considered, describe a scenario of someone who might have a very high insurance quote.

Go	Gasoline					
4.		stimate the price of gas per litre today. (You're right, it does vary a lot)				
	a)	What size gas tank does the car you have chosen have? (you may have to look it up)				
	b)	How much would it cost to fill this tank, based on today's price?				
	c)	Look up the fuel consumption for your car. It will be given as litres per 100km. There should be a value for city, and for highway driving.				
	d)	Suggest why you might get better fuel efficiency on the highway.				
	e)	Choose a destination in Canada that you would like to drive to. Use www.mapquest.ca to determine the distance in km to get there.				
	f)	If the whole way was highway driving, how many litres of gas would you need to get there?				

g) How much would this cost, based on today's price of gas?

Unit 4 - Personal Finance

Other Expenses
1a) How often is it recommended to get an oil change?
b) Use the internet to look up the price for a basic oil change. Be sure to include tax. You can choose any place that does them.
c) If you drive and average of 25000km per year, how much will you spend on oil changes?
2a) When do you need to purchase a licence plate sticker?
b) How much does it cost?
c) Where can you get it from?
d) How often do you have to get a drive clean emissions test?
e) How much does it cost?

- 8			
- 8	Dant 2.	Densina Valancia	
- 1	rart 2:	Buying Vs Leasing	
		I this is a market till	

1.	s a)	tate which option (buying or leasing) would be better in the following scenarios, and why? A 30 year old business man who drives around all day from client to client.
	b)	A college student who doesn't have much of a down payment, and a very limited amount to spend per month.
	c)	An auto mechanic, who enjoys doing all of his own repairs.
	d)	Newlyweds, who want a sports car for now, but are planning on starting a family in a few years and will need a bigger car.
	e)	Jeremy wants to put in a huge new stereo system and change the paint job.
2.	٨	Make a bill for the car you choose on page 1.
	8	Base price: \$
	C	Cost of all options: \$
	C	Other costs (freight, PDI) \$ <u>1000.00</u> (estimate)
	ŀ	HST 13%: \$
	=	Fotal: \$

Financing		

- 1. Suppose you have \$5000 to use as a down payment on the car you are financing.
 - a) Subtract that from the price of your car. This is the amount you will need to finance.
 - b) Assume your bank is giving you a rate of 2.3% compounded monthly, and paid over a 5 year term. Use the TVM solver of the graphing calculator to determine what your monthly payments will be. Enter these values:

N = 60

1% = 2.3

PV = 0.00

PMT = 0.00 (this is what you are finding) →

FV = the amount you are financing

P/Y = 12

C/Y = 12

What will the monthly payments be?

d) At the end of the 5 year term, how much will you have paid in total? (Don't forget to include the \$5000 down payment.)

Total amount = monthly payment $\times 60 + 5000$

e) How much have you paid in interest?

Lec	sing	
1.	t a	o to <u>www.canadiandriver.com</u> . From the menu at the left, choose "calculators" and follow the steps o calculate how much a monthly lease would be for the car you chose. Use a 5 year (60 month) term and a \$5000 down payment. Your monthly lease payment is \$
	b)	How much have you paid after the 5 years. Don't forget to add the down payment again.
2.		n your lease agreement you are allowed 25000 km per year for free. After that you are charged 0.08 per km. After your 5 year term, you drove a total of 140000 km. How much do you owe?
Par	+ 3	New or Used Car
4 .		tate whether it would be better to buy a new or used car, and why, based on the following
		cenarios. Sue lives in Ontario, but got a 1 year job contract in BC. She needs a car there, but wants to sell it before she moves home again.
	b)	Eric has his heart set on a certain high performance car. Only a few hundred are made each year and he is very particular about which options he wants.
	c)	Jackie commutes about 100km each way to work, and is terrified about her car breaking down.
	d)	The Reed family is interested in purchasing an extra vehicle for doing errands.

Depreciation

2. Depreciation is the major disadvantage to buying a new car. A car depreciates in value the most during its first year. Here is a chart that shows the typical depreciation rate of a car.

Time	Depreciation (% of previous year's value)
1 st year	30%
2 nd year	15%
3 rd year	15%
4 th year	10%
5 th year	8%

A Car that costs \$25000 new is only worth \$17500 after 1 year.

$$A = 25000 (1 - 0.3)$$

To figure out the value after the 2^{nd} year it would be 17500(1 - 0.15)

- a) Use this chart to figure out how much your car would be worth after each of the first five years?
- b) Based on this information, when is the best time to purchase a used car to get the best value? Why?

3. One of the major disadvantages to buying a used car, is you may not know what you are getting. If you were going to buy a used car, what are 5 important questions you might want to ask the previous owner?

Personal Finance Practice Test

1. Fill in the following table calculating the tax on each item, total cost of each item. In the last row, add up the total of each column.

Item	Price (\$)	GST (6%)	PST (8%)	Total
Jeans	89.99			
T-Shirt	32.49			
TOTAL:				

- 2. You purchase an item online for \$18.75. On top of the purchase price, you must pay a 15% shipping fee and a handling charge of \$2.50. What is the total of the purchase?
- 3. Your parents give you \$40 for gas money. How many litres can you buy if the price of gas is 104.7¢/L?
- 4. A minivan was purchased last year for \$22,000. This year, it is only worth \$18,000. Calculate the percent depreciation, accurate to one decimal place.

5. Calculate the value of a \$2300 C-Bond after 3 years, 5 months and 26 days.

								-/		
Year	1	2	3	4	5	6	7	8	9	10
Interest Rate (%)	3	4	5	6	6.5	6.75	7	7.25	8	8.75

6. Complete the table below to find the amount owing on the credit card billing date of September 12th.

Posting Date	Item	Amount	Number of days to billing date	Interest
August 18	Cash Advance	\$200		
August 22	Footwear	\$114.99		

Total amount owing on September 12:

- 7. Calculate the amount owing on the following billing date of October 12 if the bill is not paid.
- 8. An infomercial is advertising a treadmill for 6 monthly payments of \$150. You have seen the same treadmill at Fitness Depot for \$850.
 - a) How much interest is the infomercial charging so that you can pay in installments rather than all at once?
 - b) What is the annual interest rate being charged by the TV advertisers?
- 9. Explain two ways how bonds are different from stocks. Which of those options requires more investment knowledge? Why?

Personal Finance Practice Test

1. Calculate the missing simple interest values and fill in the chart. Use the space below the chart for your rough work.

Principal P	Interest Rate r	Time †	Interest I	Amount A
\$3550	6%	5 years		
\$7420		242 days	\$504.25	
	12%	15 months	\$1875	

- 2. For each compounding frequency, calculate the number of compounding periods per year.
 - a) quarterly b) bi-weekly c) weekly d) semi-annual e) monthly
- 3. Solve for i as it would appear in the compound interest formula:
 - a) 12.5% quarterly

- b) 6% semi-annually
- 4. Solve for n as it would appear in the compound interest formula:
 - a) monthly for 7 years
- b) annually for 12 years
- 5. Convert the following lengths of time to years. Round to three decimal places.
 - a) 102 months
- b) 68 weeks
- c) 165 days
- 6. How much interest is earned on \$5310, invested at 13.5% simple interest for 65years?
- 7. What principal is required to earn \$329.88 worth of simple interest at a rate of 6.5% over 3.5 years?
- 8. What is the annual rate of <u>simple interest</u> required to have an initial investment of \$15,200 grow to \$16,000 in 10 years?
- 9. Kyle invested \$1440 at 4.35%, compounded monthly for 3 years.
 - a) How much will the investment be worth at the end of the 3 years?
 - b) How much interest did Kyle earn?

Personal Finance Practice Test

- 10. Sherysse owes \$1345 that must be repaid in one year. The bank will accept payment today at a discounted rate of 12.5%, compounded semi-annually.
 - a) How much will Sherysse have to pay back if she pays her loan today?
 - b) How much money will she save if she pays today, rather than in one year?
- 11. Jonathan will need \$3058.05 in 2 years to pay off a student loan. He has \$2300 today. What rate of interest does he need if compounding is quarterly?
- 12. Jake needs \$4447.25 to pay a debt as soon as possible. He has \$3250 today.
 - a) How long will it take until he can pay off the debt if the money is invested at 10.5%, compounded monthly?
 - b) Would it take more or less time to pay off the debt if the compounding was semi-annually? Explain.
- 13. You have two options with which to invest your \$300 bonus cheque:

Option A – Invest at 5% simple interest over 6 years

Option B - Invest at 4.5% interest, compounded monthly, over 6 years

Which is the best option for you? Provide a full solution in the space below to justify you answer.

- 14. A sum of \$5000 grew to \$5962 in 2 years at 9%, compounded semi-annually. Without a calculator answer the following.
 - a) Would the interest rate be lower or higher if the compounding had been annual? Explain.
 - b) Would the interest rate be lower or higher if the compounding had been monthly? Explain.

MBF 3C Unit 5 - Geometry

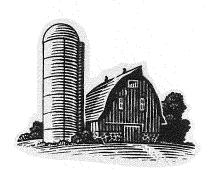
Section	Date	Lesson Title	Handouts	Homework Assigned
1		Real-Life applications of geometric shapes and figures	U05A01	U05A02
2		Imperial and Metric systems of Measurement	U05A03	U05A04
3		Representing 3-D figures using orthographic and isometric methods	U05A05	U05A06
4		Representing 3-D figures using scale models	U05A07	Supplemental Worksheet
	2000			(not provided)
5		Representing 3-D figures using a net, pattern or plan	U05A08	U05A09
6		Creating plans	U05A10	
7		Creating an individual design problem	U05A11	
8	333113	Group Design Challenge	U05A12	
9		Optional Assignment Period	U05A13 or U05A14 or U05A15	
10		Review Day	Supplemental Worksheet (not provided)	
11		Practice Test	U05A16	
12		Unit Test		

Applications of Geometry

Geometry is the foundation behind the design of many real world things. How geometry is considered in designing something can affect what the object looks like and how it works. Geometry can be seen in a variety of different contexts. We are going to look at some of these contexts in this activity.

Look at the main object in the two following pictures. Identify as many geometric shapes as you can in these objects. List them below.





Now think of the *advantages* of the geometric shapes that you identified. Identify at least one way that the geometric shape in the object affects how the object looks and how the object works.

	Geometric shape and how it effects how it looks	Geometric shape and how it effects how it works
Car		
Farm		

MBF3C U05A01-P2

A few specific areas in which we see geometry are in architecture, fashion, product design. In all of these categories, geometry can affect both how the object looks and works.

Geometry in Architecture:

Example: The Eiffel Tower in Paris

- -What is the repeated shape?
- -How might the use of these repeated shapes affect the structure?
- -How do these shapes affect the appearance?



Geometry in Fashion:

Example: A baseball cap

- -What is the primary geometric shape?
- -How is this shape useful?
- -How does the shape affect the appearance?



Geometry in product design:

Example: A box of cereal

- -What is the primary shape?
- -How is this shape useful for this product?
- -How does the shape affect the appearance?



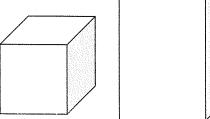
1. List all of the geometric shapes found in this object.



2. Identify two geometric shapes in this object and describe how each of these shapes effect how the object works and/or looks.



3. The two shapes shown are being considered for the packaging of a new cereal. They both have the same volume, meaning they will hold the same amount of cereal. Discuss why one of the two shapes may be a better choice for packaging and why.

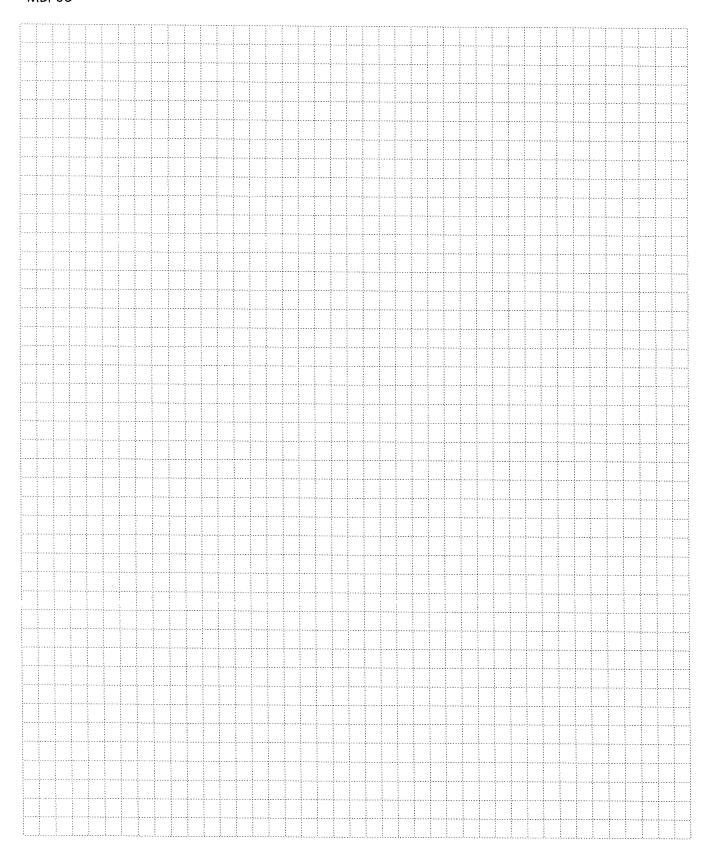


4. Pick an object of your choice and write a paragraph to explain how geometry was used in the design of the object.

Answers:

- 1. Rectangle (house, door, chimney), trapezoid (roof), pentagon (upper windows), squares (lower windows), circle (door handle and top of trees)
- 2. Sphere shape allows for the ball to roll, Pentagon repeated pattern fits together to form structure and gives consistent appearance.
- 3. The rectangular prism on the right is better. Athough it holds the same amount, it looks larger and the customer might think there is more cereal. Also, there is greater surface area on the front for the logo and other information to stand out.
- 4. Answers will vary.

MBF3C



Imperial vs. Metric Measuring Systems

Systems of measurement are used to measure the length, volume, mass or temperature of an object.

The Metric System		
	f the world use the <i>metric</i> system of main unit of measure for each categ	
Length	Volume	
Mass	Temperature	
Some of the commonly used units a	nd conversions in the metric system	are as follows:
Length	Volume	Mass
10 mm = 1 cm	1000 mL = 1 L	1000 g = 1 kg
100 cm = 1 m		1000 kg = 1 †
1000 m = 1 km		

1. If a wall is measured to be 450 cm long, what is the measurement in metres (m)?

2. If a container has a volume of 2.6 L, what is the volume in millilitres (mL)?

3. Consider the following examples of objects that could be measured. Match the examples with the most appropriate unit of measurement by placing the letter from Column A in the answer column.

	Column A	Answer	Column B
a)	volume of a cooler		170 cm
b)	mass of an average person		22°C
c)	temperature inside a room		10 mm
d)	thickness of a magazine		75 kg
e)	height of an average person		20 L
f)	distance around a running track		400 m

The Imperial Syster	m
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Some other countries, particularly the United States, use a different system of measurement called the *imperial* system. Although it is not recognized as Canada's main system of measurement, why is it still important for us to be able to understand and work with the imperial system?

In the case of the imperial system, fill in at least on	e example of a unit of measure for each category:
Length	Volume
Mass	Temperature

Some of the commonly used units and conversions in the imperial system are as follows:

Length	Volume	Mass
12 inches = 1 foot	16 fluid ounces = 1 pint	16 ounces = 1 pound
3 feet = 1 yard	2 pints = 1 quart	2000 pounds = 1 ton (US)
1760 yards = 1 mile	8 pints = 1 gallon	

- 4. If a wall is measured to be 144 inches long, what is the measurement in feet?
- 5. If a container has a volume of 6 quarts, what is the volume in pints?
- 6. Consider the following examples of objects that could be measured. Match the examples with the most appropriate unit of measurement by placing the letter from Column A in the answer column.

	Column A	Answer	Column B
a)	volume of a cooler		$\frac{1}{2}$ in. (inches)
b)	mass of an average person		5'10" (5 feet, 10 inches)
c)	temperature inside a room		5 gal (gallons)
d)	thickness of a magazine		175 lb. (pounds)
e)	height of an average person		200 yd. (yards)
f)	distance around a running track		72°F

Converting Between the Metric and Imperial Systems

The following are approximate conversions between commonly used metric and imperial measurements:

Length	Volume	Mass
30.48 cm = 1 foot	29.574 mL = 1 fluid ounce	28.35 g = 1 ounce
2.54 cm = 1 inch	0.473 L = 1 pint	0.454 kg = 1 pound
1.6 km = 1 mile	3.785 L = 1 gallon	0.907 t = 1 ton (US)

7. If a wall is measured to be 14 feet long, what is the measurement in cm?

8. If a container has a volume of 4 L, what is the volume in gallons?

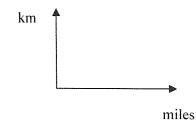
- 9. In the following questions you will be creating a conversion chart, and then graphing your data.
 - a) Inches to Centimetres (1 inch = 2.54 cm)

Inches	1	2	3			
cm				12.7	17.78	127



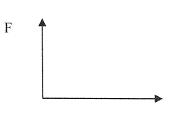
b) Miles to Kilometres (1 mile = 1.6 km)

Miles	1	2	3			
km				10	20	30

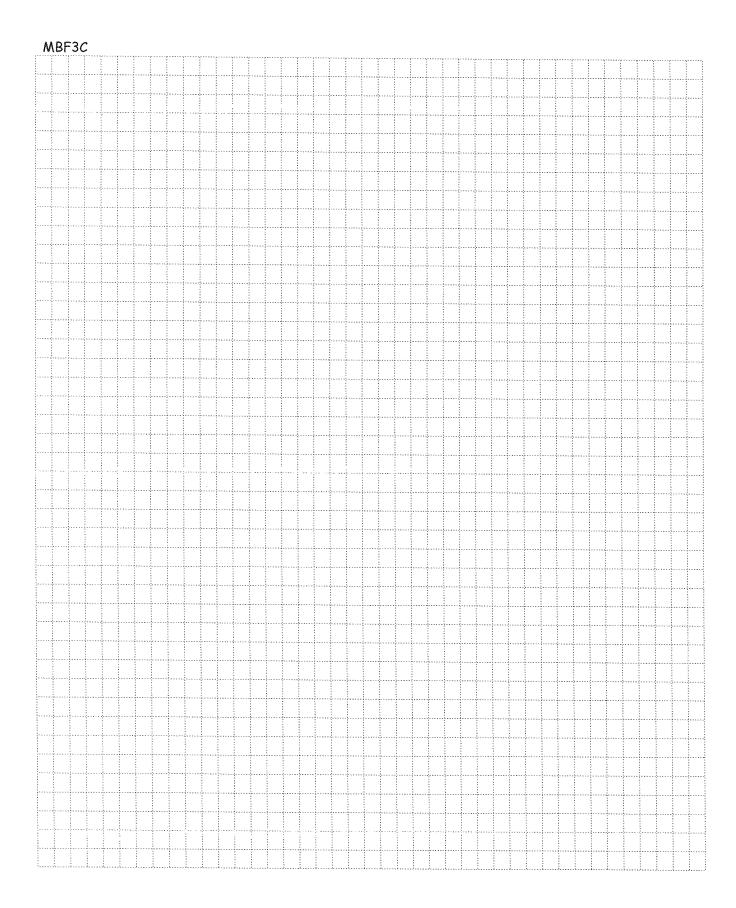


c) Celsius to Fahrenheit ($F = \frac{9}{5}C + 32$)

F	0		100			
Celsius		0		20	26	37.4



C



Exercises

1.	Convert	the	following	metric	measures:
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2. Convert the following imperial measures:

3. Convert the following metric and imperial measures:

4. Estimate the following measures using an appropriate unit of measure:

a) the length of your foot:

b) the volume of a carton of milk:

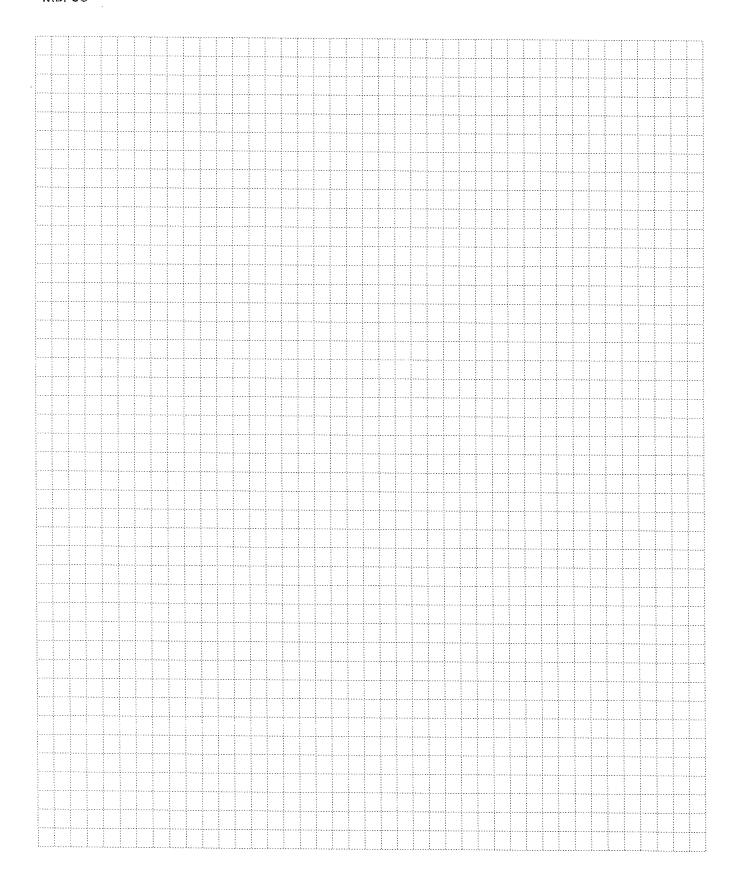
c) the height of the classroom:

d) the width of your thumbnail:

5. Jesse needs to order flooring for his room. He measured the dimensions of the room to be 300 cm by 375 cm. However, the flooring company needs to know these dimensions in feet. Find the dimensions of the room in feet.

Answers:

- 1. a) 2.4 km b) 340 mm c) 5000 mL d) 3.2 kg
- 2. a) 64 ounces b) 13 pints c) 3.5 feet d) 5280 yards
- 3. a) 91.44 cm b) 24.9 miles c) 37.85 L d) 4.94 ounces
- 4. a) 20 cm (will vary) b) 1 L (will vary) c) 10 feet (will vary) d) 15 mm (will vary)
- 5. 10 feet by 12.5 feet

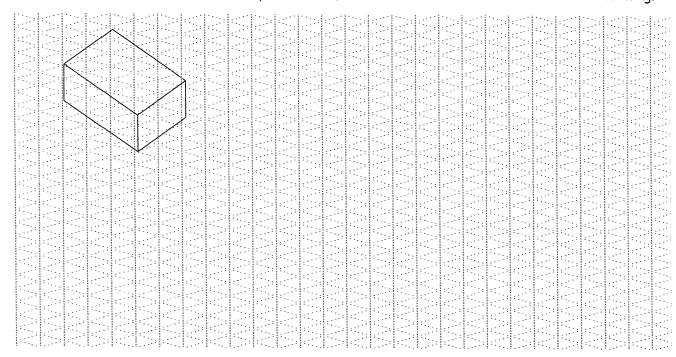


ISOMETRIC AND ORTHOGRAPHIC DRAWINGS

- Consider a rectangular prism with these dimensions: 3m (length) \times 2m (width) \times 1m (height)
- Sketch a drawing of this figure. Compare your drawing with someone next to you. How do they compare?

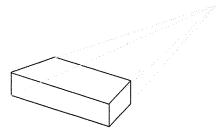
Isometric Drawings

- An isometric drawing is a 2-D representation of a 3-D figure. The drawing looks like the 3-D figure has been rotated to a corner view so that you can see the top, front, and right side of the figure on an angle.
- This is what the given rectangular prism would look like as an isometric drawing with a scale of 1 unit = 1 metre. The scale must always be stated, or the dimensions should be shown on the drawing.



Isometric Perspective Drawings

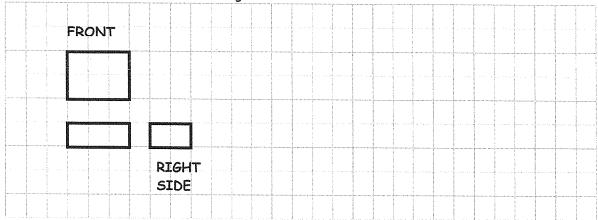
- An isometric perspective drawing is another way to represent a 3-D figure in a 2-D drawing. It is similar to an isometric drawing, but all of the parallel sides of the figure are slanted off to a point to make the object look more realistic.
- This is what the rectangular prism would look like as a perspective isometric drawing.



- To create an isometric perspective drawing, draw the front view of the object as you would with an isometric drawing. Then pick a point away from the object and angle all other lines towards this point.
- Try drawing the 2m x 4m x 5m rectangular prism in this way in the space below.

Orthographic Drawings

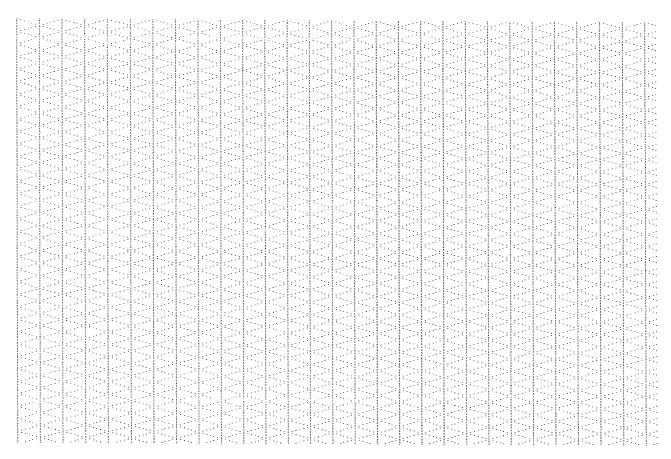
- An orthographic drawing gives a 2-D representation of a 3-D object by displaying what it looks like when viewed directly from the top, front, and right side.
- This is what the rectangular prism would look like in an orthographic drawing. It is drawn on graph paper with a scale of 1 unit = 1 metre. The scale must always be stated, or the dimensions of each side should be shown on the drawing.



TOP

- To create an orthographic drawing, you need to complete an individual scale drawing of each of the top, front and right views. These three views then must be oriented to that you can see which edges line up, as shown in the example above.
- Try to draw the $2m \times 4m \times 5m$ rectangular prism this way in the space above.

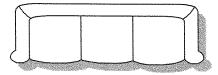
Isometric Paper



Exercises

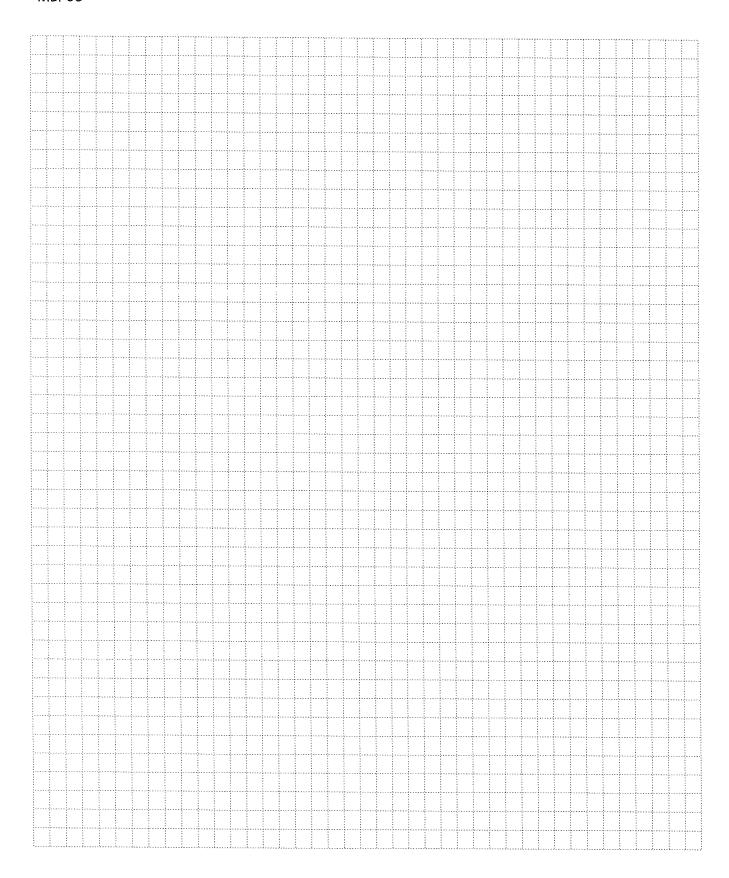
Copy and complete the following questions in your notebook.

- 1. Draw a rectangular prism with dimensions $3m \times 2m \times 5m$ in the following forms:
 - a) isometric
- b) isometric perspective
- c) orthographic
- 2. Draw an isometric drawing of a $30 \text{cm} \times 30 \text{cm} \times 30 \text{cm}$ cube with a scale of 1 unit = 3 cm.
- 3. Draw an isometric drawing and orthographic drawing of a large rectangular prism with a smaller rectangular prism sitting on top. State the scale used.
- 4. The figure shown is the top view of a couch. Sketch the front and side view of the couch.



- 5. Complete the following drawings for a piece of furniture in your home of your choice (i.e., a table). State the scale.
 - a) isometric
- b) isometric perspective
- c) orthographic

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SCALE MODELS AND NETS

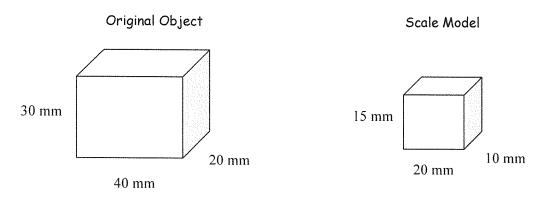
A scale model is a representation or copy of an object that is larger or smaller than the actual size of the object being represented. Very often the scale model is smaller than the original and used as a guide to making the object in full size.

Terminology: A scale is written in the form of a ratio, where the first number represents the number of units on the model and the second number represents the corresponding number of units on the actual object.

Example:

A Hot Wheels toy car is a 1:64 scale. Therefore, 1 cm on the toy represents 64 cm on the real-life vehicle.

An example of a 1:2 scale would be the following:



Note that each dimension on the model is twice as small as the original, as given by the scale.

Example: Applying a Scale Factor

a) Find the length of a toy car that models a real-life car of length 288 cm at a 1:72 scale.

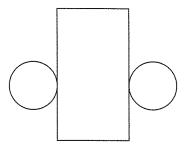
b) Find the length of a toy car that models a real-life car of length 288 cm at a 1:64 scale.

c) Which of the two scales above is considered to be larger?

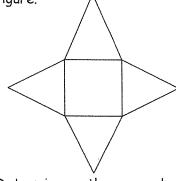
Creating Nets

A net is a 2-D representation of a 3-D figure that results when the figure is unfolded. When unfolded, the original figure becomes a flat shape where all of the edges meet to keep the net as one piece.

One example of a net arises from "unfolding" a soup can. If you flipped up the two circular tops and then cut down the side and flattened it out, this would be the resulting net:



Given the following net, think of what it would form if it was folded up into a 3-D figure. Sketch the figure. \land

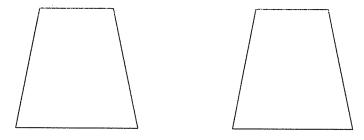


Determine another example of a net. Name the figure and then draw the corresponding net in the space below.

1. Patterns

A pattern is another 2-D representation of a 3-D figure. It results when the figure is separated into two or more individual shapes.

One example of a pattern arises when you take a piece of clothing apart at the seams. Consider a basic skirt. If you were to separate it into different pieces, this would be the resulting pattern, representing the front and the back:



In patterns, you often have to account for seams, or flaps, that allow for the separate pieces to be attached together. Where would the seams have to be in the above example?

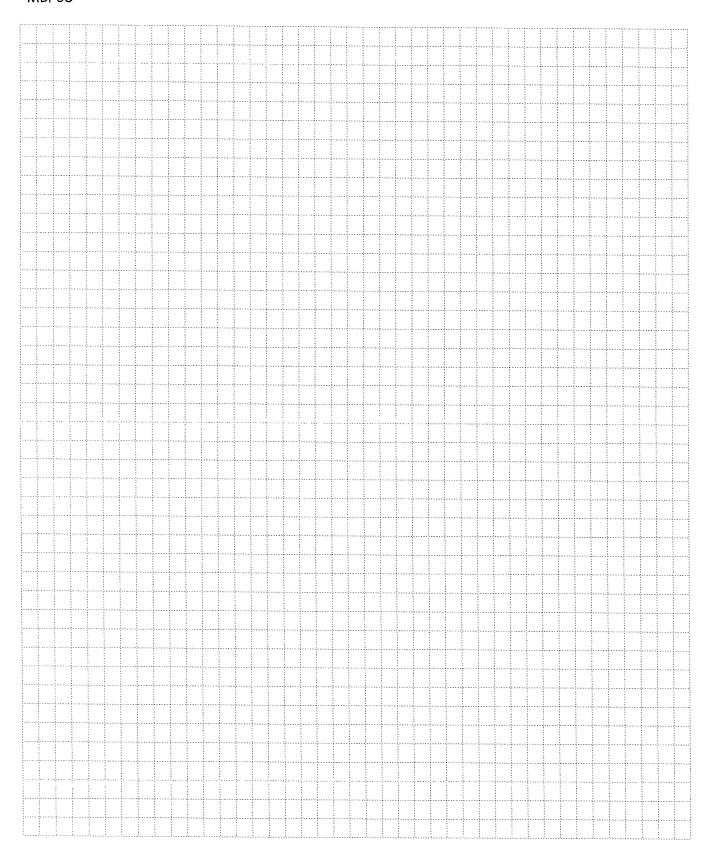
Determine another example of a pattern. Name the figure and then draw the corresponding net in the space below.

2. Nets vs. Patterns

Consider a basic cereal box. Sketch a net for the cereal box on graph paper. Sketch a pattern for the cereal box on graph paper. Include approximate dimensions and scales.

Does a net or a pattern make more sense in this situation?

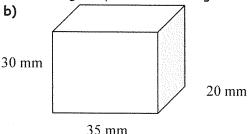
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Exercises

Complete the following exercises in your notebook.

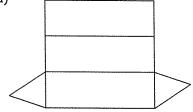
- 1. List three different types of scale models.
- 2. Why would a scale model be considered useful in designing very large or very small objects?
- 3. a) If the wingspan of a model aircraft was 15 cm, and the model was at a 1:144 scale, how long would the wingspan of the real-life aircraft be? (check to make sure your answer makes sense!)
 - b) Convert your answer in part a) to metres.
- 4. A computer's microchip is very small. To simplify the design process, engineers make a scale model of the chip. Would it make more sense for the scale to be 200:1 or 1:200? Explain.
- 5. Draw the net of the following objects using appropriate scales on graph paper.
 - a) a triangular prism with triangle side lengths 10cm and prism height 30cm.



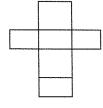
- 6. Create a pattern for a cube with all dimensions 5 cm. Allow for flaps to join the parts together. Assemble the pattern to form the cube.
- 7. How would the net of a box with a closed top differ from the net of the same box with an open top? Sketch the net for each case to demonstrate the difference.
- 8. Draw a net that may appear to fold up into a closed 3-D object, but really does not. Explain why it does not create a closed object.

Answers:

- პ. 5.
- a) 2160 cm
- a)

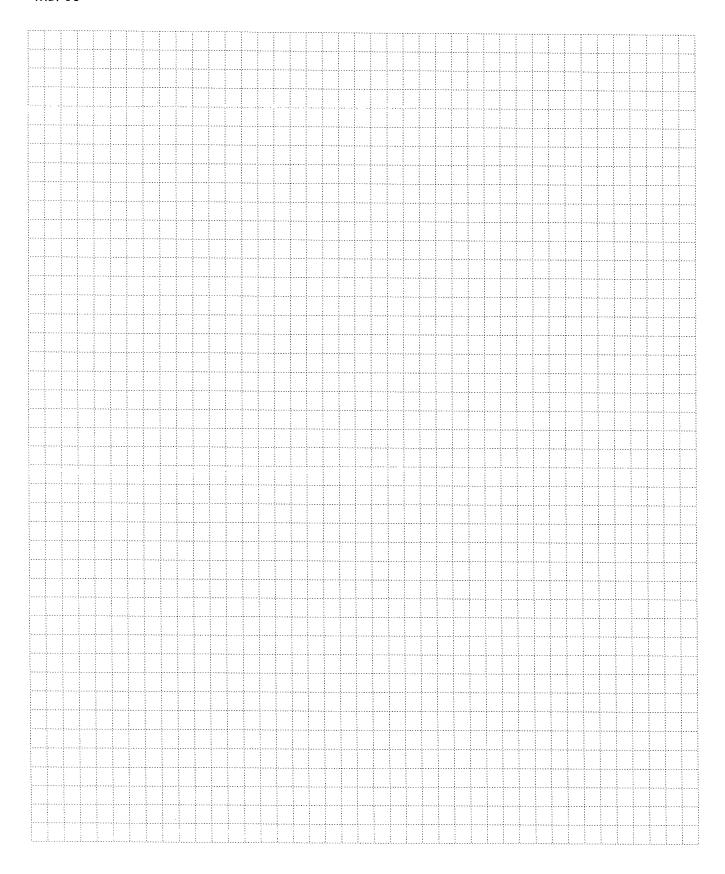


- b) 21.6 m
- b)



- 6. A closed box would look similar to 1b). An open box would have one less square to represent one less side. Any of the squares on the top or bottom or two outsides could be removed.
- 7. Results will vary. It could be a net that looks like those above, but with dimensions on the edges that do not allow the sides to match up.

MBF3C



Creating Plans

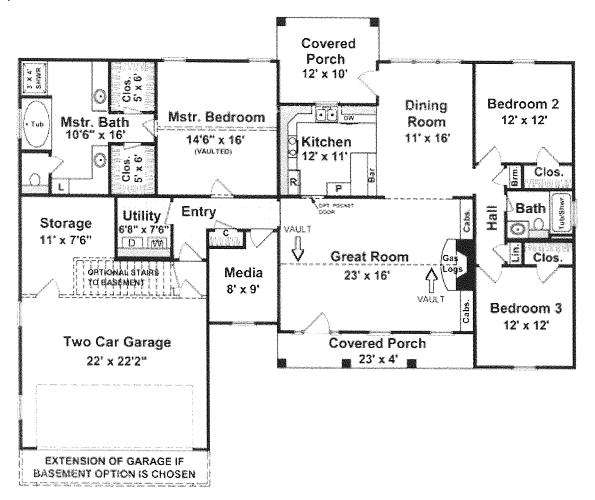
Using the following house layouts provided, choose one and determine all the information that you can about the house. Share your findings with someone else in the class.

Option #1:

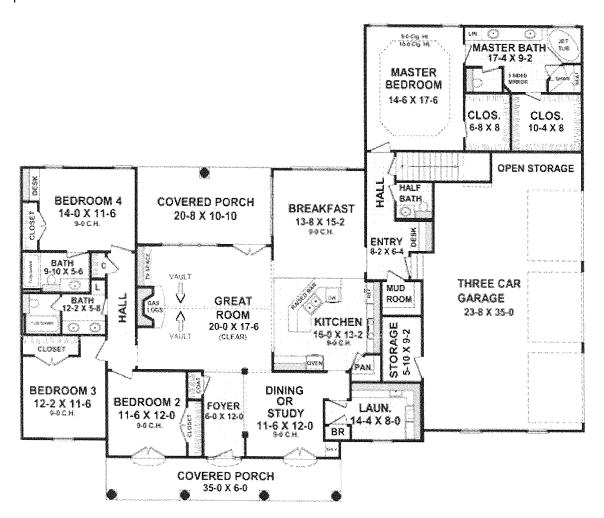


Creating Plans Continued

Option # 2:



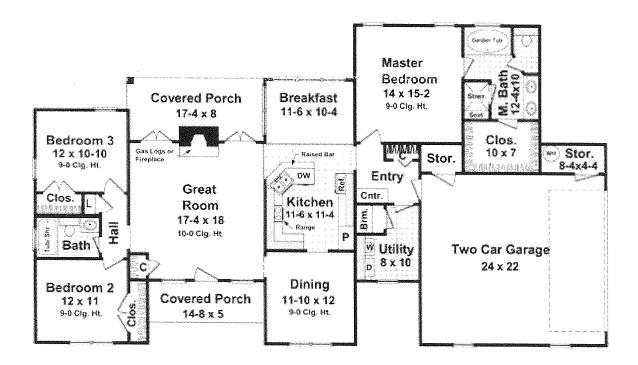
Creating Plans Continued



Where else do we see plans in real life? Why is it important to have accurate plans?

Creating Plans Continued

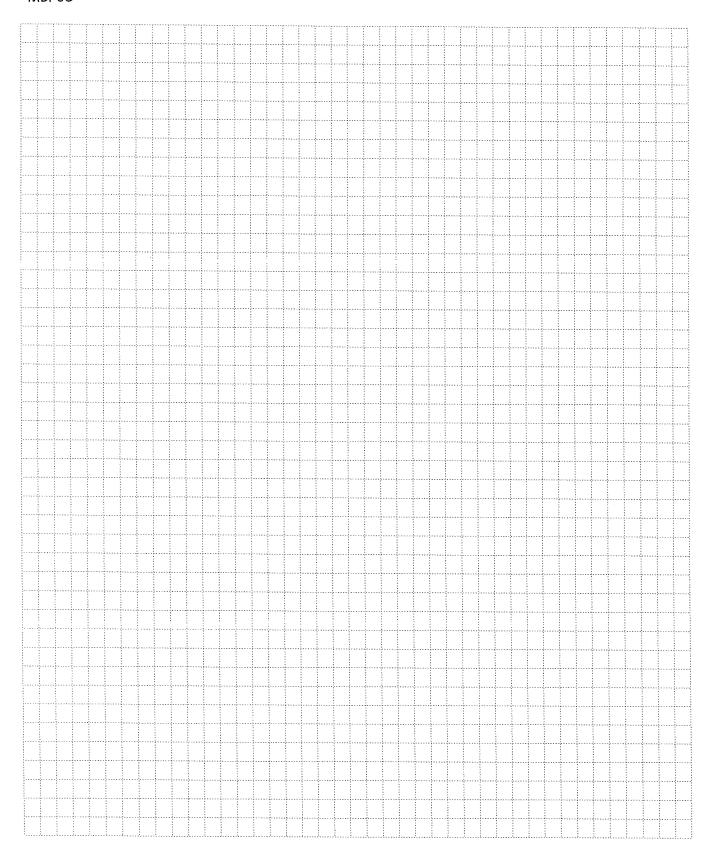
Using the top half of the graph paper on the next page, reproduce the house plan given below. Make sure the plan is accurate and to scale.



What information would you need to create a plan of your dream bedroom?

Homework: On the bottom half of the graph paper on the next page, create a plan of your dream bedroom.

MBF3C



U05A11

In-Class Assignment

- this project is to be completed in pairs or individually ONLY
- each group must have their own unique design
- each student is to submit his/her own assignment
- this project must be completed in pencil
- lines must be drawn using a straight edge
- answer all questions in the space provided
- show all mathematical calculations, as required, for full marks

K&U	APP	COM	P&T (COM)
10	15	8	5
		Model Level	

GEOMETRY DESIGN PROBLEM

Design a container that will hold 24 cans of vegetables. Each of these cans is a cylinder with radius 4 cm and height 12 cm. The cans may be arranged in any way.

KNOWLEDGE AND UNDERSTANDING

1. What shape will your container be? Briefly explain your choice.

[2]

2. State the dimensions of your container. Show your calculations as to how you arrived at these dimensions in the space below.

[6]

3. Which material(s) will the container be made out of? Explain your choice(s).

[2]

In-Class Assignment Continued

APPLICATION

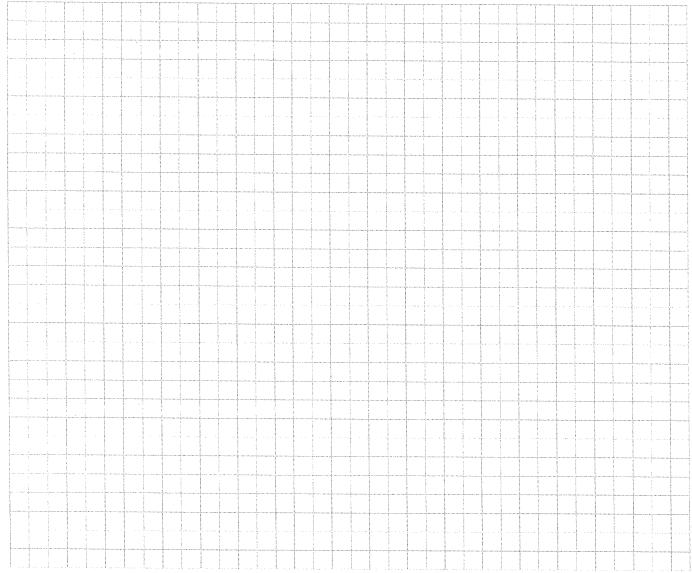
In the following section, you will be asked to draw orthographic and isometric sketches of your design, as well as a net. Use a ruler for all drawings, and keep the size of the grid in mind when you are figuring out what scale to use.

4. Choose an appropriate scale to use when drawing your sketches. All drawings will be to the same scale, so choose carefully! State your scale.

[1]

5. Draw the net on the grid below. Be sure that your diagram is drawn to scale.

[6]

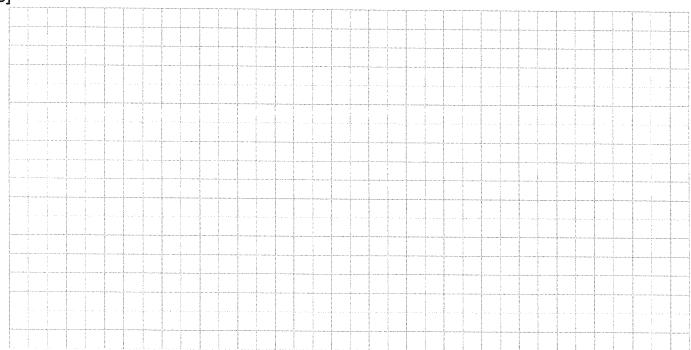


U05A11-P3

In-Class Assignment Continued

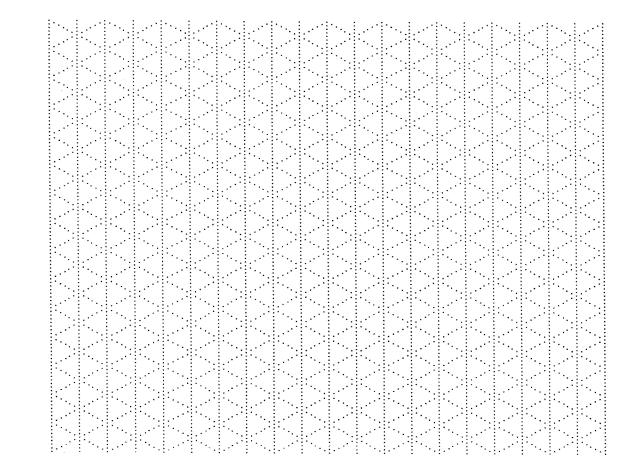
6. Create an orthographic drawing below. Be sure that your diagram is drawn to scale.

[5]



7. Create an isometric drawing below. Be sure that your diagram is drawn to scale.

[3]



In-Class Assignment Continued

	UNICATION
State	your answer clearly and use proper sentence form.
8.	You are trying to sell your design to a soup packaging company. Explain two advantages of your design in detail.
9.	What factors did you take into account when designing your container?
	when designing your container;
10	. What factors might you have overlooked when designing your container?

11. Create a scale model taking all features of your design into consideration and submit it with this assignment. The model will be marked using a rubric.

Group Design Challenge

Designing a Mini-Putt Hole

Scottsdale Engineering has hired you and your partner on contract to create a mini-putt hole for their traveling golf course. Complete the following tasks and submit in order to receive payment of a free round of golf.



Task 1: Create a plan for the design of your mini-putt hole.

Task 2: Create a prototype of the hole using cardboard for its structure and other appropriate materials(or paint) for grass, water, sand etc.

Task 3: Draw a scale diagram of the top, front, back and side views of the hole on grid paper. Label diagrams. Indicate key measurements and the scale used.

Task 4: Determine the ratio of the volume of the hole to the volume of the ball. How does your ratio compare with a ratio of the hole to ball in a miniature golf course already in existence?

Task 5: Determine the surface area of exposed cardboard.

Task 6: Determine the total cost of building a wooden-structure similar to the prototype including costs of other materials that would be used. Source the information for obtaining costs of materials needed to build the mini-putt hole.

Task 7: Write a reflection paper on the process of creating your mini-putt hole. Reflection paper should include how ideas changed and why they changed for building your prototype. The paper is to include as well any difficulties with the project and what you did to overcome those challenges.

U05A12-P2

Rubric Evaluation

Designing a Mini-Putt Hole

	Level 1	Level 2	Level 3	Level 4
Knowledge/ Understanding Scale drawing Volume Surface area	-constructs scale drawing with limited accuracy - calculates volume with limited accuracy - calculates surface area with limited accuracy	-constructs scale drawing with some accuracy -calculates volume with some accuracy -calculates surface area with some accuracy	-constructs scale drawing with considerable amount of accuracy -calculates volume with considerable amount of accuracy -calculates surface area with considerable amount of	-constructs scale drawing with a high degree of accuracy -calculates volume with a high degree of accuracy -calculates surface area with a high degree of accuracy
Thinking planning creation of prototype	-uses planning skills with limited effectiveness -uses limited problem solving strategies effectively in creating the prototype	-uses planning skills with some effectiveness -uses some problem solving strategies effectively in creating the prototype	-uses planning skills with considerable effectiveness -uses considerable problem solving strategies effectively in creating the prototype	uses planning skills with a high degree of effectiveness -uses a high degree of problem solving strategies effectively in creating the prototype
Communication Information on diagrams reflection paper	-labels diagrams with limited clarity -writes report with limited clarity	-labels diagrams with some clarity -writes report with some clarity	-labels diagrams with clarity -writes report with clarity	-labels diagrams with high degree of clarity -writes report with high degree of clarity
Application comparison of volumes calculation of costs	-applies ratios when comparing volumes with limited effectiveness -applies concepts of measurement in calculating costs with limited effectiveness	- applies ratios when comparing volumes with some effectiveness -applies concepts of measurement in calculating costs with some effectiveness	- applies ratios when comparing volumes with considerable effectiveness -applies concepts of measurement in calculating costs with considerable effectiveness	- applies ratios when comparing volumes with a high degree of effectiveness -applies concepts of measurement in calculating costs with a high degree of effectiveness

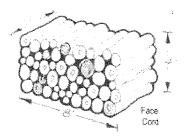
1. To conserve water in the garden mulch is spread over the surface of the soil. This prevents the soil from drying out so that less watering is required. Pine bark mulch 6 inches deep is to be spread in each of the beds shown above. Mulch is sold by the cubic yard. To the nearest cubic yard, how much mulch will need to be purchased.

15' diameter flower bed 2' x 25' flower bed

2' x 25' flower bed

4' x 25' flower bed

- 2. Ed works for an environmental protection agency which deals with land areas that have been contaminated by toxic waste. The contaminated soil covers an area of 1.62 acres. Ed must remove the top 18 inches of soil in this area.
 - 1 acre = 43,560 square feet 1 square yard = 9 square feet
 - a) What is the total volume of the contaminated soil in cubic yards?
 - b) If each truck can haul 10 cubic yards of soil, how many full truckloads of contaminated soil will Ed be removing?
- 3. Small woodlot owners usually harvest sustainably meaning that they can cut trees every year and by the next year just as many trees will have started to grow to replace them. The woodlot can be harvested forever because not too much is cut every year. If a woodlot owner cuts down 6 trees with trunks about 1 foot in radius and 20 feet tall estimate the amount of wood obtained.



use not too much is cut every
down 6 trees with trunks about 1
estimate the amount of wood

A face cord of wood is a neat
and tightly packed stack of cut

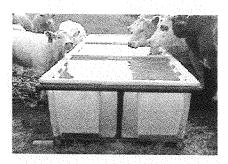
and tightly packed stack of cut and split wood measuring 8 feet wide by 4 feet tall by 16 inches deep. How many face cords of wood will the 6 trees make?

Geometry and Farming Assignment

- 1. A cylindrical well head has a diameter of 1 foot and a height of 25". Calculate its volume.
- 2. Our little calf friend is about to chow down. How much food does the trough hold if it is 1.8 m long and 35 cm deep? (image from: http://www.douglasdale.co.za/forthekids.asp)

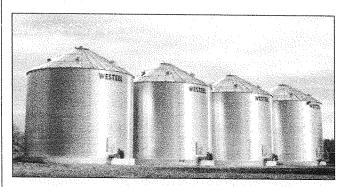


3. Promold advertises that "Each trough is $8' \times 4' \times 2'$ high and holds 400 imp. gal." How close the advertisement is to the truth. (Image from: http://www.promoldmarketing.com/Pasture.html)



3. On farms, grain is stored in grain bins. An example is shown below.

Model 2105 Wide Corr Farm Bin



- Is the weight conversion correct? 1.
- 2. Is the diameter conversion correct?
- How much steel was used to manufacture each grain bin?
- What is the approximate mass of 1 m^2 of 4. steel?
- 5. The capacity is given in tonnes and in bushels.
 - a) How many bushels make one tonne?
 - Calculate the capacity (volume) in b) cubic metres?

Product Specifications

Diameter:

6.37 Meters

Capacity:

160 Tonnes

Eave Height: Overall Height:

5.64 Meters

7.4 Meters

Approximate Weight: 1958.1 Kilograms

Product Specifications

Diameter:

20 Feet, 10 1/2 Inches

Capacity:

5900 Bushels

Eave Height:

18 Feet, 6 Inches

Overall Height:

24 Feet, 3 1/2 Inches

Approximate Weight: 4300 Lbs

Geometry and Patios Assignment

1.	A precast concrete patio stone is 2 feet square and 2 inches deep. What was the volume of concrete used to make it? Hint: to make the next question easier, convert all measurements to yards before calculating the volume.
2.	A proper base for a patio consists of a depth of 6 inches of gravel and stone dust. If you are ordering 80 patio stones to make a patio and walkway, about how many cubic yards (usually called "yards" when you order it) of gravel should you order?
3.	When reading about building a patio you read that it may be unsafe to work at temperature over $85^{\circ}F$. If your thermometer reads $32^{\circ}C$ should you be working?
4.	You are so proud of your patio that you decide to build a round fire pit. You dig a hole 2.4 m across and 8 inches deep to provide a safe base. How many cubic feet of soil did you excavate? If your wheelbarrow has a capacity of 6 cubic feet, how many loads of dirt did you cart away?
5.	Your outdoor living space is now so beautiful you are tempted to add a hot tub. Do some research to learn more about hot tubs. Be sure to consider different sizes and shapes, the volume of water that each will hold, their dimensions in metric and imperial units, and any interesting features.
6.	What else would you like to add? a pool? a trampoline? a garden shed? a sauna? Prepare a scale diagram of your fully tricked out backyard patio entertainment area.

Geometry Practice Test

1.	Andrew received a race car in a box that has a square base with side length 8 inches, and with a side length 5 inches. The box must be 6 inches high.	a square top
	a.) Draw an isometric diagram and label side lengths (3 dimensional diagram)	[2K]
	b.) Draw an orthographic diagram and label (Top Left Side Right Side)	[2K]
	c.) Draw a net that can be used to make a model of the box and the lid. Label all sides.	[2K]
	d.) Find total surface area of the box.	[2A]
		. ,
2.	Dana is designing a hexagonal (6 sided polygon) cottage containing six equal triangular	rooms that
	meet at the centre. The base of the cottage will be poured concrete 10 centimetre thick. T	The walls will
	be equal squares (its length is equal to its height) and the roof is flat. a. Draw an isometric diagram (3 dimensional sketch) of the cottage. [2 K]]

U05A16-P2

b.	Draw the net for this cottage, and label all sides. [2]	[]
c.	Find the area of the base of the cottage if the equal sides of the hexagon measure [4]	
d.	Find the volume of the concrete needed to pour the base of the cottage. [2]	A]
÷		
e.	the office of	ase is a
f.	Find the total surface area of the outer walls this cottage needs. [3.4]	A]

U05A16-P3

g. Find the total cost for the wood the side walls and the roof will need. The type of wood used for the walls and the roof costs $$20/m^2$. Note that the roof's dimensions match that of the concrete base. [2 A 3 C]

- 3. A new energy drink is being designed such that the ratio of the height to the diameter is equal to the golden ratio. The diameter will be 8 centimetres.
 - **Note: Golden Ratio** = the ratio of two lengths is approximately 1.618 to 1 This means that if the width were to be 1 cm than the length would be 1.618 cm.
 - a. Draw a net that could be used to make a scale model of the can. Label diagram. [2 K]

b. Determine the volume of the energy drink that the can will hold.

[3A]

MBF 3C Unit 6 - Statistics

Section	Date	Lesson Title	Handouts	Homework Assigned
1		One-variable data Book Computer Lab	U06A01	
2		Sampling Types and Techniques	U06A02	
3		Identify and Graphing One-Variable Data	U06A03	U06A04
4		Common Distribution Properties and Questionnaire Design	U06A05	
5		Collecting and Organizing One-Variable Data	Supplemental Worksheet (not provided)	
6		Measures of Central Tendency	U06A06	
7		Measures of Spread	U06A07	U06A08
8	-	Analyzing One-Variable Data	U06A09	U06A10
9		Review Day	U06A11	
10		Practice Test	U06A12	
11		Unit Test		

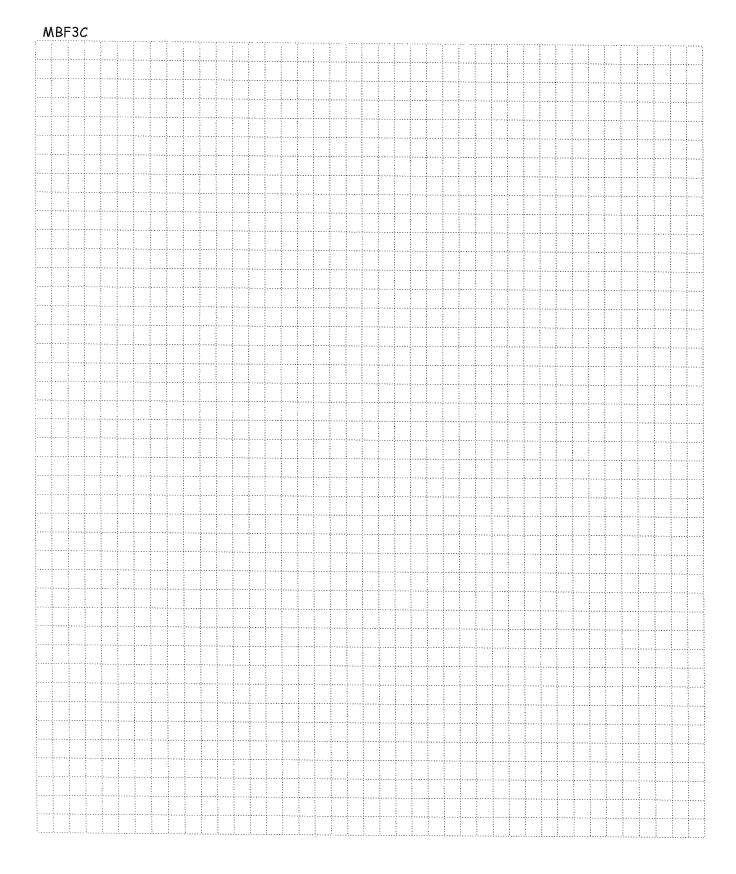
Statistics Canada-Precipitation Data

Canadian Statistics Precipitation Temperatu res Tables by... · subject province or territory · metropol itan area Alphabetic al list What's new Definition symbols oplicable <u>Veterrological</u> nternational data, limate Normais 061-1960 ast modified. 605-02-16

Search Canadian Statistics Key Word(s) Search

Related tables: Weather conditions.

	Annuc	al avera	ge		**************************************	
	Snowfall (cm)	Total precipit (mm)	ation	Wet	days
St. John's	3	322.1		1,482		21
Charlottetown	3	38.7		1,201		17
Halifax	á	261.4		1,474		17
Fredericton	2	94.5		1,131		15
Québec	3	37.0		1,208		17
Montréal	á	214.2		940		16
Ottawa	á	221.5		911		15
Toronto	1	35.0		819		13
Winnipeg	:	114.8		504		11
Regina	1	107.4		364		10
Edmonton	1	129.6		461		12
Calgary	1	135.4		399		11
Vancouver		54.9		1,167		16
Victoria		46.9		858		15
Whitehorse	1	145.2		269		12
Yellowknife	1	143.9		267		11
International comparisons						
Beijing, China	30	ć	523			66
Cairo, Egypt		•••			22	
Capetown, South Africa		(552			95
London, England					594	10
Los Angeles, U.S.A.		•••			373	3
Mexico City, Mexico					726	13
Moscow, Russia		161			575	18
New Delhi, India		•••			715	4
Paris, France		•••			585	16
Rio de Janeiro, Brazil					1,093	13
Rome, Italy					749	7
Sydney, Australia					1,205	15
Tokyo, Japan		20			1,563	10
Washington, D.C.		42			991	11



Complete the following exercises.

- 1. Create a frequency distribution and histogram for each of the following using the data from Stats Canada:
 - a) Annual average precipitation (mm) in Canada
 - b) Annual average precipitation (mm) in international cities
 - c) Number of wet days in international cities
- 2. Go to the World Cup of Soccer website

(www.world-cup-info.com/statistics/world_cup_games_played.htm)

- a) enter the data into Fathom
- b) create a graph of the number of points scored per country by dragging the graph icon into your document and dragging the needed columns from your case table.

Or

- a) Go to the Toronto Maple Leafs website (<u>www.mapleleafs.com/team/playerstats.html</u>) and enter the data into Fathom.
- b) Create a graph with the x-attribute representing the number of games played (GM) and the y-attribute representing the points per game (PPG)

Write a concluding statement based on your graph.

Questions 3 to 7 are based on the following information.

The pulses of 30 people were taken for 1 minute and recorded. These are the results:

66	79	53	81	84	76	76	67	64	83	92	56	67
				86								
		72										

- 3. Why is it hard to spot the trends in the data as it appears?
- 4. a) Make a frequency distribution table for the above data including a cumulative frequency column. Start with 50.5-55.5 as your first interval.
 - b) Construct a histogram based on your frequency distribution.
- 5. Use your graph to answer each question:
 - a) In which interval does the most common pulse occur?
 - b) In which interval does the least common pulse occur?
- 6. What percentage of the people have a pulse over 85.5?
- 7. a) If you record the pulse for 300 people, how many would you expect to have a pulse in the interval 75.5-80.5? Give reasons for your answer.
 - b) What assumptions are you making?

Questions 8 to 12 are based on the following information.

An English class had the following grades on a test (out of 100).

26	63	73	82	32	73	35	63	56	87	40	51	55
	53											
76	56	67										

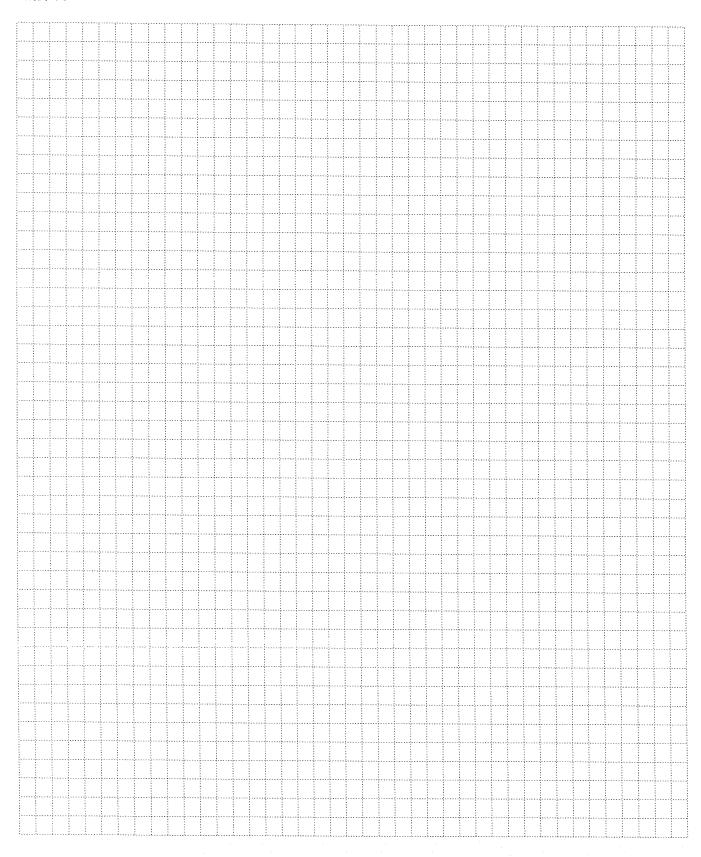
- 8. Start with the interval 20.5-30.5. Create a frequency distribution.
- 9. a) Create a histogram.
 - b) Which interval has the greatest frequency?
- 10. a) What percentage of the class received an A (80% or better)?
 - b) What percentage of the class failed (under 50%)?
- 11. The same class wrote a second test. These are their marks.

66	62	14	41	45	89	59	43	67	37
31	65	50	43	53	57	54	84	68	74
61	54	34	70	45	64	76	70	65	

Repeat questions 8 and 9 for the set of marks above.

- 12. Compare the two histograms created.
 - a) What differences are there? b) What similarities are there? c) What information do the differences indicate to the teacher?

MBF3C



U06A02

Sampling

- 1. In order to find out which songs are the most popular downloads, a survey was sent out to a number of teenagers.
 - a) What are some advantages of using a survey to collect data?
 - b) What are some disadvantages to this method?
 - c) What would be another way to get this same information?
- 2. Sometimes it is better to ask all of the population before making a decision. For each scenario, state whether a sample should be used or a census.
 - a) Testing the quality of the air in airplanes.
 - b) Determining the popularity of a particular website.
 - c) Determining the number of potential buyers of a new MP3 player.
 - d) Determining the chemical composition of a good barbeque sauce.
 - e) Checking the air pressure of the tires on a car.
 - f) Determining the effectiveness of a new laser-eye surgery.
- 3. Given the following four options, which would be most effective in predicting the outcome of the upcoming municipal election for mayor, and why?
 - a) 100 completed surveys that were handed out randomly through the city.
 - b) 100 phone calls made to different parts of the city.
 - c) 100 people interviewed at a local neighbourhood-watch party.
 - d) 100 surveys completed by children at a local middle school.
- 4. A school board received a load of 10 000 graphing calculators to pass out to their high schools. They were concerned with the state of the delivery and therefore with the number of defective calculators. They decided to check them out.

First, 20 calculators were checked and all worked perfectly.

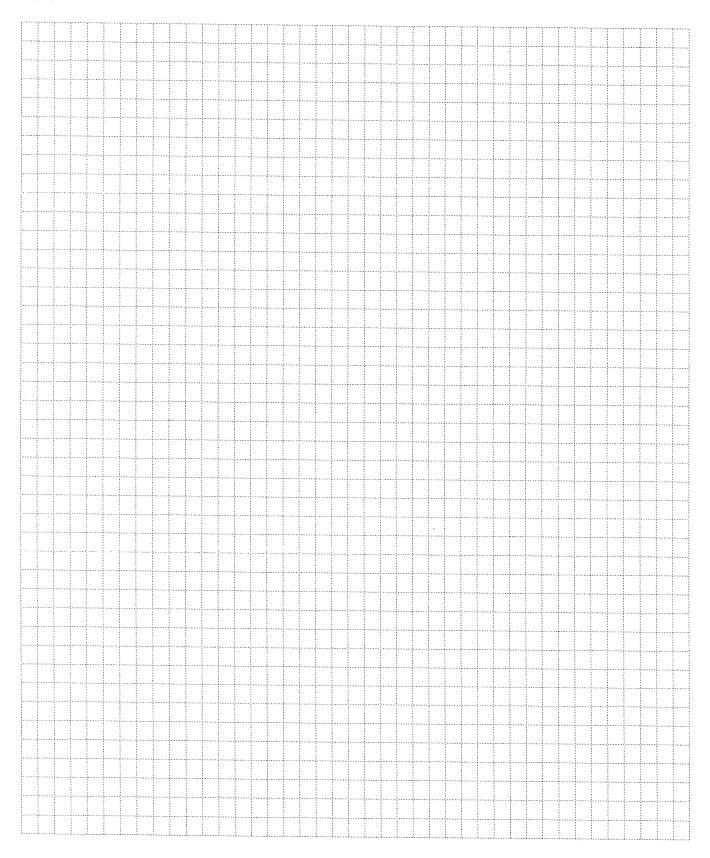
Second, 100 calculators were tested and 2 were broken.

Third, 1000 were tested and 15 were broken.

- a) After the first test, would it be fair to say that none of the calculators were broken? Why or why not?
- b) Whose statement is likely more accurate? Sami: 2% are defective Sima: 1.5% are defective
- c) In the shipment of 10 000, how many would you estimate to be defective? Explain.
- 5. Gelman's Rent-All want to see if they should open up a second shop at a neighbouring plaza. They conduct a poll by leaving sheets at the entrance of the plaza and asking people to fill them in.
 - a) What type of sample is this?
 - b) What are some of the pros of this method?
- 6. A local high school has 600 students in grade 9, 400 in grade 10, 300 in grade 11 and 200 in grade 12. A sample of 100 students is used to choose which brand of chocolate bar should be sold in the vending machine. How many of the 100 surveys should be handed out to
 - a) Grade 10s?
- b) Grade 11s?
- c) What type of sample is this?

- 7. For each scenario, state whether a stratified sample should be used. Explain your reasoning.
 - a) Canada wants to hold a general referendum to decide a major political issue. A sample of 10 000 people is chosen to predict the outcome.
 - b) A shipment of 35 000 clear plastic rulers is to be checked for defects.
 - c) There are 250 women and 750 men working at Harpo studios. A sample of 20 is taken to determine what type of end-of year party should be planned.
 - d) The director of a local community centre is supposed to decide if any of her budget should be spent on pool maintenance.
 - e) At a Tai Chi club, an opinion poll is to be conducted on the quality of the equipment.
- 8. For each of the following samples, the cluster technique was used. Which would result in a fair sample (F) and which would result in a poor sample (P)?
 - a) Asking ER-nurses about the value of a new triage approach.
 - b) Going to a high school to determine the most popular brand of jeans.
 - c) Asking only senior students about the prom location.
 - d) Asking Smart-Car owners about a hot environmental issue.

MBF3C



Identifying and Graphing One-Variable Data

Data can be recorded in several different ways; there are three types that we will be looking at.

- i) Categorical Data (Qualitative)
 - Usually recorded as a label and not a number
 - Ex. Checking male/female on a survey, listing the type of car you drive, eye colour
 - Sometimes categorical data is recorded as a number, but the value of the number is not meaningful (it is more important that we understand what it represents)
 - Ex. You might record a number 1 to mean "Yes" and a 0 to mean "No"
 - Ex. Sometimes surveys ask you to say if you "strongly agree" which is recorded as a 1 and a range is used up to "strongly disagree" which is recorded as a 5.
- ii) Continuous Data (Quantitative)
 - Any number including decimals and fractions are allowed
 - Ex. All measurements such as mass, volume, length, time and temperature
 - Ex. Recording the masses of Cadbury mini egg packages coming down a production line in units of grams.
- iii) Discrete Data (Quantitative)
 - Also a number but only whole #'s are allowed. No decimals or fractions.
 - Ex. # of pizza toppings, goals scored in the hockey game
- Eg. 1: For each of the following, state the type of data we are talking about;
- a) Number of mugs of coffee drank in a typical morning [discrete, b/c it would be a whole #]
- b) Type of pet at home (eg. Dog, cat, bird, rodent, reptile) [categorical, b/c it is recorded as a label]
- c) Number of pets at home. [discrete, b/c it would be a whole #, you can't have 2.3 fish]
- d) Amount of coffee Mr. Rudolf drinks in mL in a typical morning at school. [continuous, b/c it varies]

 Different types of Graphs can be used to display our data.
- i) Histogram
 - > Used to represent continuous data b/c bars touch each other.
 - Intervals are often used instead of individual items
- ii) Bar Graph
 - Same as a histograph but there are spaces between the bars.
 - Used for discrete data
 - Ex. # of males vs. # of females
 - Ex. # of Smarties of each colour in a box

Eg. 2: Create a bar graph of the following hockey all-time regular season goal scorers.

Name	Goals Scored
Wayne Gretzky	894
Gordie Howe	801
Brett Hull	741
Marcel Dionne	731
Phil Esposito	717
Mike Gartner	708
Mark Messier	694
Steve Yzerman	692
Mario Lemieux	690
Luc Robitaille	668

iii) Circle Graph

- > Used for numerical data when examining data as a portion of the whole.
- > Percents

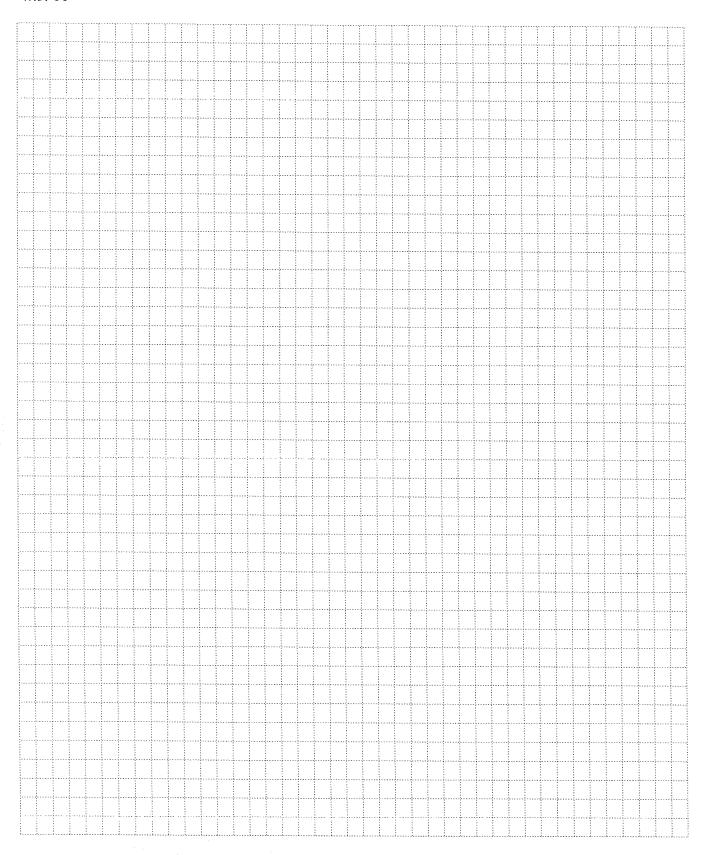
Eg. 3: Given the following monthly budget, create a circle graph.

	Cost	
Item	(\$)	Degrees in Circle
		(900/2100) x 360 =
Rent	900	154.3
		(400/2100) x 360 =
Transportation	400	68.6
	,	(500/2100) x 360 =
Food	500	85.7
		(100/2100) x 360 =
Clothing	100	17.1
		(200/2100) x 360 =
Entertainment	200	34.3
Total	2100	360

iv) Pictographs

- Good way to visualize data, not as precise as other methods.
- Commonly used with discrete data
- Difficult to represent part numbers

MBF3C



U06A04

Graphing

- 1. The size of your school is stated in several different ways. For each measurement, state if it is discrete, continuous or categorical.
 - a) The height of the building b) The number of rooms

c) The number of floors

- d) The sum of the areas on each floor in m²
- 2. Identify each variable as discrete, continuous or categorical.
 - a) favourite TV show
- b) paint colour in bedroom
- c) English grade

- d) volume of IPod
- e) age

f) calories in a meal

g) number of cancer deaths last year

- h) monthly unemployment rate
- 3. The following is a list of ways to state the size of a book. Which variable can be continuous?
 - a) thickness of binding used

b) number of words

c) length of pages

- d) number of pages
- 4. An emergency room technician assesses each patient that comes in. She records the following for each: blood pressure, age, gender, number of previous ER visits in the current year.
 - a) How many of these variables are likely measured as continuous variables?
 - b) How many are discrete?
- 5. Paul is determining whether or not his privately owned gas station will make it in his town. He asks automobile owners to name the station where they last bought gas. Is the variable that he is measuring discrete, continuous or categorical?
- 6. a) Construct a circle graph to illustrate the following data. First, you may want to complete the table.

b) Construct a bar graph with the same data.

Percentage of Canadian Travellers Passing Through
CanFly Airport per Day by Age

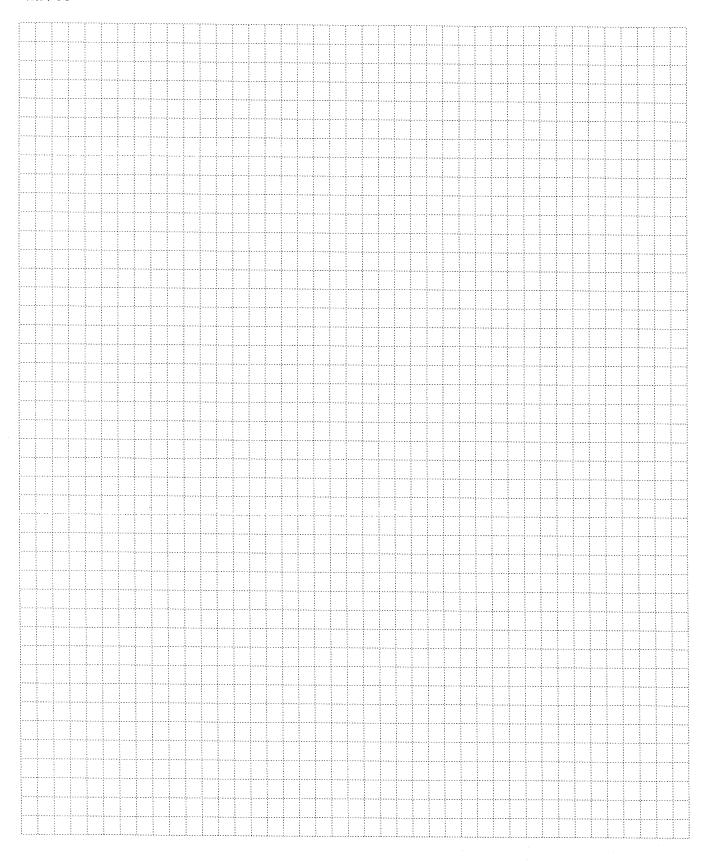
Age (Years)	Percent	Degrees in Circle
0-12	8	
13-18	12	
18-25	17	
26-40	32	
41-65	24	
65+	7	
Total		

7. Construct a pictograph to represent the following data.

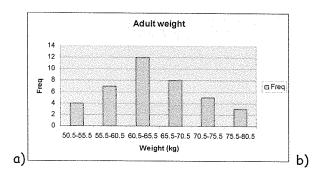
Percentage of People Who Believe in Each Creature

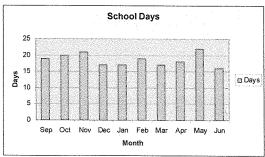
Creature	People (%)
Loch Ness	32
Monster	
Big Foot	44
Одородо	25
Aliens	78

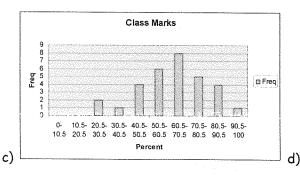
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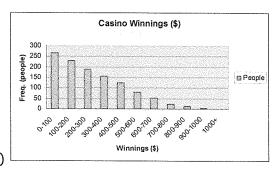


1. Label each graph as normal, bimodal, uniform, left-skewed or right-skewed.









- 2. Create a 5 question survey on one of the following topics:
 - a) professional athlete earnings
 - b) top movie songs
 - c) favourite fair ride
 - d) favourite summertime activity
- 3. The left-hand lane of the 401 express corridor is being restricted to vehicles with at least two passengers in order to reduce traffic congestion. Outline your strategy for collecting data to determine if the traffic congestion is really reduced.
 - i) Include at least 3 steps in your process.
 - ii) Identify the time(s) that you would conduct this experiment.
 - iii) Indicate how you would record your data.
 - iv) Indicate how you would use your data to answer the question.

MBF3C U06A05-P2

Example of survey on immigration:

1.	From	which	continent	did	you	originate?
----	------	-------	-----------	-----	-----	------------

South America

Europe

Asia

Africa

other

2. What do you believe is the main reason for which immigrants leave their place of origin?

war

poverty

lack of employment

abuse

other

in specific field of training

3. What do you believe is the main reason for which immigrants come to Canada?

family

climate

employment

democratic

other

opportunities government

4. Which province/territory do you think would provide the greatest opportunity for an immigrant?

British Columbia

Alberta

Ontario

Quebec

other

5. In what way do you think that immigration benefits the general Canadian population?

 $\hfill \mathbf{s}$ the absence of

able to learn

able to try

increases

other

trained professionals

new language new foods

cultural tolerance

Please note:

Students will collect and analyze data for each question in survey. Emphasize to students that their survey must be able to produce some form of data that may be analyzed with a graph, etc. Selections within the survey may be perceived biased, but "other" is provided to allow for all viewpoints to be indirectly addressed

Statistic Charts

a) Golf Player Earnings

	Golf Player Earnings				
Rank	Player	Events	Money		
1	Tiger Woods	10	\$4,263,563.00		
2	Jim Furyk	17	\$4,174,516.00		
3	Phil Mickelson	16	\$4,123,005.00		
4	Geoff Ogilvy	16	\$4,003,049.00		
5	Vijay Singh	17	\$3,328,970.00		
6	Trevor Immelman	17	\$3,030,746.00		
7	Stuart Appleby	16	\$2,903,211.00		
8	Adam Scott	13	\$2,712,183.00		
9	Chad Campbell	18	\$2,424,507.00		
10	Rory Sabbatini	17	\$2,411,584.00		
11	David Toms	15	\$2,400,544.00		
12	Carl Pettersson	20	\$2,372,482.00		
13	Stephen Ames	16	\$2,227,035.00		
14	Luke Donald	14	\$2,188,642.00		
15	Retief Goosen	12	\$2,117,378.00		
16	Brett Wetterich	16	\$2,117,006.00		
17	Rod Pampling	17	\$2,092,767.00		
18	Zach Johnson	19	\$2,064,268.00		
19	Jose Maria Olazabal	14	\$1,953,102.00		

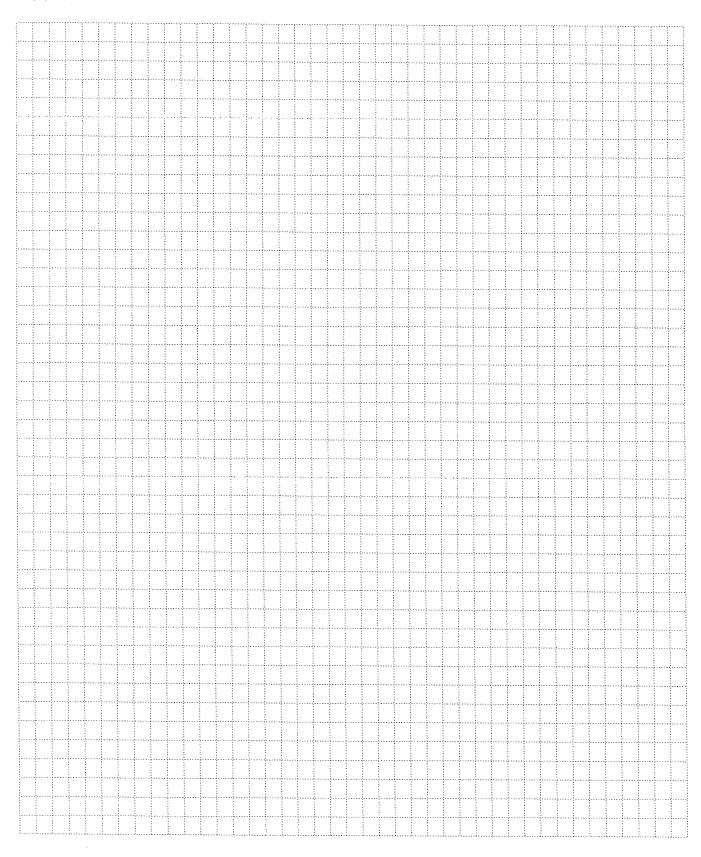
Source: http://sportsillustrated.cnn.com/golf/pga/2006/stats/moneyleaders/

Nascar Nextel Cup Driver Earnings

Rank	Player	Money Won
1	Jimmie Johnson	\$5,959,217
2	Matt Kenseth	3,922,738
3	Jeff Burton	2,859,067
4	Kyle Busch	2,873,403
5	Kevin Harvick	3,396,052
6	Mark Martin	2,500,953
7	Kasey Kahne	3,717,698
8	Denny Hamlin	2,654,297
9	Jeff Gordon	3,599,247
10	Tony Stewart	4,047,555
11	Dale Earnhardt Jr.	3,124,598
12	Greg Biffle	2,707,056
13	Kurt Busch	2,942,046
14	Carl Edwards	2,673,001
15	Casey Mears	3,392,785

 $Source: \underline{http://sports.si.cnn.com/default.asp?c=cnnsi\&page=nascar/stat/nascar-driver-stats.htm}$

b) AFI top movie songs Source: http://www.afi.com/tvevents/100years/songs.aspx 1 Over the Rainbow, PERFORMER Judy Garland WIZARD OF OZ, THE 1939 As Time Goes By, PERFORMER Dooley Wilson CASABLANCA 1942 Singin' in the Rain, PERFORMER Gene Kelly SINGIN' IN THE RAIN 1952 4 Moon River, PERFORMER Audrey Hepburn BREAKFAST AT TIFFANY'S 1961 5 White Christmas, PERFORMER Bing Crosby HOLIDAY INN 1942 6 Mrs. Robinson, PERFORMERS Paul Simon, Art Garfunkel GRADUATE, THE 1967 7 When You Wish Upon A Star, PERFORMER Cliff Edwards PINOCCHIO 1940 8 Way We Were, The, PERFORMER Barbra Streisand THE WAY WE WERE 1973 9 Stayin' Alive, PERFORMER The Bee Gees SATURDAY NIGHT FEVER 1977 10 | Sound of Music, The, PERFORMER Julie Andrews SOUND OF MUSIC, THE 1965 11 Man That Got Away, The, PERFORMER Judy Garland STAR IS BORN, A 1954 12 Diamonds Are a Girl's Best Friend, PERFORMER Marilyn Monroe GENTLEMEN PREFER BLONDES 1953 13 People, PERFORMER Barbra Streisand FUNNY GIRL 1968 14 My Heart Will Go On, PERFORMER Céline Dion TITANIC 1997 15 Cheek to Cheek, PERFORMERS Fred Astaire, Ginger Rogers TOP HAT 1935 16 Evergreen (Love Theme from A Star is Born), Barbra Streisand STAR IS BORN, A 1976 17 I Could Have Danced All Night, Audrey Hepburn (voiced by Marni Nixon) MY FAIR LADY 1964 18 Cabaret, PERFORMER Liza Minnelli CABARET 1972 Some Day My Prince Will Come, PERFORMER Adriana Caselotti SNOW WHITE AND THE 7 DWARFS 1937 Somewhere, Natalie Wood (voiced by Marni Nixon), Richard Beymer (voiced by WEST SIDE STORY 20 1961 Jimmy Bryant) Jailhouse Rock, PERFORMER Elvis Presley 21 JAILHOUSE ROCK 1957 Everybody's Talkin', PERFORMER Harry Nilsson MIDNIGHT COWBOY 1969 Raindrops Keep Fallin' on My Head BUTCH CASSIDY AND THE 23 1969 PERFORMER B. J. Thomas SUNDANCE KID 24 Ol' Man River, PERFORMER Paul Robeson SHOW BOAT 1936 High Noon (Do Not Forsake Me, Oh My Darlin) HIGH NOON 25 1952 PERFORMER Tex Ritter 26 Trolley Song, The, PERFORMER Judy Garland MEET ME IN ST. LOUIS 1944 Unchained Melody, PERFORMER The Righteous Brothers 27 **GHOST** 1990 Some Enchanted Evening, Rossano Brazzi (voiced by Giorgio Tozzi) 28 SOUTH PACIFIC 1958 Born To Be Wild, PERFORMER Steppenwolf EASY RIDER 1969 30 Stormy Weather, PERFORMER Lena Horne STORMY WEATHER 1943 31 Theme from New York, New York, PERFORMER Liza Minnelli NEW YORK, NEW YORK 1977 32 I Got Rhythm, PERFORMER Gene Kelly AMERICAN IN PARIS, AN 1951 Aquarius, PERFORMERS Ren Woods, Ensemble HAIR 1979 Let's Call the Whole Thing Off, Fred Astaire, Ginger Rogers 34 SHALL WE DANCE 1937 35 America, PERFORMERS Rita Moreno, George Chakiris, Ensemble WEST SIDE STORY 1961 Supercalifragilisticexpialidocious, Julie Andrews, Dick Van Dyke, Ensemble 36 MARY POPPINS 1964 37 Swinging on a Star, PERFORMER Bing Crosby GOING MY WAY 1944 Theme from Shaft, PERFORMERS Isaac Hayes, Chorus 38 1971 Days of Wine and Roses, PERFORMER Chorus 39 DAYS OF WINE AND ROSES 1963 Fight the Power, PERFORMER Public Enemy DO THE RIGHT THING 1989 41 New York, New York, Gene Kelly, Frank Sinatra, Jules Munshin ON THE TOWN 1949 Luck Be A Lady, Marlon Brando, Ensemble **GUYS AND DOLLS** 1955 Way You Look Tonight, The, Fred Astaire SWING TIME 1936 44 Wind Beneath My Wings, Bette Midler **BEACHES** 1988 45 | That's Entertainment, Fred Astaire, Nanette Fabray, Jack Buchanan, Oscar Levant BAND WAGON, THE 1953 46 Don't Rain On My Parade, Barbra Streisand FUNNY GIRL 1968 47 Zip-a-Dee-Doo-Dah, James Baskett SONG OF THE SOUTH 1947 48 Whatever Will Be, Will Be (Que Sera, Sera), Doris Day MAN WHO KNEW TOO MUCH, THE 1956 Make 'Em Laugh, Donald O'Connor SINGIN' IN THE RAIN 1952 50 Rock Around the Clock, Bill Haley and the Comets BLACKBOARD JUNGLE 1955

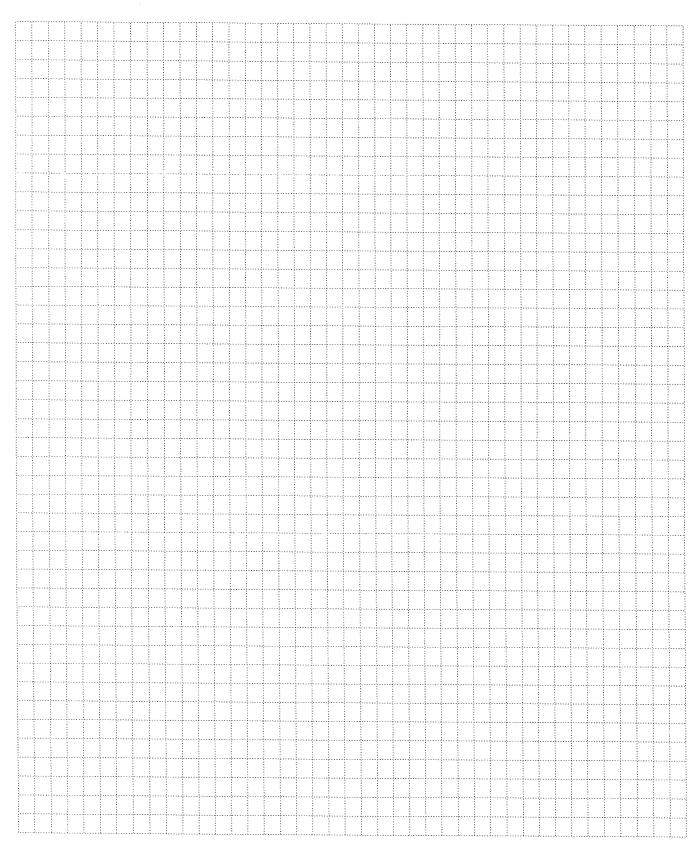


Measures of Central Tendency

- 1. Find the mean, median and mode for each set of data.
 - a) 64, 69, 72, 54, 89, 92, 54, 32
- b) 12, 0, 8, 4, 6, 3, 7, 3, 2, 9, 5, 6, 7, 7, 8
- c) 4.2, 11.5, 6.8, 5.2, 5.4, 6.3, 12.1, 11.5, 11.9, 7.8, 13.1, 5.8, 6.2
- d) 0, 2.1, 5.7, 8.3, 2.6, 7.3, 8.4, 0.5, 0.4, 2.1, 2.2, 4.3, 5.7 e) $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{3}$, $\frac{7}{2}$, $\frac{1}{6}$ (answer in fraction form)
- 2. Gabriel buys 8 DVDs at Discount Dan's DVD shop. Three cost \$10.50, 2 cost \$7.75, 1 cost \$5.25 and 2 cost \$3.50. Find the mean, median and mode of the costs of his DVDs.
- 3. The prizes in the local lottery were worth the following: 2 prizes of \$1 000 000; 7 prizes of \$350 000, 10 prizes of \$250. Find the mean, median and mode.
- 4. The masses, in kilograms, of group of Jessy Bragg's weight loss group are shown. 81, 79, 83, 76, 89, 75, 67, 83, 65, 74, 78
 - a) Find the mean, median and mode.
- b) Is the median greater than or less than the mean?
- c) Is the mode greater than or less than the mean?
- 5. The hourly rates of employees of a supermarket are given. \$9.25, \$8.50, \$22.50, \$7.85, \$8.85, \$12.65, \$10.85, \$11.50
 - a) Find the mean, median and mode.
- b) Which of your answers best represents the data? Why?
- c) Which of your answers would most misrepresent the data? Why?
- 6. a) Find the mean and median for each set of marks.
 - Suzy: 25, 36, 39, 87, 89, 94
- Ruiz: 45, 56, 88, 89, 92, 98
- b) What is the best measure of central tendency for Suzy, the mean or the median?
- c) What is the best measure of central tendency for Ruiz, the mean or the median?
- 7. State and explain whether each statement is based on the mean, median or mode.
 - a) 0.2% of light bulbs are defective.
 - b) The most popular search engine is Google.
 - c) The average university grad earns \$35 000 annually upon graduation.
 - d) Most drinking and driving accidents occur on long weekends.
- 8. You earned the following marks (each out of 50) on your first five test: 28, 36, 38, 41, 44. What mark would you have to get on the next test in order to bring your test average up to 80%?

Measures of Central Tendency Solutions

- 1. a) mean = 65.8; median = 66.5; mode = 54 b) mean = 5.8; median = 6; mode = 7 c) mean = 8.3; median = 6.8; mode = 11.5 d) mean = 3.8; median = 2.6; modes = 2.1 and 5.7 e) mean = 95/12; median = 5/6; mode = 1/6
- 2. mean = \$6.58; median = \$7.75; mode = \$10.50 3. mean = \$234 342.10; median = \$250; mode = \$250
- 4. a) mean = 77.3; median = 78; mode = 83 b) greater than c) greater than 5. a) mean = \$11.49; median = \$10.05; mode = n/a b) median; wide range of wages c) mean; data is not close together
- 6. a) Suzy: mean = 61.7; median = 63; Ruiz: mean = 78; median = 88.5
- b) median c) median 7. a) mean b) mode c) median (grads make a wide range of salaries depending on their degree/subject area) d) mode 8. a) 106% or 53 out of 50.



Standard Deviation

- 1. True or False? The standard deviation cannot be negative.
- 2. Calculate the range and standard deviation of each.
 - a) 4, 8, 6, 3, 12, 9, 7, 6
- b) 35, 38, 40, 43, 46, 23, 38
- c) 2.4, 4.3, 6.5, 1.1, 8.9, 3.6, 7.2, 9.6 d) 4.55, 3.23, 6.78, 3.54, 5.54, 6.78
- 3. The machine packaging cookies has been considered defective. The packages are labelled as containing 150g. A sample of 15 packages was selected and the masses are given.

145, 151, 152, 150, 147, 152, 149, 148, 153, 150, 146, 152, 148, 149, 151

- a) Calculate the mean.
- b) If any packages are more than 2.2g from the mean, the package is not sold. How many are defective?
- c) Should the machine be fixed?
- 4. A group of student landscapers are to keep track of their own weekly hours. They are listed below. 44, 52, 43, 39, 42, 41, 38, 43, 46, 45, 44, 39, 40, 42, 45
 - a) Find the range. Is this a useful tool for representing this data?
 - b) Find the mean.
 - c) Find the standard deviation.
 - d) What can be said about the entry of 52 hours/week?
 - e) Calculate the standard deviation again without the 52 hours/week entry.
- 5. The sale prices of the last 10 homes sold in 1985 were: \$198 000, \$185 000, \$205 200, \$225 300, \$206 700, \$201 850, \$200 000, \$189 000, \$192 100, \$200 400.
 - a) What is the average sale price?
 - b) What is the standard deviation?
 - c) Do you think that a price of \$240 000 would be considered unusual? Why or why not?
- 6. The sales price of the last 10 homes sold in 2005 were: \$345 500, \$467 800, \$289 000, \$675 000, \$398 500, \$243 000, \$899 950, \$453 000, \$239 000, \$256 000.
 - a) What is the average sales price?
 - b) What is the standard deviation?
 - c) Which year was more consistent? How do you know?

Solutions

- 1. true 2. a) range = 9; s.d. = 2.85 b) range = 23; s.d. = 7.37 c) range = 8.5; s.d. = 3.08 d) range = 3.55; s.d. = 1.55 3. a) 149.5 b) 7 4. a) 14; yes, to ensure that students are being honest when recording the number of hours worked b) 42.9 c) 3.50 d) answers may vary; for example: over-inflated e) 2.52 5. a) \$200 355 b) 11 189.04 c) yes, based on the standard deviation, it would be an extremely high value 6. a) \$426 675
- b) 214 078.1 c) 1985; smaller range of values

Measures of Central Tendency & Spread Homework

- 1. Calculate the mean, median, mode, range, variance, and standard deviation
- a) 14, 19, 22, 22, 23, 23, 24, 24, 24

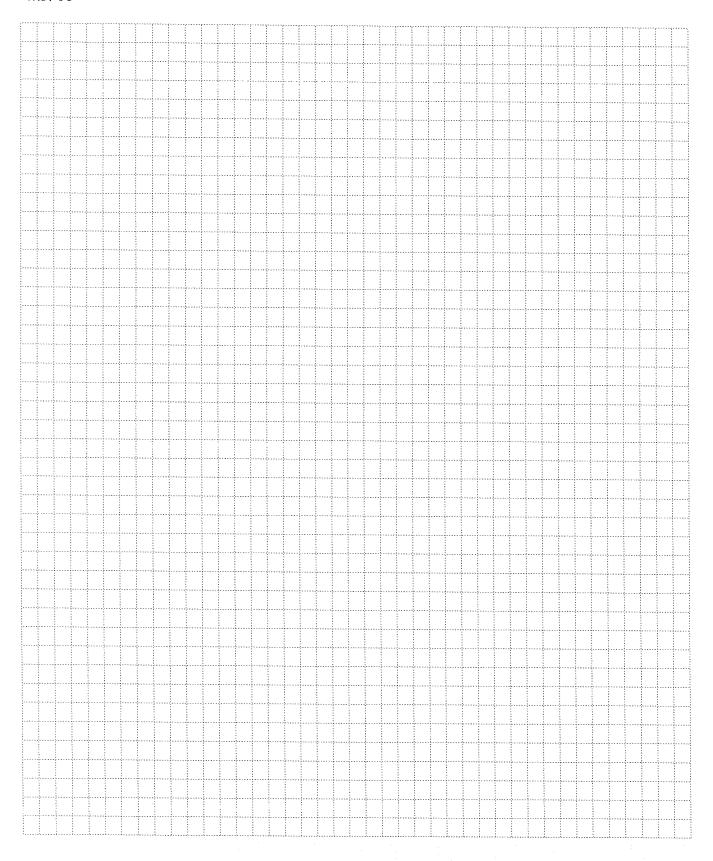
b) 17, 19, 19, 27, 27, 50

1. a)

Value	Frequency
27	3
28	5
29	9
30	6
31	1

b)

Value	Frequency
55	7
60	9
65	10
70	15
75	22
80	12
85	5



Investigation

All questions can be done by hand or with the use of technology. Compare any two of the following sets of data. Be specific when giving conclusions. Sources: MLS and Remax

Canadian Housing Prices by City (\$)

Canadian City	June 2006	June 2005	2002
Vancouver	508 435	422 843	301 473
Victoria	538 913	469 588	242 503
Calgary	367 033	245 803	198 350
Edmonton	254 240	199 409	150 165
Regina	137 022	132 054	100 751
Saskatoon	160 548	139 728	118 999
Ottawa	260 458	254 725	200 711
Toronto	358 035	345 065	275 975
Montreal	222 879	210 740	143 589
Fredericton	136 371	134 334	114 185
Saint John	127 586	125 455	104 052
Halifax	201 316	184 853	148 737

ı.	State the	two sets	of data that	you wish to	o compare t	iere:	

Find the measures of central tendency for each. State any conclusions found.	2.	Find the measures of	central tendence	y for each.	State any	conclusions found.	
--	----	----------------------	------------------	-------------	-----------	--------------------	--

Mean:

1st Set

2nd Set

Median:

1st Set

2nd Se

Mode:

1st Set

Conclusions:

2nd Set

3. Find the measures of spread for each. State any conclusions found.

Range: 1st Set

2nd Set

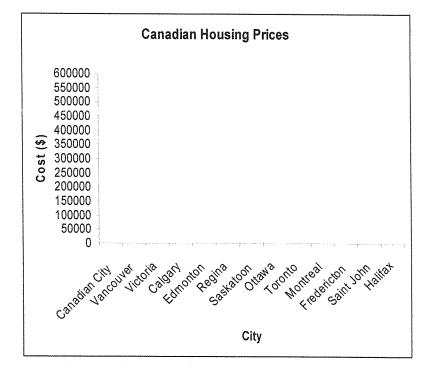
Standard Deviation:

1st Set____

2nd Set

Conclusions:

4. Graph your data on the graph to the right.



Statistics in the Media

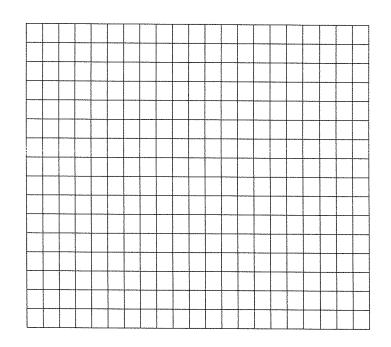
- 1. You've spent some time learning about statistics and how they are calculated and used in the media. If you were a statistician and wanted to relate to people some information about a topic of interest to you what steps would you have to follow in order to be able to report your findings?
- 2. Find two more examples of statistics in the media and generate another set of at least three questions based on the information you analyse from the statistics.
- 3. Generate at least two possible headlines for newspaper articles based on your statistic examples.
- 4. Find two examples of statistics in the media being used to mislead or misrepresent statistical information.

1. A French class had the following grades on a test.

66	62	14	41	45	89	59	43	67	37
31	65	50	43	53	57	54	84	68	74
61	54	34	70	45	64	76	70	65	

a. Make a frequency distribution table for the above data and then construct a histogram.

Grades	Frequency	Cumulative Frequency
10.5 - 20.5		
	-	
	<u> </u>	
101000000	-	



2. For each of the following state the type of data as either discrete, categorical or continuous.

a.	# of childr	en in a family.
----	-------------	-----------------

b. Favorite pizza toppings.

c. Average birth weight in Ottawa.

d. Rainfall in millimeters.

e. # of students enrolled in Math

3. Fill in the following blanks;

a. A ______ is bell shaped, and is symmetrical about the mean, median and the mode.

- b. A ______ is not symmetrical and the majority of the data is on the left side of the graph with a long tail to the right. The mean is being affected by high outliers.
- c. In a _____ there are two mounds at either ends of the graph representing two different cluster of data.
- d. A ______ is not symmetrical and the majority of the data is on the right side of the graph with a long tail to the left. The mean is being affected by low outliers.
- 4. Gelman's Rent'All wants to see if they should open up a second shop at a neighboring plaza. They conduct a poll by leaving sheets at the entrance of the plaza and asking people to fill them in.
 - a. What type of sample technique is this?
 - b. What are some of the pros of this method?
 - c. What are some cons for this method?
- 5. For the following data, calculate the three measures of central tendency (mean, median, mode), the three measures of spread (range, variance, standard deviation) and the Interquartile Range (Recall, IQR = Q3 Q1).

a)

64	69	72	54	89	92	54	362
32	54	59	64	69	72	89	92

b)

	,											
0	2.1	5.7	8.3	2.6	7.3	8.4	0.5	0.4	2.1	2.2	4.3	5.7

Statistics - Practice Test

1. These are the final grades from the MBF3C class last semester.

73	44	50	63	56	65	62	74	85	62
82	56	64	64	74	45	54	56	64	76
78	81	73	86	70	66	67	75	52	65

- a) Organize the data into the chart below by starting with the first interval at 39.5%-49.5%.
- b) Create a histogram to show the frequency distribution of the marks.
- c) What type of data distribution is this?
- 2. An load of recyclables was examined at the transfer station. It contained the following materials:

	Paper	Plastic	Metals	Unusable Waste
-	40 kg	12 kg	26 kg	32 kg

Display this information in a circle graph.

:	Mass	Percentage	Degrees
Paper			
Plastic			
Metals			
Unusable Waste			
Total			

3. The masses of six boys and six girls are given in kilograms:

		Gi	rls			
112 112		114	118	120	132	
			ys			
110	118	119	121	128	130	

- a) Find the range of each data set: Girls:
- b) Calculate the mean of each data set: Girls:_____
- c) Calculate the *median* of each data set: Girls: _____

Boys:

Boys:

- 4. Identify each variable below as discrete, continuous or categorical:
 - a) number of lockers in the school b) volume of pop in a bottle
 - c) favourite ice cream flavour
- d) number of people at a baseball game
- 5. A local high school has 325 students in grade 9, 350 in grade 10, 275 grade 11 and 250 in grade 12. A sample of 100 students is used to choose which brand of chocolate bar should be sold in the vending machine. How many of the 100 surveys should be handed out to:
 - a) Grade 9s?

b) Grade 12s?

c) What type of sample is this?

- 6. The golf scores of 25 professional golfers are compared to the scores of 25 students from a John McCrae gym class.
 - a) Which group would likely have the higher mean? Explain.
 - b) Which group would likely have the higher standard deviation? Explain.

U06A12-P2

7. What does the range represent? Explain in a few sentences, using an example to aid your explanation.

8. The top scorers on the Toronto Raptors are Chris Bosh and T.J. Ford. Their points scored over the last 6 games were recorded:

Bosh	22	28	18	25	23	26
Ford	12	14	10	13	12	16

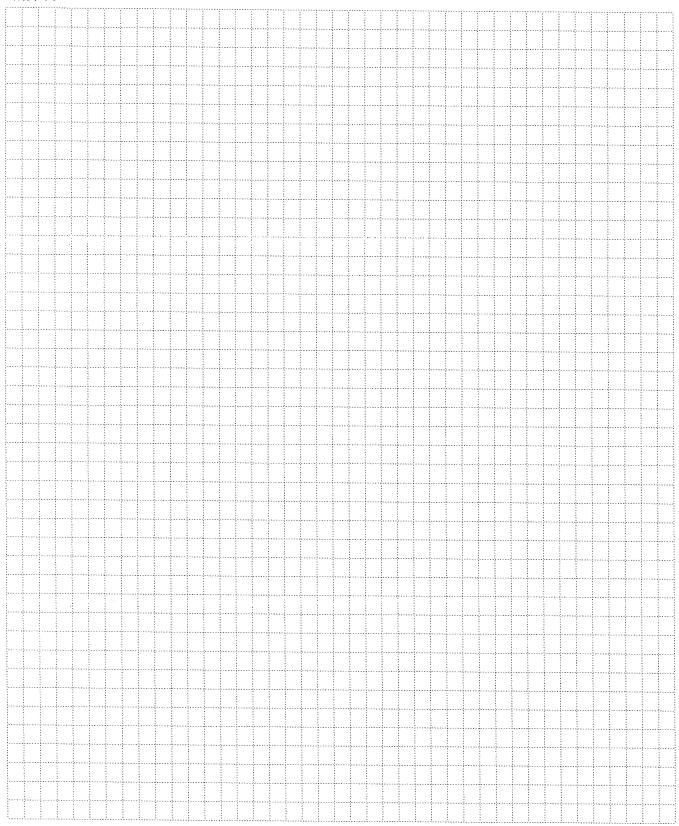
- a) Find each player's average score, rounded to the nearest whole number.
- b) Find each player's standard deviation.
- c) Using your answer from part (b), which player is more consistent? Justify your answer.
- 9. Matching exercise. The following is a list of the five sample types examined in class. Match them with the definitions below. Write the name of the sample type beside the appropriate definition.

Cluster	Stratified	Convenience	Voluntary		Random	
						ĺ

- in this sample, people are chosen from the population because they are readily available; e.g. friends, relatives, teammates
- in this sample, all people from a population chosen have an equal chance of being selected
- in this sample, people are divided into subgroups, then random members are chosen from each subdivided group
- in this sample, people choose themselves to participate in the sample; e.g. filling out an online questionnaire
- in this sample, every person in a randomly chosen group is surveyed
- 10. The computer system for a large hospital contains the records of 60,000 patients. The records are numbered sequentially from 1 to 60,000. The hospital administrator wishes to determine the patients' level of satisfaction with hospital food. For her sample, she selects the records numbered 100, 200, 300, and so on.
 - a) How many people will be surveyed?
 - b) What type of sampling was used?
 - c) Will this sample be representative of the population? Why or why not?
- 11. The school cafeteria is thinking of introducing a vegetarian pizza to the menu. Which sampling technique would you recommend to determine which of three possible toppings should be used? Explain.

MBF 3C Unit 8 - Probability

Section	Date	Lesson Title	Handouts	Homework Assigned
1		Introduction to Probability	U07A01	U07A02
2		Theoretical Probability	U07A03	U07A04
3		Theoretical Probability - Day 2	U07A05	Supplemental Worksheet (not provided
4		Compare Experimental & Theoretical Probability	U07A06	
5		Investigation using Technology – Comparing Experimental & Theoretical Probability	U07A07	
6		Interpreting Statistics from the Media	U07A08	
7		Review Day	Supplemental Worksheet (not provided)	
8		Practice Test	U07A09	
9		Unit Test		
7,170,100				



Experimental Probability

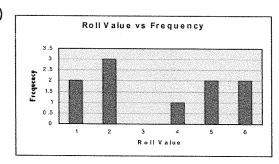
- 1. Perform an experiment to investigate the experimental probability of rolling a single die.
 - (a) Roll the die 10 times. Record the results of each roll
 - (b) Create a frequency distribution table and graph of the results of the 10 rolls.
 - (c) Determine the experimental probability of rolling a 1 after 10 rolls. Write this probability as a fraction, a decimal, and a percent.
 - (d) Roll the die another 40 times to make 50 rolls in total. Record the results of each roll.
 - (e) Create a frequency distribution table and graph of the results of all 50 rolls.
 - (f) Determine the experimental probability of rolling a 1 after 50 rolls. Write this probability as a fraction, a decimal, and a percent.
 - (g) Compare and contrast the two results of the two experimental probabilities.
 - (h) Are the results what you would expect?

Solutions:

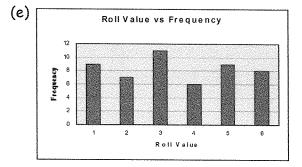
Results will vary; have students compare their results.

Here is an example of possible results:

(a) 10 rolls Roll value Frequency 1 2 2 3 3 0 4 1 5 2 6 2



- (c) In the above example: Fraction:
- Decimal: 0.20 Percent: 20%
- (d) 50 Rolls Roll value Frequency 1 9 2 7 3 11 4 6 5 9 8



- (f) In the above example: Fraction: $\frac{9}{50}$ Decir
 - $\frac{9}{50}$ Decimal: 0.18 Percent: 18%
- (g) Answers here will vary \Rightarrow In this case with more rolls the probability dropped a little.
- (h) Answers here will vary \rightarrow Just honestly give your opinion about what you expected.

U07A02

Probability Experiment: Tossing Three Coins

Toss three coins. Record the number of tails of each toss in the table. Record the number of points: 3 points for all three tails, 1 point for any 2 tails, 0 points for 1 or no tails. If you receive 3 or 1 points in a turn, you get to go again. If you receive no points, switch to your partners turn. Play until one person reaches 15 points. (If you run out of space on the table, continue your points on the back)

Your Points

Toss #	# of	Points
	Tails	
1		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		

Partner's Points

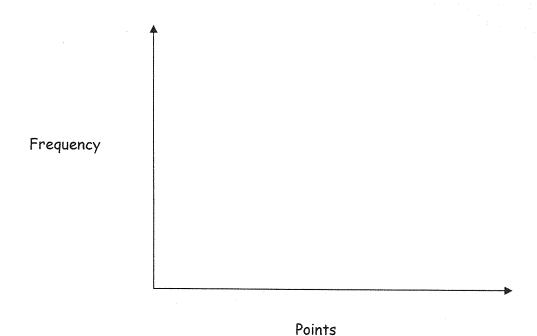
Toss #	# of	Points
	Tails	
1		
2		
2 3 4		
4		
5		
5 6 7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		

Your total points:	Partner's total points:
--------------------	-------------------------

Create a frequency distribution using all of the rolls from you and your partner.

Event	Frequency (Number of times event occurs)
3 points	
1 point	
0 points	

Create a frequency distribution graph.

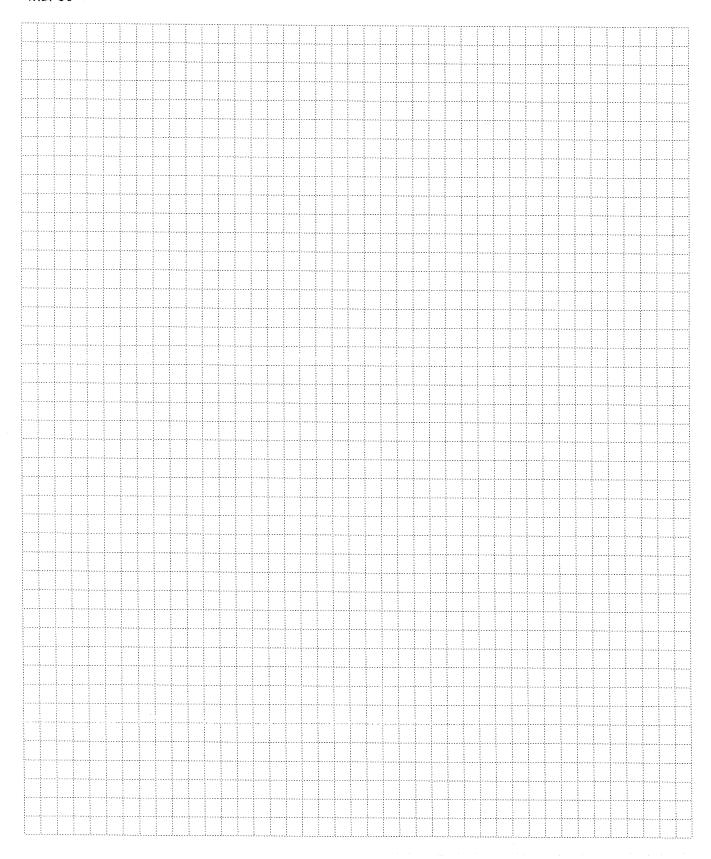


Now using the frequency distribution table or graph, determine the **experimental probability** of obtaining 3 points, 1 point, or 0 points. The experimental probability is the ratio of the number of times an event occurs and the total number of trials.

Probability of Event A (3 Points): P(A) =

Probability of Event B (1 Point): P(B) =

Probability of Event C (O Points): P(C) =



Theoretical Probability

- 1. Find the probability of each of the following situations:
- (a) You toss a coin \rightarrow what is the probability of seeing tails come up?
- (b) You toss two coins → what is the probability of seeing both coins show tails?
- (c) You toss three coins → what is the probability of seeing only one tail on all three coins?
- (d) You toss three coins \rightarrow what is the probability of seeing at least one tail on all three coins?
- 2. Find the probability of each situation of rolling a six-sided die:
- (a) What is the probability of rolling a 5?
- (b) What is the probability of rolling a 1 or a 2?
- (c) What is the probability of rolling an odd number?
- (d) What is the probability of rolling a number greater than 2?
- 3. A standard deck of cards contains 52 cards these cards are identified as follows: There are 4 suits: Spades, Hearts, Clubs and Diamonds. Each suit contains 13 cards: Ace (often valued at 1), numbered cards 2, 3, 4, 5, 6, 7, 8, 9, and 10, and then a Jack ("J"), a Queen ("Q") and finally a King ("K"). Spades and Clubs are both black coloured cards and the Hearts and Diamonds are red coloured cards. The Jack, Queen and King cards are also often referred to as face cards as they have a face on them. Based on the above description of a standard deck of cards calculate the probability for the following situations based on an experiment of drawing one card from a well-shuffle deck:
- (a) What is the probability of drawing a red card from the deck?
- (b) What is the probability of drawing a heart card from the deck?
- (c) What is the probability of drawing an even numbered card (2, 4, 6, 8, 10 of any suit) from the deck?
- (d) What is the probability of drawing a face card from the deck?

Solutions:

(Note: after the first question only one method of showing the probability will be used -- any are acceptable.)

- 1. (a) 1/2, 0.5, or 50%, (b) 1/4, 0.25 or 25%, (c) 3/8, 0.375, or 37.5%, (d) 7/8, 0.875, or 87.5%
- 2. (a) 1/6 (b) 1/3 (c) 1/2 (d) 2/3
- 3. (a) 1/2 (b) 1/4 (c) 5/13 (d) 3/13

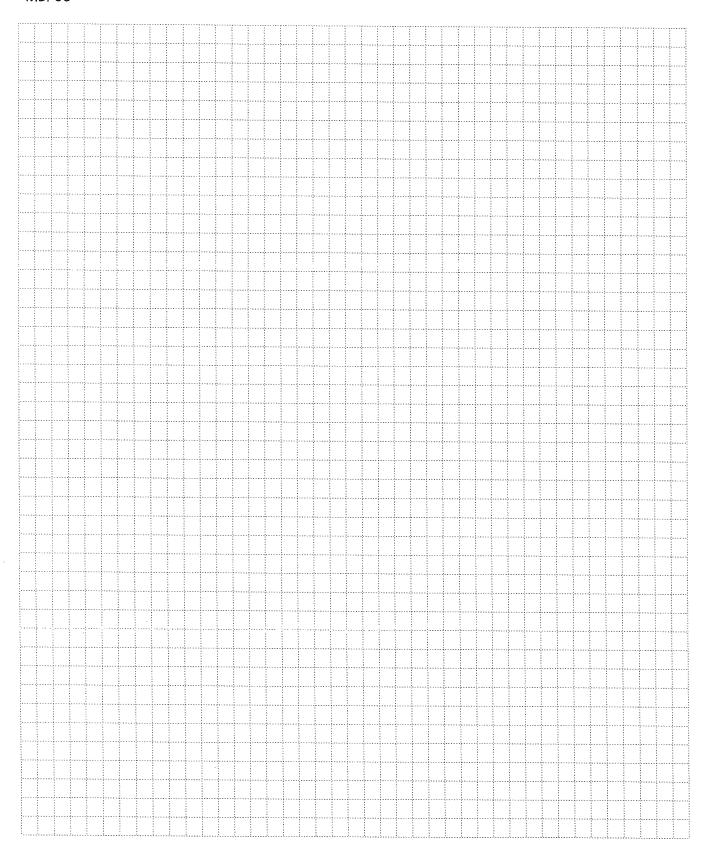
Probability

- 1. Using a standard deck of cards, consider the following possibilities:
- (a) What is the probability of picking a 7, 8, or 9?
- (b) What is the probability of picking a heart or a face card?
- (c) If the deck is now split for the game of Euchre (only 9, 10, J, Q, K, A exist in the deck; all other cards are removed) what is the probability of picking an Ace?
- 2. Using the table of possible sums from rolling a pair of dice answer the following questions:
- (a) What is the probability of rolling sum that is a multiple of 3?
- (b) What is the probability of rolling sum that is a multiple of 5?
- (c) What is the probability of rolling sum that is 7 or 11?
- (d) Ignoring the sums for this question what would be the probability of rolling doubles? (Doubles occur when both numbers on the die match i.e. 1st die shows a 1 and so does the 2nd.)
- 3. Jesse needs to get ready for school. He has two pair of pants to choose from: one black and one brown. He has three shirts to choose from: one red, one green, and one white.

 Any combination of pants and shirts is equally likely. (hint: use a tree diagram)
- (a) What is the probability that he will wear the green shirt and the brown pants?
- (b) What is the probability he will wear the black pants with any of the shirts?

Homework Solutions:

- 1. (a) 3/13 (b) 11/26 (c) 1/4
- 2. (a) 1/3 (b) 7/36 (c) 2/9 (d) 1/6
- 3. (a) 1/6 (b) 1/2



Investigation

Suppose that your final exam has 10 multiple choice questions, each with possible answers of A, B, C or D.

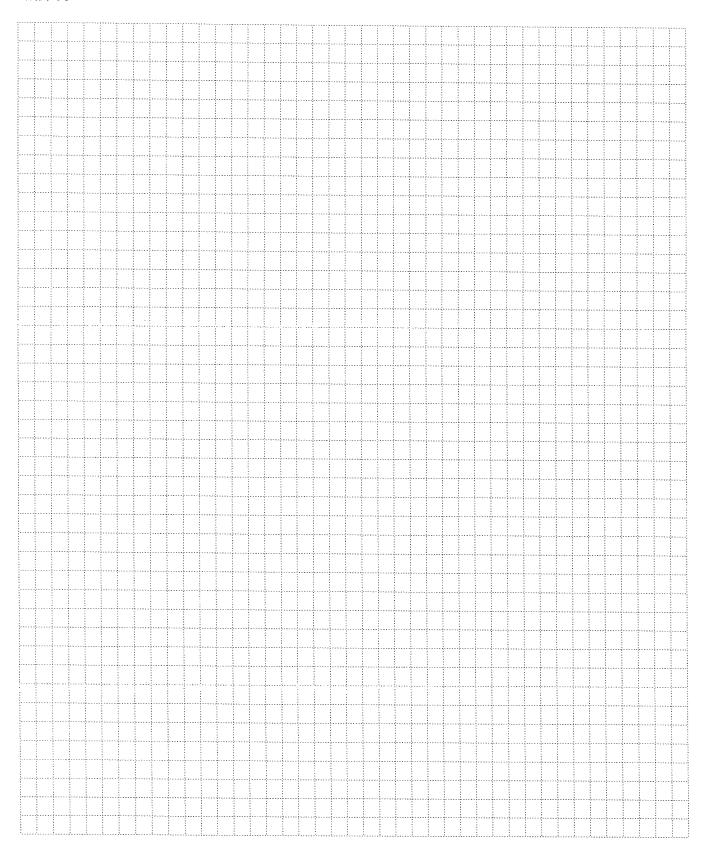
1.	Jse a deck of cards to simulate the probability of passing this portion of the ex	(am
	imply by guessing.	

a)	Choose	a suit	to	represent	the	correct	answer.	Name	it	here:	
----	--------	--------	----	-----------	-----	---------	---------	------	----	-------	--

- b) Draw 10 cards one after the other, replacing each card between drawings.
- c) Record whether or not you passed in the chart below.
- d) Repeat all 10 draws 19 more times.

Trial	Number of Cards of Chosen Suit (out of 10)	Pass or Fail
	Experimental Probability= 10	
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

- 2. Calculate your experimental probability of passing the test.
- 3. Given that the theoretical probability that you would pass is 7.8%, how close were you to this value?
- 4. Explain why your simulation may not have been exact.



Experimental and Theoretical Probability

- 1. The centre on a high school basketball team has a free-throw success rate of 85%.

 Describe a simulation that would help to determine the probability that he might miss 5 shots in a row. Be specific in your description.
- 2. Describe a simulation that would estimate the probability of having 3 girls in a family of 3 children. Be specific in your description.
- 3. An event occurs every 8 out of 19 times on average during a simulation. What is the experimental probability of this event?
- 4. A coin is flipped 15 times to simulate a family having a boy. Heads were used to represent boys and tails for girls. Twelve heads were recorded. Is the experimental probability of 4/5 close to the theoretical probability?
- 5. A casino introduces a new game called *Test Your Luck*. To play, you draw a card from a deck. If it's a heart, they will match your bet. If it's black, they keep your bet. Explain why this is not a good game to play.

Solutions

- 1. Make a spinner with sector angles 306E and 54E. Spin it 5 times recorded the number of times that it hits the 54E section. Repeat 100 times.
- 2. Flip a coin 3 times. Assign heads to girls. Record whether all 3 are girls. Repeat 100 times.
- 3. $\frac{8}{19}$ 4. No, the theoretical prob. is $\frac{1}{2}$.
- 5. The probability of losing is twice the probability of winning.

U07A07

Investigation

- 1. Is the coin toss for gas a fair game? Explain.
- 2. How much should you expect to pay in a 4-week month?
- 3. What is the probability that you will never pay in a 4-week month? Show your work.
- 4. What is the probability that you will pay at least once in a 4-week month? Show your work.
- 5. Repeat the in-class simulation on the graphing calculator. Record your results.

Steps: A) Go to the MATH function. Cursor over to the PRB menu (<<<).

- B) Select 5: randInt to access the random integer function.
- C) Assign O for heads and 1 for tails.
- D) On your screen you will see randInt(type 0,1,2) It is important to include the commas as written.

 This tells the calculator to create lists of the numbers 0 and 1, 2 at a time.
- E) Press ENTER four times to simulate a 4-week month.
- F) Record your results in the chart provided. Remember, you win if you see either {0 0} or {1 1}.
- G) Repeat 23 times.

Month	Wins	Losses	Cost
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			

6. What was your average cost using the results in Question 5?

U07A07-P2

Investigation Continued.....

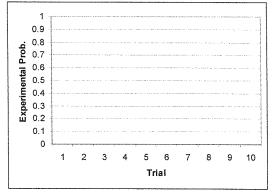
- 7. How does your answer to Question 6 compare with your answer to Question 2?
- 8. Using the results in Question 5, how often did you never pay in a month?
- 9. How does your answer to Question 8 compare with Question 3?
- 10. Explain your answers to Question 7 and Question 9.

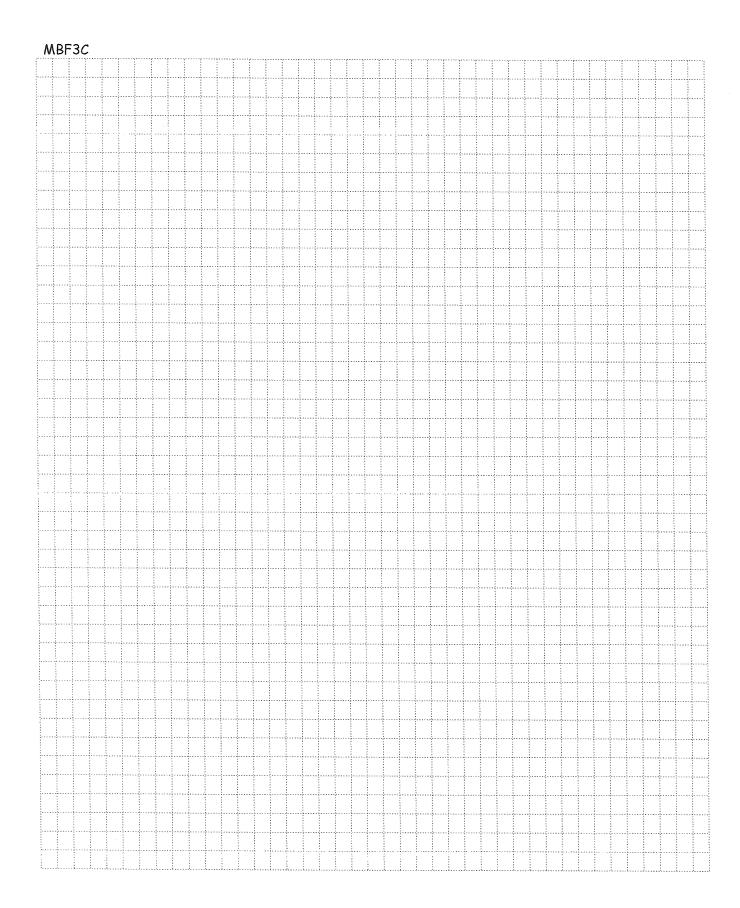
Simulation 2

- 11. What is the probability of rolling a 2 on a numbered cube?
- 12. Use the randInt function on the graphing calculator with the numbers (1,6,10) to generate 10 numbers between 1 and 6.
- 13. Record how many of each set of 10 are 2's. Use the chart below. Repeat by pressing ENTER until you've completed the chart.

Trial	Number of 2's	Experimental Probability # of 2's 10
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

14. Complete the graph and describe any trends.





U07A08

Investigation: Probability in the Media

Based on the discussions begun in class, search through your example(s) of media sources for examples of statistical information. (*Either newspapers, magazines, or the internet*.)

Once you have at least three examples you feel you can use, do the following for each:

1) Analyse the statistical information in your examples. E.g. find the mean, median and mode if appropriate, using totals and specific values to generate the probabilities of those specific values occurring, etc.

2) From this analysis generate some questions that could be used to get another student to analyse your statistical example. You should come up with at least three questions for each of your statistical examples.

3) Find a partner and share your statistic examples and questions. You work on their questions while they work on yours. Once you've completed the questions, discuss possible results of your analysis – e.g. develop appropriate headlines for newspaper articles based on your findings.

- 1. Classify the following events as I for independent or D for dependent.
 - a) A card is chosen from a deck and then replaced. A second card is chosen.
 - b) Two teams are being chosen by random selection. As each person is chosen, he or she joins a team and leaves the draw.

2. Re-write each of the probabilities as either a decimal, fraction or percent.

	Fraction	Decimal	Percent
a)	$P(A) = \frac{1}{4}$		
b)		P(B) = 0.42	

- 3. An event occurs every 6 out of 113 times on average during a simulation. What is the experimental probability of this event?
- 4. What is the probability of:
 - a) flipping two coins and getting at one heads and one tails? b) rolling a die and getting a 3?
- 5. Out of a total of 1000, how many:
 - a) hits should one get if their batting average is 0.343?
 - b) saves will Emery make if his save percentage is 0.944?
 - c) goals will Gerber let in if his save percentage is 0.875?
- 6. Jesse has two shirts to choose from: one green and one brown. He has three ties to choose from: one red, one blue, and one white. Any combination of shirts and ties is equally likely.
 - a) Draw a tree diagram.
 - b) What is the probability that he will wear the green shirt and the blue tie?
 - c) What is the probability that he will wear the blue tie with any of the shirts?
- 7. John has challenged Johanna to a "die dual". John's die is a six-sided die with the numbers 2, 2, 2, 3, 3, 6 on it and Johanna's die is a five-sided die with the numbers..... 1, 4, 7, 8, 9 on it. They each roll their own die and whosever roll is the largest....wins. If the rolls are the same, they re-roll.
 - a) What is the probability that there is a 'winner' after a roll?
 - b) What is the probability that the two rolls add up to 6 or less? Show your work.
 - c) What is the probability that John wins?
- 8. Find the probability of flipping a tail with a coin and throwing a 3 with a die.
- 9. Using a standard deck of cards, consider the following possibilities:
 - a) What is the probability of picking a 7, 8, or 9?
 - b) What is the probability of picking a heart or a royal card (Jack, Queen, Ace, King)?
- 10. From a standard deck of cards, one card is drawn. What is the probability that the card is black and a face card (Jack, Queen, King)?

- 11. A casino introduces a new game called *Test Your Luck*. To play, you draw a card from a normal deck of cards. If it's a spade, nobody wins, if it's a club, they will match your bet. If it's red, they keep your bet.
 - a) What is the probability of winning? Losing?
 - b) Should Casinos be legal in Canada? Think through your answer critically. Back up your argument with valid arguments. (i.e. don't just say that yes they should because if people get addicted it is their own fault. At the same time, don't say that casinos shouldn't be legal because the casino owners make too much money. Your answers need to show critical reasoning.)
- 12. A coin is flipped 15 times to simulate a family having a boy. Heads were used to represent boys and tails for girls. Ten heads were recorded. Explain if the experimental probability is close to the theoretical probability?
- 13. GM's OnStar service is a wireless communication system between a central call centre and an OnStar enabled car, allowing drivers to call a representative.
 - 1 300 'Good Samaritan' calls; Orange Alerts; someone in trouble etc. per month
 - 75 stolen vehicle location assists per year
 - 15 000 remote unlock calls per month
 - 4 000 requests for roadside assistance per month
 - 32 000 requests for route assistance per year
 - a) If you were an OnStar Operator, what would be the probability of the next call being a person needing some roadside assistance?
 - b) What type of call has the highest probability? What is its probability?
- 14. Shaquille O'Neal is absolutely terrible at the free throw line. In fact, many teams look at his success rate of 38% and intentionally foul him so that he has to shoot from the free throw line. Describe a <u>simulation</u> that would help to determine the probability that he might miss 5 shots in a row. Be specific in your description.
- 15. Calculate the probability for each scenario:
 - a) On a TV station, 8 out of 13 commercials are for food products. Estimate the probability that out of the next 4 commercials shown, there will not be any commercials for food products.
 - b) What is the probability that out of the next four commercials shown, at least one commercial will be for a food product.
- 16. What is the probability of tossing two die with a sum of eight or getting doubles? Show your work using the equations learned in class.

Appendix

Content:

Activity Solutions

Conversion Tables

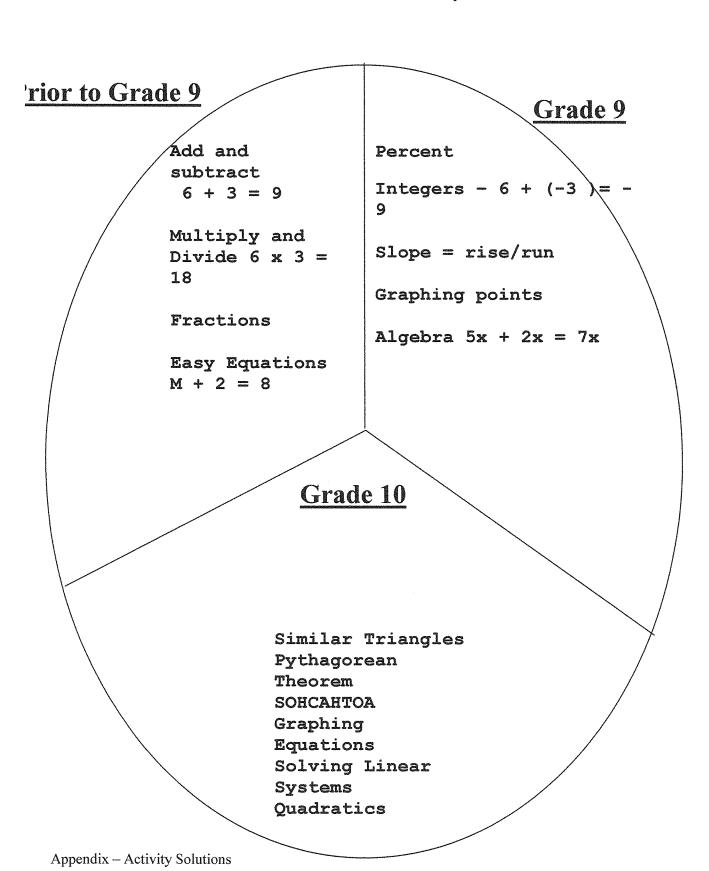
Graph Paper

Formula Sheets

Sample Summative

Sample Exam

Math I Know! (Sample Answers)



U00A02-SOLUTIONS

Find Someone Who (Sample Answers)

A kite is 10 m up in the sky. The angle the string makes with the ground is 50 degrees. How long is the kite string?

SIN50 =
$$\frac{10}{H}$$
, $H = 57.59$ m

Can Evaluate:

$$-6 + -3 = -9$$

2 - (-3) = 5_

Name:

Can solve for p:

$$P - 4 = 13$$

$$P-4+4=13+4$$

$$P = 17$$

Can Evaluate:

$$6 \times 3 = _18_$$

$$9 \times 9 = 81$$

Can round to the nearest hundred

3x + 6

Can expand 3(x + 2)

Name

Can use the Pythagorean theorem to solve for b given c = 13 and a = 12

$$B^2 = 13^2 - 12^2$$

$$B^2 = 169 - 144$$

$$B^2 = 25$$

$$B = 5$$

Can represent slope in two ways Slope = rise div by run or slope is m in y = mx + b

Can round to the nearest tenths

a)
$$7.22 = 7.2$$

b)
$$3.024 = 3.0$$

c)
$$2.56 = 2.6$$

Name

Can solve: If you build a skateboard ramp whose ratio of height to base must be 2:3, what is the base if the height is 4.5 m?

$$2/3 = 4.5/x$$

$$2x = 13.5$$

$$x = 6.75$$

Name

Can expand and simplify (x+1)(x-1)

$$(X + 1)(X - 1)$$

$$= x^2 - 1x + 1x - 1$$

$$= x^2 - 1$$

Name

Can expand and simplify $(x + 1)^2$

$$= (x+1)(x+1)$$

$$= x^2 + 1x + 1x + 1$$

$$= x^2 + 2x + 1$$

Name

Can use the Pythagorean theorem to solve for c given a=3 and b=4

$$C = 5$$

Appendix – Activity Solutions

Can Divide:

100 by 10 = _10_ 144 by 12 = __12_

Name:

U00A03-SOLUTIONS

Math MATCHING

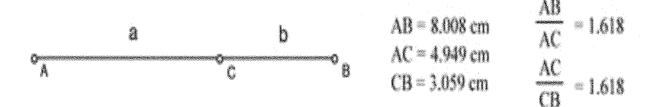
Match the letter from Column B with the most appropriate number in Column A. Be certain to $\underline{show\ ALL\ work.}$

	Column A	Column B
C 1.	Expand and simplify 2(x + 3)	A. 24.4 m
H 2.	Use the Pythagorean Theorem to solve for c given a = 8 and b = 6	B. 15.56 m
7 3.	Expand and simplify $(x + 3)(x + 2)$	C. 2x + 6
A 4.	Use the Pythagorean Theorem to solve for c given a = 20m and b = 14m	D. 7.5 m
D 5.	If you build a skateboard ramp whose ratio of height to base must be 2:3, what is the base if the height is 5 m?	E. 17.68 m
G 6.	A kite is 15 m up in the sky. The angle the string makes with the ground is 50 degrees. How long is the kite string?	F. x ² + 5x + 6
B 7.	A kite is 10 m up in the sky. The angle the string makes with the ground is 40 degrees. How long is the kite string?	G. 19.58 m
E 8.	Bob has a kite and his string is 25 m long. The angle the kite makes with the ground is 45 degrees. How far horizontally is the kite from Bob?	H. 10

U00A04-Solutions

The Golden Ratio

<u>Definition</u>: Let a segment AB of length l be divided into two segments by the point C (see figure below). Let the lengths of AC and CB be a and b respectively. If C is a point such that l:a as a:b, C is the "golden cut" or the golden section of AB. The ratio of l/a or a/b is called the **golden ratio**. In the terminology of the early mathematicians AB is divided by C in "extreme and mean ratio." Kepler called it "**the divine proportion**."



Length (AB) = l

U00A05-P2-SOLUTIONS

Sample Solution:

Standard Two Step Equation

$$2x + 7 = 13$$



Steps

- 1) get rid of what is on the same side as the variable but not connected to it (use opposite operations of + and) Be certain that whatever you do to the L.S. you Do to the R.S.
- 2) get rid of what is connected to the variable (use opposite operations of multiply and divide) Be certain to balance the equation.

Solving a Linear Equation with Variables on both sides 3x - 6 = 2x - 2

Steps:

- decide which side to collect your variables onto (usually the side with the largest coefficient)
- 2) refer to steps in quadrant "A"

Solving Equations with Brackets 5(x-2) = 4x - 3

Steps:

- 1) use the distributive property
- 2) simplify
- 3) refer to the steps in quadrant "A"

Solving Equations with Fractions $\frac{x+3}{2} + \frac{x+1}{5} = 8$

Steps

- 1) multiply all parts through by what would be the CD (common denominator)
- 2) use the distributive property if necessary
- 3) collect like terms
- 4) refer to the steps in quadrant "A"

Similar Triangles (Sample Answers)

1) Name the Triangle, the angles, and the sides.



Triangle Names:

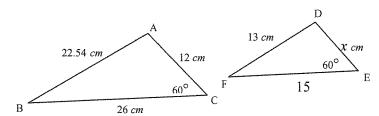
 \triangle ABC, or \triangle BCA, or \triangle CAB.

Side Names: BC or a; AC or b, and AB or c.

Angle Names are $\angle A$ or $\angle BAC$ or $\angle CAB$,

 \angle B or \angle ABC or \angle CBA, and \angle C or \angle ACB or \angle BCA

2) Why are the following triangles similar? Solve for x.



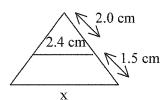
The corresponding angles C and E are equal at 60 degrees. The ratio of the corresponding sides $\frac{BC}{FE} = \frac{BA}{FD} = 1.733$.

The side x can be found by solving the equation $\frac{BC}{FE} = \frac{AC}{DE}$

$$\frac{26}{15} = \frac{12}{x}$$

Thus x = 6.92 cm

3) The two triangles are similar. Determine the length represented by x.



Remind students to take the smaller triangle off of the larger triangle. Then the ratio is "little triangle to big triangle", that is

$$\frac{x}{2.4} = \frac{2}{(2+1.5)}$$

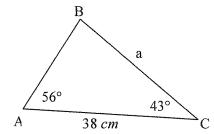
$$\frac{x}{2.4} = \frac{2}{3.5}$$

$$x = 1.37 \text{ cm}$$

U01A08-Solutions P1

Making Decisions in Trigonometry

1)



First we need to find $\angle B$ we can do this using the sum of the angles in a triangle. \angle B = 180° - 56° - 43°

Therefore, $\angle B = 81^{\circ}$

Now we can solve for a using the Sine Law

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B}$$

$$\frac{a}{\sin 56^{\circ}} = \frac{38}{\sin 81^{\circ}}$$

$$a = \frac{38}{\sin 81^{\circ}} \cdot \sin 56^{\circ}$$

$$a = \frac{38}{0.98768834} \cdot 0.82903757$$

$$\therefore$$
 a = 31.9 cm

Solving for the other sides:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

$$\frac{38}{\sin 81^{\circ}} = \frac{c}{\sin 43^{\circ}}$$

$$c^2 = (31.9)^2 + (38)^2 -$$

2(31.9)(38) cos 43°

$$c = \frac{38}{\sin 81^{\circ}} \cdot \sin 43^{\circ}$$
 $c^2 = 1017.61 + 1444 - 1444$

$$c^2 = 1017.61 + 1444$$

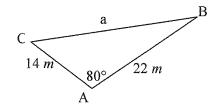
2424.4(0.7313537)

$$c = \frac{38}{0.98768834} \cdot 0.68199836$$

$$c^2 = 688.5160858$$

$$c = 26.2 cm$$

$$c = 26.2 cm$$



$$a^{2} = b^{2} + c^{2} - 2bc \cdot \cos A$$

$$a^{2} = (14)^{2} + (22)^{2} - 2(14)(22) \cdot \cos 80^{\circ}$$

$$a^{2} = 196 + 484 - 616 \cdot 0.173648178$$

$$a^{2} = 196 + 484 - 106.96727744$$

$$a^{2} = 573.03272256$$

$$\sqrt{a^{2}} = \sqrt{573.03272256}$$

$$\therefore a = 23.9 \text{ m}$$

Solving for the other angles.

Solving for the other angles.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 80^{\circ}}{23.9} = \frac{\sin B}{14}$$

$$\frac{\sin 80^{\circ}}{23.9} \cdot 14 = \sin B$$

$$\sin B = \frac{0.98480775}{23.9} \cdot 14$$

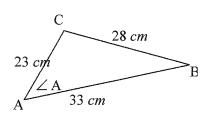
$$\sin B = 0.57687483$$

$$\angle B = 35.231034^{\circ}$$

$$\angle B = 35^{\circ} \rightarrow \text{this gives us } \angle C = 65^{\circ}$$
(i.e. $180^{\circ} - 80^{\circ} - 35^{\circ}$)

Making Decisions in Trigonometry

3)



$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$
 $(28)^2 = (23)^2 + (33)^2 - 2(23)(33) \cdot \cos A$
 $784 = 529 + 1089 - 1518 \cdot \cos A$
Rearranging:
 $1518 \cdot \cos A = 529 + 1089 - 784$
 $1518 \cdot \cos A = 834$
 $\cos A = \frac{834}{1518}$
 $\cos A = 0.54940711$
 $\angle A = 56.67365194$
 $\therefore \angle A = 57^\circ$

Solving for the other angles:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 57^{\circ}}{28} = \frac{\sin B}{23}$$

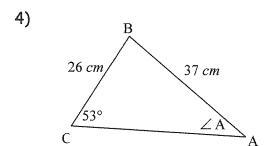
$$\frac{\sin 57^{\circ}}{28} \cdot 23 = \sin B$$

$$\sin B = \frac{0.83867057}{28} \cdot 23$$

$$\sin B = 0.688907967$$

$$\angle B = 43.543727^{\circ}$$

$$\therefore \angle B = 44^{\circ} \rightarrow \text{this gives us } \angle C = 79^{\circ}$$
(i.e. $180^{\circ} - 57^{\circ} - 44^{\circ}$)



$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin A}{26} = \frac{\sin 53^{\circ}}{37}$$

$$\sin A = \frac{\sin 53^{\circ}}{37} \cdot 26$$

$$\sin A = \frac{0.79863551}{37} \cdot 26$$

$$\sin A = 0.56120333$$

$$\angle A = 34.13905704$$

$$\therefore \angle A = 34^{\circ}$$

Solving for the other sides $\angle B = 180^{\circ} - 34^{\circ} - 53^{\circ}$ $\therefore \angle B = 93^{\circ}$

Sine Law vs. Cosine Law
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$b^{2} = a^{2} + c^{2} - 2ac$$

$$\cos B$$

$$\frac{b}{\sin 93^{\circ}} = \frac{37}{\sin 53^{\circ}}$$

$$2(26)(37) \cos 93^{\circ}$$

$$b = \frac{37}{\sin 53^{\circ}} \cdot \sin 93^{\circ}$$

$$b^{2} = (26)^{2} + (37)^{2} - \frac{1}{\sin 53^{\circ}}$$

$$b^{2} = 676 + 1369 - 1924 (-100)$$

$$0.05233596)$$

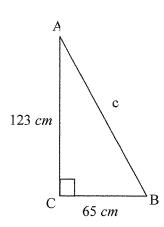
$$b = \frac{37}{0.79863551} \cdot 0.998629535$$

$$b^{2} = 2145.69437981$$

b = 46.3 cmb = 46.3 cm

Making Decisions in Trigonometry

5)



$$c^{2} = 123^{2} + 65^{2}$$

$$c^{2} = 15 \ 129 + 4 \ 225$$

$$c^{2} = 19 \ 354$$

$$c = \sqrt{19354}$$

$$c = 139.1$$

Solving for angles:

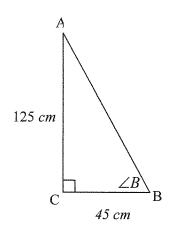
$$\tan B = \frac{123}{65}$$

$$\tan B = 1.8923$$

$$\angle B = 62^{\circ}$$

$$\therefore \angle A = 28^{\circ}$$

6)



$$\tan B = \frac{125}{45}$$
$$\tan B = 2.7778$$
$$\angle B = 70^{\circ}$$

Solving for the other angle and side.

$$c^2 = 125^2 + 45^2$$

= 15 625 + 2025
= 17 650
 $c = 132.85$ cm

U01A12-P2 SOLUTIONS

Exercise Solutions

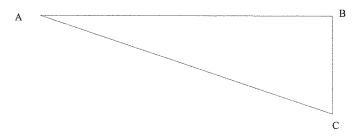
Exercise 1: (Determine the height of the classroom)



In the diagram above AD is the measured distance from the floor to the clinometer (this is equal to CE), DE is the measured distance from the wall to the clinometer (this is equal to AC), and πBAC is the angle measured by the clinometer. Using the Tan ratio you can find BC. The height of the wall is BC + CE.

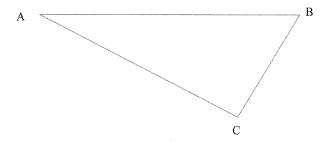
Exercise 2: (Determine the width of a classroom wall)

(a)



In the diagram above the measured distance BC is the distance from the wall to the clinometer and π BCA is the angle measured by the clinometer. Using the Tan ratio you can solve for AB which is the width of the wall.

(b)

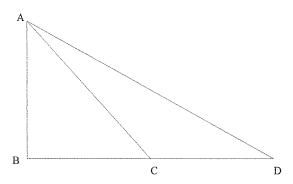


In the diagram above πACB is the angle measured by the clinometer. You could then measure CB and CA. Now use the cosine law to find AB.

U01A12-P3 SOLUTIONS

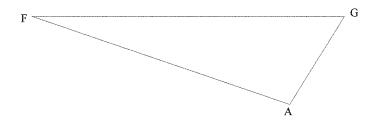
Task Solutions

Task 1: Finding the Height of an object without approaching it

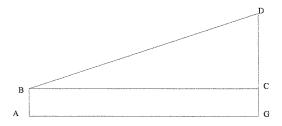


In the diagram above AB is the height of the wall. This is to be determined. C is the starting point and BC is not to be measured. First use the clinometer to find πBCA and then subtract this from 180 to get πACD . Back up to point D (you determine point D) and measure the distance CD. Now use the clinometer to measure πCDA . You now have the information needed to solve for CA using the Sine Law. With CA and πBCA find AB using the Sin ratio.

Task 2: Finding the Width of an object without approaching it



In the above diagram point A is your starting point away from the wall. FG is the width of the wall. In exercise 2 (b) you were able to measure the distances AG and FG. This time you can't. π GAF can still be measured with the clinometer. To determine AG and AF we use the height of the wall, the clinometer and the Tan ratio. Shown in the following diagram is the distance AG at the bottom. The height of the wall is DG. AB is the distance from the floor to the clinometer. Subtract BA = CG from DG and measure π DBC with the clinometer. Now use the Tan ratio with DC and π DBC to find BC = AG.



Repeat this process to find AF in the first diagram. Now use AF, AG, and π GAF in the Cosine Law to find GF = the width of the wall.

Properties of a Parabola - Solutions

Equation	Vertex	Step Pattern From Vertex	Direction of Opening
$y = (x - 2)^2 + 1$	(2,1)	1, 3, 5	Up
$y = -(x + 4)^2 + 6$	(-4,6)	-1, -3, -5	Down
$y = 4(x - 4)^2 - 1$	(4, -1)	4, 12, 20	Up
$y = 3(x + 7)^2 - 4$	(-7, -4)	3, 9, 15	Up
$y = -2(x - 10)^2 + 100$	(10, 100)	-2,-6,-10	Down
$y = (x - 4)^2 + 15$	(4, 15)	1, 3, 5	Up
$y = -2(x + 2)^2 + 64$	(-2,64)	-2,-6,-10	Down
$y = 5(x - 10)^2 - 11$	(10, – 11)	5, 15, 25	Up
$y = 2(x + 3)^2 - 3$	(-3, -3)	2, 6, 10	Up
$y = -(x - 20)^2 - 10$	(20, –10)	-1,-3,-5	Down

Unit 2 Quadratics - Solutions

2.1 Answers will vary

2.2

	Account to the second s	
Number of Cheers	Price per Cheer	Total Money Paid (1 st × 2 nd columns)
7	\$2.30	\$16.10
8	\$2.10 + 10¢ = \$2.20	\$17.60
9	\$2.00 + 10¢ = \$2.10	\$18.90
10	\$2.00	10 × \$2.00 = \$20.00
11	\$2.00 - 10¢ = \$1.90	\$20.90
12	\$1.80	\$21.60
13	\$1.70	\$22.10
14	\$1.60	\$22.40
15	\$1.50	\$22.50
16	\$1.40	\$22.40
17	\$1.30	\$22.10
18	\$1.20	\$21.60

The maximum money of \$22.5 is paid when you do 15 math cheers

2.3

Rectangle Label	If the length of the pool area is	Then the width is	And the area is (units are sections ²)
Α	1 section	9 sections	1 × 9 = 9
В	2 sections	8	16
С	3 sections	7	21
D	4 sections	6	24
E	5 sections	5	25
F	6 sections	4	24
G	7 sections	3	21
Н	8 sections	2	16
I	9 sections	1	9

The maximum area of 25 occurs when the area is 5 sections long and 5 sections wide

<u>2.4</u>

Side Length	Surface Area	Side	e Length	Surface Area	
1	6		5	150	
2	24		6	216	
3	54		7	294	
4	96		8	384	

2.5

1. 50 members, 2. approx \$1250, 3. \$800, 4. \$0 revenue, 5. no, can't give have a negative admission price!

2.6 Question 1

· · · · · · · · · · · · · · · · · · ·		
If the length of the display is	Then the width is	And the area is (units are cm²)
10cm	50	500
20cm	40	800
30cm	30	900
40cm	20	800
50cm	10	500
60cm	0	0

Question 2

Ticket Price	Number of People	Total Money From Tickets
23	270	6210
24	260	6240
25	250	6250
26	240	6240
27	230	6210
28	220	6160
\$29	210	6090
\$30	200	\$30 × 200 = \$6000
31	190	5890
32	180	5760

2.7

A. 27 or 93 (approx), B. 60 coffees, C. - \$250, D. 50 or 70 coffees 1. (60, 110) 2. 27 and 93, 3. x = 60, 4. \$110, 5. - \$250

2.8

True and False: F, T, T, T, F, F

The graph of a quadratic is called a parabola

2.9

1. 35 feet, 2. 10 seconds, 3. - 7 feet, 4. approx 4.5 seconds and 15.5 seconds, 5. (10, -7), 6. No at some point the flight will change its course (it will have to come back to earth at some time)

2.10

Basic Parabola

y - values: 9, 4, 1, 0, 1, 4, 9

Vertex (0,0) Direction UP

Step Pattern Over 1, Up 1

Over 1, Up 3

Over 1, Up 5

Step Pattern also written as 1, 3, 5,...

Parabola Investigation #1 y - values: 11, 6, 3, 2, 3, 6, 11

Vertex (0,2) Direction UP

Step Pattern Over 1, Up 1

Over 1, Up 3

Over 1, Up 5

Appendix – Activity Solutions

```
Parabola Investigation #2 y - values: 6, 1, -2, -3, -2, 1, 6
                            Vertex (0,-3)
                            Direction UP
                            Step Pattern Over 1, Up 1
                                          Over 1, Up 3
                                          Over 1, Up 5
The number causes the vertex to move up or down.
Parabola Investigation 3
                            y - values: 9, 4, 1, 0, 1, 4, 9
                            Vertex (3, 0)
                            Direction UP
                            Step Pattern Over 1, Up 1
                                          Over 1, Up 3
                                          Over 1, Up 5
Parabola Investigation #4 y - values: 9, 4, 1, 0, 1, 4, 9
                            Vertex (-4, 0)
                            Direction UP
                            Step Pattern Over 1, Up 1
                                          Over 1, Up 3
                                          Over 1, Up 5
The number causes the vertex to go left or right (but opposite of the sign) + goes left, - goes right.
Parabola Investigation #5 y - values: 18, 8, 2, 0, 2, 8, 18
                            Vertex (0,0)
                            Direction UP
                            Step Pattern Over 1, Up 2
                                          Over 1, Up 6
                                          Over 1, Up 10
Parabola Investigation #6 y - values: -27, -12, -3, 0, -3, -12, -27
                            Vertex (0.0)
                            Direction UP
                            Step Pattern Over 1, Down 3
                                          Over 1. Down 9
                                          Over 1, Down 15
The number causes the step pattern to be multiplied, but also flips the parabola down if it is a negative
number
                                                  2.11
              Vertex (-1, -8), opens up, step pattern 2, 6, 10,...
Parabola 1:
       Zeroes: 1 and -3, y-intercept: -6, optimal value: -8, axis of sym: x = -1
              Vertex (3, 4), opens down, step pattern -1, -3, -5,...
Parabola 1:
       Zeroes: 1 and 5, y-intercept: -5, optimal value: 4, axis of sym: x = 3
```

2.12

Money, Money, Money: (a) \$450 000, (b) 200cars (c) break-even points (zeroes)

Sub's Way: (b) y = 2(x - 3)2 - 18, (c) yes

The Golden Arch: (b) 8m, (c) yes, using the graph this can be seen!, (d) 1.25m and 6.75m

2.13

Equation	$y = 3(x - 4)^2 - 8$	$y = -2(x + 1)^2$	$y = -(x + 2)^2 + 10$
Vertex			(-2, 10)
	(4, -8)	(-1, 0)	
Direction of	Up	Down	Down
Opening			
Step Pattern	3, 9, 15,	-2, -6, -10,	-1, -3, -5,
Max or Min?	min	Max	Max
Optimal Value	-8	0	10
Axis of Symmetry	X = 4	X = -1	X = -2

2.14

Parabola Investigation #1 y-values: 8, 3, 0, -1, 0, 3, 8

Direction: up Zeroes: -1 and 1 Axis: x = 0

Step Pattern: 1, 3, 5, ...

Parabola Investigation #2 y-values: 5, 0, -3, -4, -3, 0, 5

Direction: up Zeroes: -1 and 3 Axis: x = 1

Step Pattern: 1, 3, 5, ...

Parabola Investigation #3 y-values: -10, 0, 6, 8, 6, 0, -10

Direction: down Zeroes: -5 and -1 Axis: x = -3

Step Pattern: -2, -6, -10, ...

The zeroes are the opposites of the numbers in the brackets with the x's. The multiplier again controls the direction of opening and the step pattern.

2.15

Equation	y = 3(x - 3)(x + 5)	y = -(x + 2)(x + 6)	y = x(x + 8)
Zeros	3 and -5	-2 and -6	0 and -8
Direction of Opening	Up	Down	Up
Axis of Symmetry	X = -1	X = -4	X = -4
Step Pattern	3, 9, 15,	-1, -3, -5,	1, 3, 5,

Vertex of middle parabola is (-4, 4)

2.16

1.

Equation	y = 2(x - 5)(x + 9)	$y = -(x + 2)^2 + 6$	y = 4(x+2)(x+8)
Zeros	5 and -9	Not given by equation	-2 and -8
Direction of Opening	Up	Down	Up
Axis of Symmetry	X = -2	X = -2	X = -5
Step Pattern	2, 6, 10	-1, -3, -5,	4, 12, 20,
Vertex	Not given by equation	(-2, 6)	Not given by equation

2. (a) 1 and 9, (b) when the cannonball has a height of zero, (c) t = 5 when the ball reaches max height, (d) (5, 48) the max height of ball is 48 feet

3. break-even: 10 or 500 units

max profit: \$30 012.5 at 255 units

 $\begin{tabular}{ll} 2.17 \\ Use this table to record which group has done which question \\ \end{tabular}$

#	Solutions	Group 1	Group 2	Group 3	Group 4	Group 5
1	4 and -8	•		·		
2	5 and - 17					
3	X = -4					
4	X = 4					
5	-5 and -1					
6	(-2, -36)					
7	(-8, -6)					
8	Vertex (5, -2)					
	Step Pattern -2, -6, -10					
	Opens down					
9	(0, -5)					
10	(0, 45)					
11	\$80 000 at 3000 units					
12	\$1 444 000 at 4200 units					
13	400 and 8000 units					
14	125 feet at 5 seconds					
15	Vertex (1, -4)					
	Step Pattern 1, 3, 5					
	Opens up					
16	Vertex (4, -8)					
	Step Pattern 3, 9, 15					
	Opens up					
17	$Y = (x - 2)^2 + 3$					
18	 Equations have x2 					
	2. graph is a parabola				- Andreas	
	3. 2 nd differences are the same					

Solutions to Planet X Problem

Part (a)

"Maximum height" indicates that we should solve for the vertex.

h= 48t - 16t2 is in standard form

Rewrite as

$$h = -16t^2 + 48t$$

Common factor h = -16t(t - 3)

From factored form h = -16t(t - 3) the zeroes are 0 and 3

Axis of symmetry is t = 1.5 (at 1.5s the object is at its max height)

Sub t = 1.5 into equation

$$h = -16(1.5)^2 + 48(1.5)$$

$$h = -16(2.25) + 48(1.5)$$

$$h = -36 + 72$$

$$h = 36$$

The maximum height reached by the object is 36m

Part (b)

Solve for t when h = 32m

Original equation

Sub 32 for h

$$32 = 48t - 16t^2$$

Rearrange equation

$$0 = -16t^2 + 48t - 32$$

Common factor

$$0 = -16(t^2 - 3t + 2)$$

Trinomial factor

$$0 = -16(t - 2)(t - 1)$$

The solutions are t=1 or t=2. Thus the object is at 32m of height at 1s and 2s.

Part (c) Solutions to Planet X Problem ... Continued

The ground has a height of zero. Thus, find t when h=0. Or, find the zeroes of the parabola.

 $h=48t-16t^2$ is in standard form

Rewrite as $h = -16t^2 + 48t$

Common factor h = -16t(t - 3)

From factored form h = -16t(t - 3) the zeroes are 0 and 3

Thus the object hits the ground at 0s (launch) and 3s.

Part (d)

Vertex form needs vertex, step pattern and direction of opening.

h= 48t - 16t² is in standard form

Rewrite as $h = -16t^2 + 48t$

Here we see a = -16 (this tells us the step pattern is 16, 48, 80 and that the parabola opens down!)

Common factor h = -16t(t - 3)

From factored form h = -16t(t - 3) the zeroes are 0 and 3

Axis of symmetry is t = 1.5 (at 1.5s the object is at its max height)

Sub t = 1.5 into equation

$$h = -16(1.5)^2 + 48(1.5)$$

$$h = -16(2.25) + 48(1.5)$$

$$h = -36 + 72$$

$$h = 36$$

The vertex is (1.5, 36) and thus vertex form is $h = -16(t - 1.5)^2 + 32$

1.
$$x^2 + 7x + 12$$
, 2. $x^2 + x - 12$, 3. $x^2 - 5x + 6$, 4. $x^2 + \frac{17}{4}x + 1$, 5. $x^2 - 6x + 9$, 6. $6x^2 + 7x - 3$, 7. $4x^2 - 16x + 16$, 8. $45x^2 - 30x + 5$, 9. $\frac{3}{2}x^2 + 2x - 2$

PART B

Area =
$$6x^2 - x - 2$$
, perimeter = $10x - 2$

PART C

1.
$$y = x^2 + 2x - 15$$
, 2. $2x^2 - 12x + 23$, 3. $y = -3x^2 - 6x - 5/2$

Parabola Investigation #1:

y-values: 5, 0, -3, -4, -3, 0, 5 Direction of opening: up Step Pattern: 1, 3, 5, ...

Y-intercept: -3

Parabola Investigation #2:

y-values: -10, 0, 6, 8, 6, 0, -10 Direction of opening: down Step Pattern: -2, -6, -10

Y-intercept: -10

Parabola Investigation #3:

y-values: 24, 9, 0, -3, 0, 9, 24 Direction of opening: up Step Pattern: 3, 9, 15, ... Y-intercept: 9

Standard form gives you the step pattern, direction and opening and the y-intercept.

3.3

Examples: $x^2 + 6x + 9$, $x^2 - 24x + 48$, $3x^2 - 6x - 24$

Practice: 1. $x^2 - 3x - 10$. 2. $2x^2 - 4x + 5$. 3. $2x^2 - 4x - 6$. 4. $2x^2 + 16x + 29$. 5. $3x^2 + 3x - 6$. 6. $-x^2 + 6x - 11$

3.4

1.	
1. $2x(x-3)$	A. 2x + 8
$2x^2 - 6x$	
2. $4x - 2(x - 4)$	B. $x^2 + 3 - (3 + 6x - x^2)$
2. 17. 2(1. 1)	B. x / 3 = (0 / 0x = x)
2x + 8	$2x^2-6x$
2	2X - 0X
3. $3(x^2 - 4x + 2)$	C. $x(x-6) + 2(x^2 - 3x + 1)$
$3x^2 - 12x + 6$	$3x^2 - 12x + 6$

- 2a) $y = 2x^2 12x + 16$
- b) $y = -x^2 + 6x 8$; $y = 3x^2 6x 24$; $y = 2x^2 12x + 16$; $y = 2x^2 4x 6$ The third expression simplifies to the same result as in a).
- c) y = 2(x-4)(x-2) has the same parabola as the one in a) the both opens up, have a step pattern 2, 6, 10, .. and y intercept of -6...

A.
$$y = 2x^2 + 2x - 12$$
, B. $y = -3x^2 - 9x - 6$, C. $y = -x^2 + 12x - 24$, D. $y = x^2 + 10x + 26$

- 1. The y-intercept is (0, -5)
- 3. The equation of the ultra function is $U = x^2 + 18x 10$

1(a)
$$y = x^2 + 3x + 2$$
, (b) $y = x^2 - 6x + 6$, (c) $y = 2x^2 + 2x - 24$, (d) $y = 2x^2 - 12x + 8$, (e) $y = -3x^2 - 3x + 18$, (f) $y = -3x^2 - 3x + 18$, (f) $y = -3x^2 - 3x + 18$, (g) $y = -3x^2 - 3x + 18$, (h) $y = -3x^2 - 3x + 18$, (e) $y = -3x^2 - 3x + 18$, (f) $y = -3x^2 - 3x + 18$, (f) $y = -3x^2 - 3x + 18$, (g) $y = -3x^2 - 3x + 18$, (h) $y = -3x^2 - 3x + 18$, (e) $y = -3x^2 - 3x + 18$, (f) $y = -3x^2 - 3x + 18$, (f) $y = -3x^2 - 3x + 18$, (f) $y = -3x^2 - 3x + 18$, (f) $y = -3x^2 - 3x + 18$, (g) $y = -3x^2 - 3x + 18$, (e) $y = -3x^2 - 3x + 18$, (f) $y = -3x^2 - 3x + 18$, (f) $y = -3x^2 - 3x + 18$, (g) $y = -3x^2 - 3x + 18$, (g) $y = -3x^2 - 3x + 18$, (h) $y = -3x^2 - 3x + 18$,

2. Yes. Answers will vary

3.8

- 1. (a) -12, (b) -4, -12, (c) 3, (d) -9, (e) 2, 7
- 2. (a) (x-4)(x+1), (b) (x-7)(x-4), (c) (x+3)(x+4), (d) (x+4)(x-8)
- (e) (x-6)(x-7), (f) (x-2)2
- 3. (a) (0, 15), (b) y = (x + 3)(x + 5), (c) -3 and -5, (d) (-4, -1) (e) x = -4

1. (a)
$$y = 2(x-3)(x+1)$$
, (b) 3 and -1, (c) $x = 1$, (d) $(1, -8)$, (e) 2, 6, 10...

- (g) $y = 2(x-1)^2 8$
- 2. from a-value: direction of opening, step pattern. Then factor and get zeros. From zeros get axis of symmetry and then sub in to get optimal value (vertex). Then graph!
- 3. (a) h = -5t(t-4) b) 0 m, 15m, 20m c) 4 seconds d) 20m

- 1. (a) y = -2(x-5)(x-1), (b) 5 and 1, (c) x = 3, (d) (3, 8), (e) -2, -6, -10... (g) y = -2(x-3)2 + 8
- 2. (a) 105 yards b) 45 yards c) at -1 and 9 seconds d) 125 yards at 4 seconds

3.11

Quadratic Quandry (a) -\$112 000, (b) 20 000 and 140 000 people, (c) \$144 000, (d) 40 000 and 120 000 people Competition (a) -\$100 000, (b) 20 000 and 100 000 people, (c) \$80 000

3.12

- 1. (a) -3 and 1, (b) (-1, -8), (c) (2, 10) and (-4, 10) (d) $y = 2(x+1)^2 8$ and y = 2(x+3)(x-1)2. (a) 84.5% at 4.5 million viewers, (b) 3 million and 6 million (c) -2 and 11 million 3. (a) $(20+2x)(30+2x)-(30)(20)=4x^2+100x$, (b) 600, (c) $4x^2+100x=600$, (d) $0=4x^2+100x-600$, (e) 5 cm thick

Graph-Fest Solutions

PART A:

2 (answers will vary) They are all lines

3. (answers will vary)

y = 2x		
-2	-4	
-1	-2	
0	0	
1	2	
2	4	
3	6	

y = -1.5x		y = 0.7	′5×+4
-2	3	-2	2.5
-1	1.5	-1	3.25
0	0	0	4
1	-1.5	1	4.75
2	-3	2	5.5
3	-4.5	3	6.25

4. First Differences: (a) 2; (b) -1.5; (c) 0.75, Second Differences: (a) 0 (b) 0; (c) 0 (answers will vary) The first differences are in the equation (slope), second differences are all zero.

PART B:

2. (answers will vary) The graphs are curves/exponential graphs

3. (answers will vary)

y = 2 [×]		
-2	0.25	
-1	0.5	
0	1	
1	2	
2	4	
3	8	

y = 0.5 [×]		
-2	4	
-1	2	
0	1	
1	0.5	
2	0.25	
3	0.125	

$y = -1.75^{\times}$		
-2	-0.3265	
-1	-0.5714	
0	-1	
1	-1.75	
2	-3.0625	
3	-16.413	

4. First Differences: (a) 0.25, 0.5, 1, 2, 4; (b) -2, -1, -0.5, -0.25, -0.125; (c) -0.2449, -0.4286, -0.75, -1.3125, -13.3505 Second Differences: (a) 0.25, 0.5, 1, 2; (b) 1; 0.5, 0.25, 0.375, (answers may vary) Don't really notice anything

PART C:

2 (answers may vary) They are all parabolas

3. (answers may vary)

y = x ²		
-2	4	
-1	1	
0	0	
1	1	
2	4	
3	9	

y = 2x²		
-2	8	
-1	2	
0	0	
1	2	
2	8	
3	18	

$y = 0.5x^2$		
-2	2	
-1	0.5	
0	0	
1	0.5	
2	2	
3	4.5	

4. First differences: (a) -3, -1, 1, 3, 5; (b) -6, -2, 2, 6, 10; (c) -1.5, -0.5, 0.5, 1.5, 2.5, Second Differences: (a) 2, 2, 2, 2 (b) 4, 4, 4, 4; (c) 1, 1, 1, 1, (answers will vary) Second differences are the same

MBF3C U03A08-SOLUTONS

<u>Solutions</u>

Γ	1.		The function
	Х	У	is linear
	1	1	because the
	2	2	first
	3	3	differences
	4	4	are the same.
	5	5	.
	6	6	Equation: y=x
	7	7	
	8	8	
	9	9	
	10	10	

У	The
20.6	function is
30.4	quadratic
30.4	because
20.6	the second
1	differences
h	are the
	same
	Equation:
	VARIES
	20.6 30.4 30.4 20.6

	3.		
	Х	У	The function is
	0	800	exponential
	1	856	because it is
	2	915.92	neither linear
	3	980.03	nor quadratic
	4	1048.64	(first and
	5	1122.04	second
'	·	· · · · · · · · · · · · · · · · · · ·	differences
			reveal no
			patterns).
			Equation:
			y=800(1.07) ^x
L			

4.		
X	У	The function
0	16 000	is exponential
1	13	because it is
	171.37	neither linear
2	10	nor quadratic
	167.99	(first and
3	6979.37	second
4	3594.08	differences
5	0	reveal no
L		patterns).
		Equation:
		VARIES

5.		
X	У	The function is
0	600	linear because first
1	618	differences are the
2	636	same.
3	654	
4	672	Equation: y=18x+600

1.

Mice				
Yr V	Pop.			
1999	23 576 √			
2000	22 986.6			
2001	22 411.9 √			
2002	21 851.6			
2003	21 305.3			
2004	20 772.7			
2005	20 253.4			
2006	19 747.1			
2007	19 253.4			

Cats				
Yr. V	Pop.			
1999	15 786 V			
2000	16 060.1 🕡			
2001	16 359.4			
2002	16 653.9			
2003	16 953.6			
2004	17 258.8			
2005	17 569.5			
2006	17 885.7			
2007	18 207.7			

[28]

Mice first differences do not reveal any parter. Mice second differences do not reveal any parter. Mice population is decreasing.

Exponential Decay (Show constant Vti V

Cats first differences do not reveal any plate in Cats second differences do not reveal any vitt in Cat population is increasing.

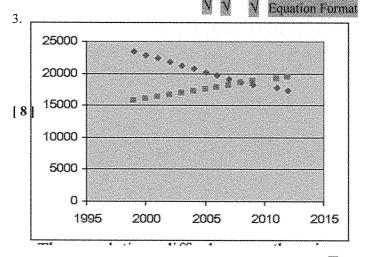
Exponential Growth (Show constant vary)

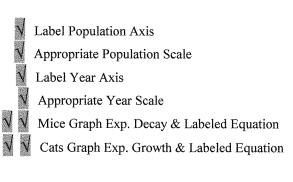
2. Mice: $y = 23576(0.975)^x$ because 23576 is the initial population, while it decreases at a rate of 0.975.

[6]

Equation Format

Cats: $y = 15786(1.018)^x$ because 15786 is the initial population, while it increases at a rate of 1.018



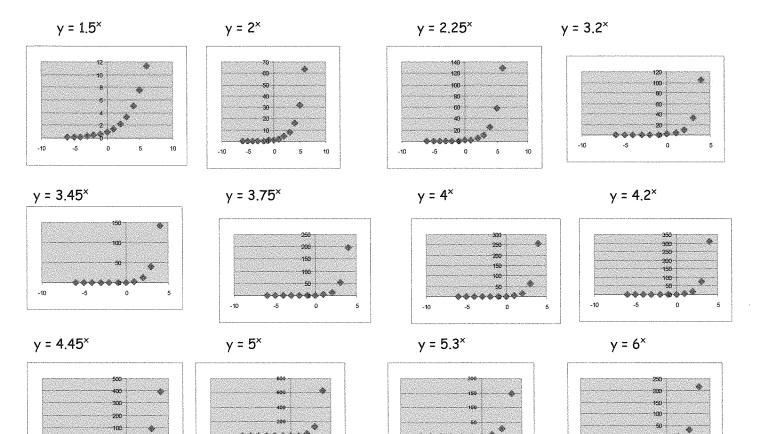


- 4. The populations differ because the mice population decreases, while the cat population increases. They are related because the cat population is the large cause for the decrease in the mice population.
- [3]
 The population of the cats will be greater than the population of the mice 2009.
- The populations would be the same sometime between 2008 and 2009 because in 2008 the mice population (18772) will be greater than the cat population (18535.4), but in 2009 the mice population (18302.7) will be less than the cat population (18869).

 Explanation Why?
- [2]
 Eventually the mice population will decline to less than one (zero), and the cat population will continue to increase.
 [2]
- 8. ANSWERS WILL VARY Write a brief paragraph summarizing your findings regarding the mice and cat populations.

[2]

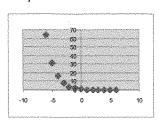
Exponential Growth Investigation



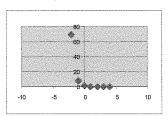
U03A12-SOLUTIONS

Exponential Decay Investigation

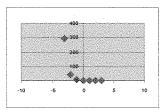
$$y = 0.5^{x}$$



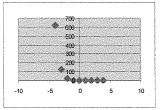
$$y = 0.12^{x}$$



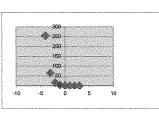
$$y = 0.15^{\times}$$



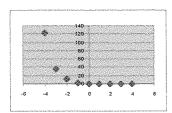
$$y = 0.2^{\times}$$

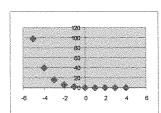


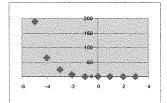
$$y = 0.25^{\times}$$



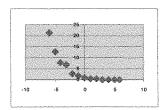
$$y = 0.3^{\times}$$







$$y = 0.6^{\times}$$



U04A02-SOLUTIONS

1. a) 0.06

b) 0.045

c) 0.0125

d) 0.0085

e) 0.32

2. a) 1.5

b) 0.30769

c) 0.24109

d) 4

e) 1

3 Complete the following chart.

Principle (\$)	Interest rate %	Time	Interest Earned (\$)	Total Amount (\$)
2000	4.5	3 months	22.5	2022.5
550	0.5	36 months	8.25	558.25
1500	1.5	14.2222	320	1820
4513.89	7.2	16 weeks	100	4613.89
2500	7.33333	18 months	275	2775
1239.20	6.75	240 days	55	1294.20
10000	10.8333	6 weeks	125	10125
780	1.3	5.72	58	

4. \$45

5. \$14.58, \$514.58 in total **6**. \$4974.36

7. 33.33%

8. 8.14 years

9. \$198.80 earned, total investment: 4698.80

10. a) \$600 is invested at 4% simple interest for 2 years. \$48 b) yes c) yes, if interest still held at 4%

U04A04-SOLUTIONS

1.(a) \$503.01 b) \$503.00

2) Complete the charts and graphs for the following information.

a) b) same each year

Year	Principle	Interest	Total amount
1	700	63	763
2	700	63	826
3	700	63	889
4	700	63	952
5	700	63	1015
6	700	63	1078
7	700	63	1141
8	700	63	1204
9	700	63	1267
10	700	63	1330

3) \$700 is invested at 9% interest compounded annually for 10 years

a)

b) It increases each year

c) Exponential Growth

c) linear growth

Year	Principle	Interest	Total amount
1	700	63	763
2	763	68.67	831.67
3	831.67	74.85	906.52
4	etc	etc	988.11
5			1077
6			1174
7			1279.6
8			1394.8
9			1520.3
10			1657.2

e) When you are paying interest on a loan.

U04A05-SOLUTIONS

1. Complete the charts and graphs for the following information.

a) \$2000 is invested at 7.5% simple interest for 10 years.

	a) \$2000 15 111005100 01 7:070 51111510 111101 001 701 10				
Year	Principle	Interest	Total amount		
1	2000	150	2150		
2	2000	150	2300		
3	2000	150	2450		
4	2000	150	2600		
5	2000	150	2750		
6	2000	150	2900		
7	2000	150	3050		
8	2000	150	3200		
9	2000	150	3350		
10	2000	150	3500		

b) \$2000 is invested at 7.5% interest compounded annually for 10 years.

Year	Principle	Interest	Total amount
1	2000	150	2150
2	2150	161.25	2311.3
3	Etc	etc	2484.6
4			2670.9
5			2871.3
6			3086.6
7			3318.1
8			3567
9			3834.5
10			4122.1

U04A08-SOLUTIONS

1) a) 1122.82 b) 1049.18

c) 1627.85

c) 3

2. a) 0.0125

b) 0.0015

b) 156

c) 0.001041667

d) 0.0016153846

e) 13

e) 0.00000136986

f) 21

f) 0.12

3. a) 24 Δ

Principle	Interest rate	Time Invested	Compounding Frequency	Amount	Interest Earned
\$300	2.3%	18 months	Semi-annually	310.47	10.47
\$1200	1.25%	2 years	Weekly	1230,37	30.37
\$1575	0.75%	85 days	Daily	1577.75	2.75
\$870	18%	3.5 years	Quarterly	1611.19	741.19
\$14000	5.45%	9 months	Annually	14568.44	568.44

d) 5

5a) \$83.63 **b)** \$157.01

c) \$10.01

6a) \$82.69

b) \$156.74 **c)** \$10.00

7a) \$84.52

b) \$157.26

c) \$10.01

8) \$699.52

9) \$836.54, \$36.54 is interest

U04A09-SOLUTIONS

- 1a) \$2010.22 b) \$992.49 c) \$300.08 d) \$75.12
- 2. Investment A \$1000 grows to \$1060.9 investment B it only grows to \$1045.7... in one year!)
- 3. \$603.66
- 4. \$7153.84
- 5. Steven (\$13693.18) made 2253.14 more than Brett (\$11441.04)
- 6. **6a)**\$3698.78 **b)** \$3216.87 **c)**\$481.91
- **7**. \$39569.05

U04A10-SOLUTIONS

Using the TVM Solver

1a) State the sequence of key strokes to get to the TVM Solver b) ALPHA, ENTER c) FV - value at end, PV value at beginning (usually negative) d) PMT e) 4

2.

Question	N	I%	PV	PMT	FV	P/Y	C/Y
(a)	14	3.5	-600	0	?	1	2
(b)	?	2.3	-4000	0	6000	1	12
(c)	20	?	-10000	0	20000	1	4
(d)	52	0.8	?	0	650	1	52

3. 4.1153% 4a) 9.5648% b) 9.53952% c) 9.536488% 5a) 138.8 years b) 34.8 years c) 12.8 years d) 7.9 years e) 3.6 years 6a) \$2923.17 b) \$2848.31 c) \$2704.28 d) \$2314.45

U04A11-SOLUTIONS Online Assignment: Part 1: Investment alternatives

- 1. Chequing used for daily banking and managing of money (paying bills etc) Savings used to build an amount of money to be used at a later date
- 2a) Youth account b) GIA account c) Interac withdrawal fees, international fees, bill payment fees d) 0.25%
- e) 3.00% f) **answers may vary** the Youth account is geared towards a younger clientele
- 3) \$0.04 interest earned. 4a) Guaranteed Investment Certificate b) There is very low or no risk c) At the bank 5a) Wait and See GIC, this allows access to the cash b) She could use a 5-year GIC to take advantage of the high interest rate, or a Triple Value has the best effective interest rate. c) 1 year Long Term GIC or a Money Market GIC. These allow him to keep his money in the account and give a good interest rate. 6a) \$3082.5 b) \$11905.58
- 7. You get a better rate of interest. For some people, not being able to "handle" it removes the temptation of spending it. 8a) Whereas stocks give investors part ownership of a company, bonds are loans made by investors to corporations or governments. b) Use the website: www.tsx.com 10. 500 multiplied by the share price 11a) \$14.552
- b) Probably not as it is not a lot of money lost and the stock could rebound. Other factors to consider: the way the market is going, any news about the company, if the economy is strong, etc

U04A12-SOLUTIONS Part 2: Credit Cards

- 1a) 19.75% b) TD GOLD Travel VISA, TD GOLD Elite VISA, TD Emerald VISA, TD USD Advantage VISA
- c) There are a lot of benefits with the Elite card, as well, perhaps the person does not want to buy a GM carl 2a) someone who does a lot of traveling, especially for business and can use the travel rewards later.
- b) Someone who performs a lot of transactions and may not pay off their bill all the time a) Someone who
- b) Someone who performs a lot of transactions and may not pay off their bill all the time c) Someone who wants a credit card, but without the annual fee

Credit card bills

3a) Previous balance - the amount of your last bill b) Credit limit - the maximum amount of money you can charge to your card c) Available credit - the credit limit minus your current balance (how much more you can spend) d) New balance - how much you owe the company right now e) Minimum payment - the least amount of payment they are willing to accept. This doesn't mean you won't be charged interest! F) Billing date (due date) - when the bill should be paid 4)\$185.37 5a) \$18.67 b) \$2778.10 c) \$3101.02 \$341.59 \$3354.42 He would owe \$2935.89 on his low-rate card which is \$418.53 less than the other card. Taking into account the \$120 annual fee, he would have saved \$298.53

6a) People who always pay off their bills b) People who hold a balance (do not pay off their bills)

Credit Rating

- 1) Go to the link for "credit education centre"
- a) A credit report is a summary of your financial history and performance (paying bills on time, etc) with organizations that lend you money (auto dealers, banks, credit cards, etc)
- b) Personal Identification, Inquiries, Public Record Information, Third-Party Collection Agency, Trade Information,

Consumer Statement, Credit Rating

a) always pay off your credit card bills, do not overextend yourself when you purchase a car or home,

Car Project

Part 1: Costs associated with owning a vehicle

- 1) ** will vary by student** Insurance, 1 (a) The amount of money you pay out at the beginning of a claim.
- b) The premiums increase a) age of driver, make of car, gender, driving record b) ** will vary by student** c) a young male with a poor driving record who is trying to insure a very expensive car

<u>Gasoline</u> 1a) ** will vary by student** b) ** will vary by student** c) ** will vary by student ** d) ** will vary by student e) There would be less stops and starts on the highway. The start up and initial acceleration usually uses the most gas f) ** will vary by student ** q) ** will vary by student ** h) ** will vary by student **

Other Expenses 1a) every 3 months or every 5000 km b) ** will vary by student ** c) ** will vary by student ** 2a) every year. b) \$74.00 c) MTO offices, kiosks in malls (Oshawa Centre) d) every other year (odd # year for even # year make of cars). Every year for older models of cars. e) ** will vary by student **

Part 2: Buying Vs Leasing

- 1a) Buying he would probably go over his km limit. b) Leasing lower monthly payment c) Buying when the vehicle is older he/she can do the repairs (when it is off warranty) d) Leasing can change your vehicle after 4 years or so
- e) Buying it's your car so you can do what you want to it. 2. ** will vary by student **

<u>Financing:</u> 1a)** will vary by student ** b) ** will vary by student ** ** will vary by student **

Leasing:

- 1. Go to www.canadiandriver.com. From the menu at the left, choose "calculators" and follow the steps to calculate how much a monthly lease would be for the car you chose. Use a 5 year (60 month) term, and a \$5000 down payment.
- a) ** will vary by student ** b) ** will vary by student ** \$1200

Part 3: New or Used Car

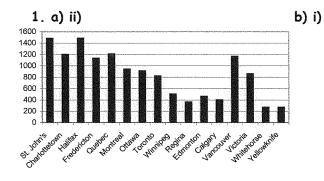
1. Used - usually you will lose a lot of money if you sell a new car in a short amount of time. a) New - can get exactly what he wants b) New - warranty will give some security for now and will have a longer lifespan c) Used - won't be used much so why spend \$ on new when used will do?

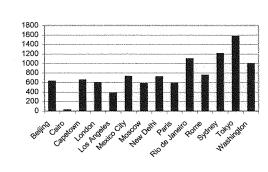
Depreciation: 2a)** will vary by student **

MBF3C U06A01-SOLUTIONS

1. a) i)

Annual average precipitation (mm)	Frequency (f)
St. John's	1482
Charlottetown	1201
Halifax	1474
Fredericton	1131
Quebec	1208
Montreal	940
Ottawa	911
Toronto	819
Winnipeg	504
Regina	364
Edmonton	461
Calgary	399
Vancouver	1167
Victoria	858
Whitehorse	269
Yellowknife	267



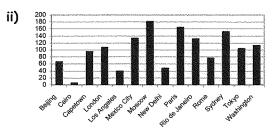


Annual average	Frequenc
precipitation (mm)	<u>У</u> (f)
Beijing	623
Cairo	22
Capetown	652
London	594
Los Angeles	373
Mexico City	726
Moscow	575
New Delhi	715
Paris	585
Rio de Janeiro	1093
Rome	749
Sydney	1205
Tokyo	1563
Washington	991

c) i)

Number of wet days (x)	Freq. (f)	Number of wet days (x)	<u>Freq.</u> (f)
Beijing	66	New Delhi	47
Cairo	5	Paris	164
Capetown	95	Rio de Janeiro	131
London	107	Rome	76
Los Angeles	39	Sydney	152
Mexico City	133	Tokyo	104
Moscow	181	Washington	112

b) ii)

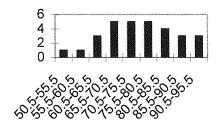


3. Not organized or ranked; difficult to compare data.

4. a)

Number of	Frequency	Cumulative
pulses (x)	(f)	frequency
50.5-55.5	1	1
55.5-60.5	1	2
60.5-65.5	3	5
65.5-70.5	5	10
70.5-75.5	5	15
75.5-80.5	5	20
80.5-85.5	4	24
85.5-90.5	3	27
90.5-95.5	3	30

b)

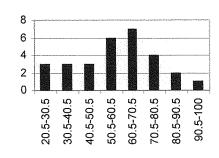


- **5**. **a)** 65.5-70.5; 70.5-75.5; 75.5-80.5 **b)** 50.5-55.5; 55.5-60.5 **6**. 20%
- 7. a) 50; Determine the percent frequency of the interval and multiply by the total number of people.
 - b) Answers may vary; for example: pulses do not change.

8.

Grades (x)	Frequency (f)
20.5-30.5	3
30.5-40.5	3
40.5-50.5	3
50.5-60.5	6
60.5-70.5	7
70.5-80.5	4
80.5-90.5	2
90.5-100	1

9. a)



b) 60.5-70.5

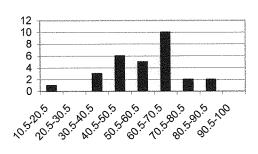
10. a) 10.3%

b) 31.0%

11. i)

Grades (x)	Frequency (f)
10.5-20.5	1
20.5-30.5	0
30.5-40.5	3
40.5-50.5	6
50.5-60.5	5
60.5-70.5	10
70.5-80.5	2
80.5-90.5	2
90.5-100	0

ii)



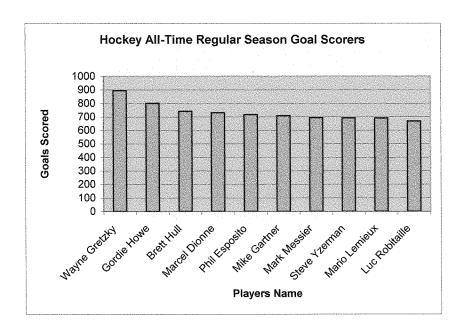
iii) 60.5-70.5

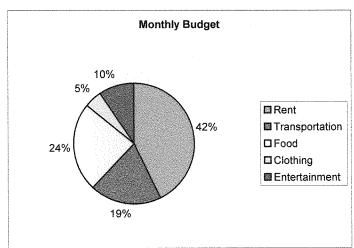
- 12. a) Answers may vary; for example: second test resulted in lowest grade (14).
 - b) Answers may vary; for example: failure rate. c) Answers may vary.

U06A02-Solutions

- a) answers may vary; for example: easy to conduct. b) answers may vary; for example: may not be representative of entire population. c) answers may vary; for example: interview, case study.
 a) sample b) census c) sample d) sample e) census (not all tires have the same pressure necessarily) f) sample
- 2. a) random; variety in responses will reflect different viewpoints
- 3. a) no; for example: 20 is not a representative value of 10 000. b) 1.5% c) 150; based on 3rd method which is more accurate.
- 4. a) convenience b) answers may vary; for example: gather helpful ideas; is not time consuming to conduct.
- 5. a) approx. 27 b) 20 c) stratified
- 6. answers may vary a) yes; samples will be proportional to the total constituents b) no; ineffective c) yes; represents both sexes fairly d) no; director should consult board members and those directly related e) no; use other sample technique
- 7. a) P b) F c) P d) F

MBF3C **U06A03-Solutions**





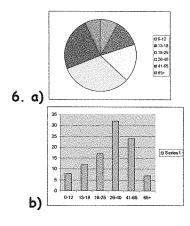
U06A04-Graphing Solutions

- 1. a) continuous
- b) discrete
- c) discrete
- d) continuous

- 2.a) categorical
- b) categorical c) discrete
- d) continuous

- e) discrete
- f) discrete
- g) discrete
- h) continuous

- 3.c) length of pages
- 4.a) 0 b) 1
- 5. categorical



7. Answers will vary

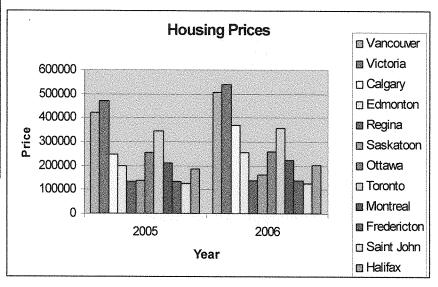
U06A05 Types of Distributions Solutions

- 1. a) normal b) uniform c) right-skewed (discuss why it may appear to be normal) d) left-skewed
- 2. answers may vary; for example: c) immigration: i) From which country have you originated? ii) What is the main reason for which immigrants leave their place of origin? iii) What is the main reason for which immigrants come to Canada? iv) Which province/territory do you think would provide the greatest opportunity for an immigrant? v) In what way do you think that immigration benefits the general Canadian population?
- 3. i) Answers may vary; for example:
 - a) Identify problem/question does the addition of the HOV (Higher Occupancy Vehicle) lane really reduce congestion on the 401?
 - b) Hypothesis the HOV lane does not reduce congestion on the 401 because people are still driving by themselves to work each day. (added to help students focus on which methods would help them with their study)
 - c) Collect data interview, survey, experiment, case study or observation.
 (i.e. interview target regular commuters only, giving them a multiple-choice questionnaire that allows them to choose a pre-determined time spent commuting each day prior to and after the addition of the HOV lane, and limit the number of respondents to 100 sample with random sampling from the GTA (Greater Toronto Area))
 - d) Analyze the data use bar graph to organize results and determine the degree of distribution
 - e) Conclusion determine if data supports the hypothesis and answers the original question
 - ii) Answers may vary slightly; for an experiment or observation, peak times would be morning and evening rush hour, i.e. 6am-9am and 4pm-7pm. For an interview, contact commuters between 7pm and 8pm.
 - iii) Answers may vary; for example: interview the interviewee would be restricted to select a predetermined response (i.e. multiple-choice). Responses would be tabulated in a spreadsheet and then organized into a bar graph.
 - iv) Answers may vary slightly; for example: bar graph compare height of bars 'prior to' and 'after' addition of HOV lane, using the properties of the distribution that may indicate if there are any relationships/correlations
 (emphasize that the sampling must ensure no bias).

MBF3C **U06A09 Solutions**

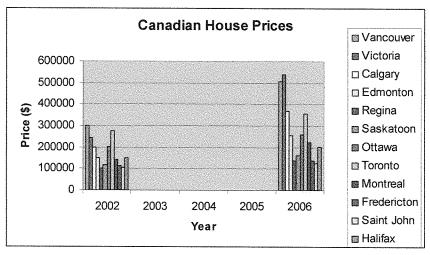
2006 vs. 2005

Canadian City	Jun-06	Jun-05
Vancouver	508435	422843
Victoria	538913	469588
Calgary	367033	245803
Edmonton	254240	199409
Regina	137022	132054
Saskatoon	160548	139728
Ottawa	260458	254725
Toronto	358035	345065
Montreal	222879	210740
Fredericton	136371	134334
Saint John	127586	125455
Halifax	201316	184853
Mean	272736.3	238716.4
Median	238559.5	205074.5
Range	411327	344133
Standard		
Deviation	141844.9	116365.1



2006 vs. 2002

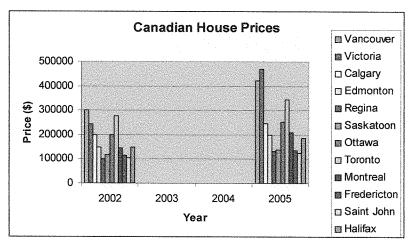
Canadian City	Jun-06	Jun-02
Vancouver	508435	301473
Victoria	538913	242503
Calgary	367033	198350
Edmonton	254240	150165
Regina	137022	100751
Saskatoon	160548	118999
Ottawa	260458	200711
Toronto	358035	275975
Montreal	222879	143589
Fredericton	136371	114185
Saint John	127586	104052
Halifax	201316	148737
Mean	272736.3	174957.5
Median	238559.5	149451
Range	411327	200722
Standard		
Deviation	141844.9	68509.17



MBF3C **U06A09-P2 Solutions**

2005 vs. 2002

Canadian City	Jun-05	Jun-02
Vancouver	422843	301473
Victoria	469588	242503
Calgary	245803	198350
Edmonton	199409	150165
Regina	132054	100751
Saskatoon	139728	118999
Ottawa	254725	200711
Toronto	345065	275975
Montreal	210740	143589
Fredericton	134334	114185
Saint John	125455	104052
Halifax	184853	148737
Mean	238716.4	174957.5
Median	205074.5	149451
Range	344133	200722
Standard		
Deviation	116365.1	68509.17



MBF3C

U06A10 Solutions

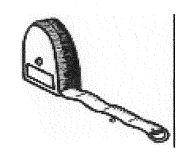
- 1. Answers may vary depending on the topic chosen: e.g. Topic idea: "Wish to report on number of car crashes at a particular intersection in a particular town." First step do some research on the particular intersection → police reports (if possible) about accidents at that intersection, include variables in the study → weather conditions, number of cars in crash, time of day, direction of vehicles involved, direction of the vehicle charged in the incident. Next step: Perform survey of the intersection, count traffic through the intersection, number of vehicles, direction of vehicles, time of day, if any crashes occur during the survey include relevant data in this survey. Next step: analyse the results, Based on the results report the information learned to the appropriate people.
- 2. Answer will vary again based on articles found \rightarrow results will be similar to the investigation performed in class as well as the example based on GM OnStar statistics outlined in class as an example.
- 3. Answer will vary based on articles → results will be similar to the investigation performed in class as well as the example based on GM OnStar statistics outlined in class.

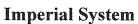
Answer will vary based on articles \rightarrow you should be able to describe how the media has misrepresented their information. Possible methods are: using inappropriate graph types for the information given, using sample sizes that are too small for the population they are attempting to represent, not giving enough information like total number of people involved in the survey compared to the population being represented, not using proper scales on the axes of the graph given, etc.

Conversions and Units

Metric System

TATCH IC DASH		
mm	millimeter	0.001 m
cm	centimeter	0.01 m
dm	decimeter	0.1 m
m	meter	1 m
dam	decameter	10 m
hm	hectometer	100 m
km	kilometer	1000 m



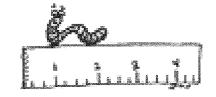




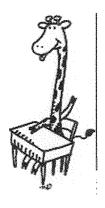
1 foot (ft) = 12 inches (in)	1 tablespoon $(T) = 3$ teaspoons (t)			
1' = 12"				
1 yard $(yd) = 3$ feet	1 cup (c)	= 16 tablespoons		
1 mile (mi) = 1760 yards	1 pint (pt)	= 2 cups		
1 sq. foot $= 144$ sq. inches	1 quart (qt)	= 2 pints		
1 sq. yard $= 9$ sq. feet	1 gallon (gal)	= 4 quarts		
1 acre $= 4840$ sq. yards	16 ounces (oz)	= 1 pound (lb)		
$= 43560 \text{ ft}^2$				
1 sq. mile $= 640$ acres	1 ton	= 2000 pounds		

Conversions

From	to	Multiply by	From	to	Multiply by
cm	in	0.3937	in	cm	2.54
m	ft	3.2808	ft	m	0.3048
km	mi	0.6214	mi	km	1.609
m^2	ft^2	10.76	ft ²	m ²	0.0929
km ²	mi ²	0.3861	mi ²	km ²	2.59



Conversions



From	to	Multiply by	From	to	Multiply by
g	oz	0.0353	oz	g	28.35
kg	lb	2.2046	lb	kg	0.4536
t	T	1.1023	Т	t	0.9072
ml	fl oz	0.0338	fl oz	ml	29.575
L	gal	0.2642	gal	L	3.785

IMPERIAL & METRIC CONVERSIONS

Units of Length				
Metric Conversions	Imperial Conversions			
1 cm = 10 mm	1 foot = 12 inches			
1 m = 100 cm	1 yard = 3 feet			
1 km = 1000 m	1 mile = 1760 yards			
1 mm = 0.1 cm	1 yard = 36 inches			
	1 mile = 5280 feet			
Imperial to Metric	Metric to Imperial			
1 inch = 2.54 cm	1 cm = 0.3937 inch			
1 foot = 30.48 cm	1 m = 39.37 inches			
1 foot = 0.3048 m	1 m = 3.2808 feet			
1 mile = 1.6093 km	1 km = 0.6214 mile			

Units of Volume or Capacity

Metric Conversions	Imperial Conversions	
1 mL = 1 cm ³	1 gallon = 277.42 cubic inches	
1 L = 1000 cm ³	1 cubic foot = 6.2288 gallons	
1 m ³ = 1000 L	1 pint = 16 fluid ounces	
1000 mL = 1 L	1 quart = 1 pint	
	1 gallon = 8 pints	
	1pound = 16 ounces	
	1 US ton = 2000 pounds	

Imperial to Metric	Metric to Imperial
1 fluid ounce = 28.4131 mL	1 mL = 0.0352 fluid ounce
1 pint = 0.5683 L	1 L = 1.7598 pints
1 quart = 1.1365 L	1 L = 0.8799 quart
1 gallon = 4.5461 L	1 L = 0.2200 gallon
1 gallon = 4546.1 cm ³	
1 US gallon = 3.785 L	

Imperial to Metric	Metric to Imperial
1 fluid ounce = 28.4131 mL	1 mL = 0.0352 fluid ounce
1 pint = 0.5683 L	1 L = 1.7598 pints
1 quart = 1.1365 L	1 L = 0.8799 quart
1 gallon = 4.5461 L	1 L = 0.2200 gallon
1 gallon = 4546.1 cm ³	
1 US gallon = 3.785 L	
1 ounce = 28.35 g (mass)	
1 pound = 0.454 kg (mass)	
1 US ton = 0.907 tonnes (mass)	

Area	Mass	Time
1 ha = 10 000 m²	1 mg = 1000 μ g (micrograms)	1 h = 60 min.
1 acre = 43 560 square feet	1 kg = 1000 g	1 min. = 60 seconds
1 square yard = 9 square feet	1 t = 1000 kg	
	1 g = 0.001 kg	

Distance

Imperial to Metric

From	То	Multiply By
Inches	Centimeters	2.54
Feet	Centimeters	30.48
Feet	Meters	0.3048
Miles	Kilometers	1.609
Yards	Meters	0.9144

Imperial to Imperial

Foot	Inches	12
Foot	Yard	0.3333
Yard	Foot	3
Mile	Foot	5280
Mile	Yard	1760

Area

Imperial to Metric

From	То	Multiply By
Square Inch	Square cm	6.45
Square Foot	Square Meter	0.0929
Square Yard	Square Meter	0.8361
Acre	Hectare	0.4047
Square Mile	Square Km	2.59

Imperial to Imperial

Square Meter	Square foot	9
Acre	Square Foot	43,560
Acre	Square Yard	4,840

Mass (Weight)

Imperial to Metric

From	То	Multiply By
Ounce	Gram	28.35
Pound	Kilogram	0.454
Ton	Metric Tonne	0.907

Imperial to Imperial

Pound	Ounce	16
Ton	Pound	2,000

Volume

Imperial to Metric

From	То	Multiply By
Cubic Inch	Cubic Cm	16.39
Cubic Foot	Cubic Meter	0.02832
Cubic Yard	Cubic Meter	0.7646
Ounce	Millilitre	29.574
Cup	Litre	0.2366
Gallon	Litre	3.785
Quart	Litre	1.136

Imperial to Imperial

Tablespoon	Teaspoon	3
Ounce	Tablespoon	2
Cup	Ounce	8
Pint	Cup	2
Quart	Pint	2
Gallon	Quart	4

Metric to Imperial

From	То	Multiply By
Centimeters	Inches	0.3937
Centimeters	Feet	0.0328
Meters	Feet	3.28
Kilometers	Miles	0.62
Meters	Yards	1.09

Metric to Metric

Centimeter	Millimeter	10
Meter	Centimeter	100
Kilometer	Meter	1000

Metric to Imperial

From	То	Multiply By
Square cm	Square Inches	0.155
Square Meter	Square Foot	10.76
Square Meter	Square Yard	1.196
Hectare	Acre	2.47
Square Km	Square Mile	0.386

Metric to Metric

Square Meter	Square Cm	10,000
Hectare	Square Meter	10,000

Metric to Imperial

From	То	Multiply By
Gram	Ounce	0.0353
Kilogram	Pound	2.2
Metric Tonne	Ton	1.103

Metric to Metric

Kilogram	Gram	1,000
Metric Tonne	Kilogram	1,000

Metric to Imperial

From	То	Multiply By
Cubic Cm	Cubic Inch	0.061
Cubic Meter	Cubic Foot	35.31
Cubic Meter	Cubic Yard	1.3
Millilitre	Ounce	0.0338
Litre	Cup	4.23
Litre	Gallon	0.2642
Litre	Quart	0.88

Metric to Metric

I	Litre	Millilitre	1,000		
	Cubic Metre	Litre	1,000		

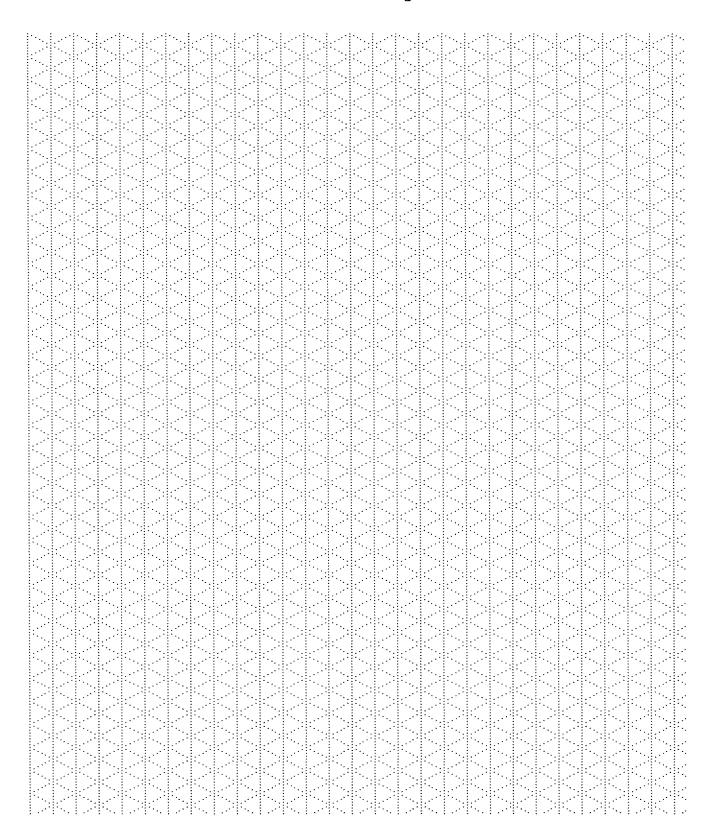
Conversion Tables

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MBF 3C Lesson

Isometric Drawings



Formula Sheet

Grade 9 Academic

Geometric Figure	Perimeter	Area/Surface Area
Rectangle	P = l + l + w + w or $P = 2(l + w)$	A = lw
Parallelogram	P = b + b + c + c or $P = 2(b + c)$	A = bh
Triangle a h c b	P = a + b + c	$A = \frac{bh}{2}$ or $A = \frac{1}{2}bh$
Trapezoid a d b	P = a + b + c + d	$A = \frac{(a+b)h}{2}$ or $A = \frac{1}{2} (a+b)h$
Circle	$C = \pi d$ or $C = 2\pi r$	$A = \pi r^2$

Geometric Figure	Area/Surface Area	Volume
Cylinder	$A_{ m base} = \pi r^2$ $A_{ m lateral\ surface} = 2\pi r h$	$V = (A_{\mathrm{base}})(\mathrm{height})$
h	$A_{ m total} = A_{ m 2\ bases} + A_{ m lateral\ surface} \ = 2\pi r^2 + 2\pi r h$	$V = \pi r^2 h$
Sphere	$A = 4\pi r^2$	$V = \frac{4}{3} \pi r^3 \qquad \text{or} \qquad V = \frac{4\pi r^3}{3}$
Cone	$A_{ m lateral\ surface} = \pi r s$ $A_{ m base} = \pi r^2$	$V = \frac{(A_{\text{base}})(\text{height})}{3}$
	$A_{\text{total}} = A_{\text{lateral surface}} + A_{\text{base}}$ $= \pi r s + \pi r^2$	$V = \frac{1}{3} \pi r^2 h \qquad \text{or} \qquad V = \frac{\pi r^2 h}{3}$
Square- based pyramid	$A_{ m triangle} = rac{1}{2}bs$ $A_{ m base} = b^2$	$V = \frac{(A_{\text{base}})(\text{height})}{3}$
b	$A_{\text{total}} = A_{4 \text{ triangles}} + A_{\text{base}}$ $= 2bs + b^{2}$	$V = \frac{1}{3} b^2 h \qquad \text{or} \qquad V = \frac{b^2 h}{3}$
Rectangular prism	A = 2(wh + lw + lh)	V = (area of base)(height)
h l w		V = lwh
Triangular prism a / c	$A_{\mathrm{base}} = \frac{1}{2} bl$	$V = (A_{\mathrm{base}})(\mathrm{height})$
h	$A_{ m rectangles} = ah + bh + ch$ $A_{ m total} = A_{ m rectangles} + A_{ m 2 \ bases}$ $= ah + bh + ch + bl$	$V = \frac{1}{2}blh$ or $V = \frac{blh}{2}$

MBF 3C SUMMATIVE

Computer programmers have created reservation systems for theatres, airplanes, and concert halls that automatically assign the best available seats to ticket buyers. Suppose seats were assigned randomly instead. Read the following example...and you decide!

1. The seats in a concert hall are arranged in 10 rows with 10 seats in each row. The 4 seats in the middle of each row are separated from the three seats on the far left and the 3 seats on the far right by aisles.

a) How many seats are in the hall?

b) How many seats are there in the **front row**?

c) How many **aisle** seats are there?

d) How many seats are in the last 5 rows?

Suppose you are the first person to buy a ticket. You are randomly assigned a seat. Which type of seat are you most likely to get? Write your answers as a fraction, a decimal and a percent. (Your answer should look like this example: $\frac{1}{2} = 0.5 = 50\%$)

- a) a front row seat:
- b) an aisle seat:
- c) a seat in the last 5 rows:

Which seat assignment is more likely, or are they equally likely? Explain your thinking.

a) one of the two best seats in the theatre, in the middle of the 6th row:

b) one of the worst seats in the theatre, the three seats near the wall at either end of the front row.

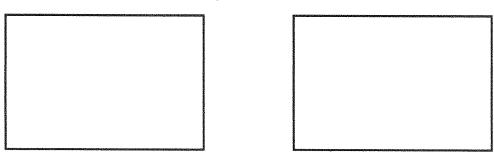
In real life, seats in a concert hall are not assigned randomly...thankfully!

2. A company is testing two egg carton designs to see which could better withstand a drop from a specified height. The two cartons were dropped 100 times each. The results are shown in the table. (Example: Carton A had 2 boxes that had 0 eggs broken.)

Number of Broken Eggs	Carton A	Carton B
0	2	0
1	12	5
2	22	27
3	28	36
4	25	28
5	8	3
6	3	1

a) Without calculating, which appears to be the better carton? Explain.

b) Draw a histogram – using the TI-83 - for the number of broken eggs in each carton. Which appears to be the better carton? Explain.



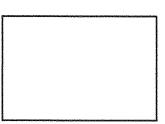
c) Calculate the mean and standard deviation for the number of broken eggs for each carton. Which appears to be the better carton? Why would mean and mode not be useful statistics in this case?

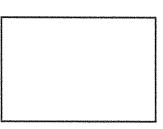
3. Three water storage tanks are allowed to drain. For each tank an equation giving the height, *h* metres, of the water in the tank is given as well as *t* for time in hours.

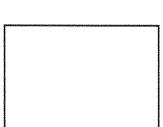
Tank A: $h = 20(0.95)^{t}$ Tank B: h = 20 - 0.5t

Tank C: $h = 0.0125t^2 - t + 20$

a) Classify each drainage pattern as linear, quadratic or exponential. Explain your thinking by showing your work – i.e. if you use tables of values, graphs, or TI-83 screens, show that.







b) Show mathematically how you know that the initial height of the water in each tank is 20 m.

Key Words by Unit

1. Quadratics

- Parabola
- Axis of symmetry
- Vertex
- Transformations
- Horizontal Translation
- Vertical Translation
- Reflection
- Vertical stretch
- Standard form
- Vertex form
- Factored form

2. Exponential Relations

- Base
- Exponent
- Power
- Exponential growth
- Growth factor
- Exponential decay
- Decay factor
- Initial value
- Exponential relations
- Doubling time
- Half-life

3. Personal Finance

- Interest
- Simple interest
- Principal (present value)
- Amount (future value)
- Compound interest
- Compounding period
- Debit cards
- Credit cards

4. Trigonometry

- Hypotenuse
- Adjacent side
- Opposite side
- Right triangle
- Sine
- Cosine
- Tangent
- Primary trigonometric ratios
- Acute angle
- Obtuse angle
- Angle of elevation/inclination
- Angle of depression
- Sine law
- Cosine law

5. Statistics

- One-variable data
- Discrete data
- Continuous data
- Categorical data
- Distribution
- Population
- Sample
- Bias
- Mean, Median, Mode
- Spread, Range, Standard Deviation

6. Probability

- Outcome
- Event
- Theoretical probability
- Sample space
- Trial
- Simulation



INSTR	UCT	IONS:
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All of the items 1-5

Choose ONE of the items 6-8

Choose ONE of the items 9-11

NAME:	

For your summative, you are to create a Portfolio.

Due:

YOU MUST SUBMIT THIS PAGE ALONG WITH A COVER PAGE.

Your portfolio must include:

Item #	Description	Submitted (check box)
1	A 2 page, double spaced type written description on "How the Math that I have learned so far in school is applicable to real life"	
2	All unit tests with the corrections done <u>on a separate piece of paper for each test</u> and stapled together.	
3	A Formula Sheet with all the formulas you will need for the exam	
4	A question you make up from each unit. You must have the question and the solution written underneath with all steps shown. Include a marking scheme for the question with a minimum of 4 – 6 marks for each unit. To help you, the units are listed below - Trigonometry - Statistics - Quadratics - Probability - Exponential Functions - Personal Finance	
5	A type written glossary of terms including definitions for the key words by unit. (see handout of "Key Words by Unit")	
6	Create a crossword puzzle with the course content. (min. 25 words) (Provide completed solution as well)	
7	Create a word search with the course content. (min. 25 words) (Provide completed solution as well)	
8	Create a three fold brochure to help students in the years to come understand how to use the graphing calculators for things such as resetting the memory, entering equations to graph, adjusting windows, setting up a table of values, graphing linear, quadratic and exponential equations, financial applications, etc	
9	Create a card game (ie. Go fish, or concentration) dealing with a topic covered in the course (ie. Exponential Functions, Personal Finance, etc)	
10	Create a review game for the entire course content (i.e. Jeopardy, Who Wants to Be a Millionaire, Wheel of Fortune). Include instructions to play the game and solutions to the questions.	
11	Create a piece of artwork that is inspired by course content (EG: a sketch of the golden gate bride that was quadratics inspired). Write a few sentences to describe how you were inspired and the content covered.	

Portfolio Evaluation

Completeness	5	All components of the portfolio are included in a neat , well organized fashion with a table of contents included.
	4	All components of the portfolio are included but not as well organized or neat.
	3	Most components of the portfolio are included (1 missing item)
	2	Several components of the portfolio are missing (2 missing items)
	1	The portfolio is largely incomplete (3 missing items)
	0	The portfolio is incomplete (4 or more missing items)
Written Answers	5	All components of written answers are completed. Student shows clarity of thought and insight . Typed and properly formatted.
	4	All components of written answers are completed. Student responses show evidence of thought and careful consideration. Typed but lacking in some formatting
	3.	Most components of written answers are completed or student responses show only <u>some</u> evidence of thought and careful consideration. Typed but poorly formatted.
	2	Several components of written answers are missing or student responses show little evidence of thought and careful consideration. Possibly not typed and very little formatting.
	1	The written answers are largely incomplete or student responses indicate little effort to complete.
	0	The written answers were not completed
Included Items	15 - 13	Included portfolio items indicate an excellent level of work or a significant improvement throughout the term or a <u>consistent effort to learn from previous evaluations</u>
	12 - 10	Included portfolio items indicate a good level of work or an improvement throughout the term and an effort to learn from previous evaluations
	9 - 7	Included portfolio items indicate a satisfactory level of work or an improvement throughout the term and some effort to learn from previous evaluations
	6 - 4	Included portfolio items indicate a satisfactory level of work or an improvement throughout the term and/or a minimal effort to learn from previous evaluations
	3 - 1	Included portfolio items indicate a poor level of work and/or no effort to learn from previous evaluations
	0	There were no portfolio items included.

TOTAL:

Anywhere Secondary School

Department of Mathematics

Name:

Date:

Time:

Course Title:

Grade 11 Collage Mathematics

Course Code:

MBF 3C

Sections:

Subject Teacher:

Instructions to Proctors:

- Please distribute foolscap for rough work.
- Please pick up all student work, including the foolscap pages. No student may leave with any paper.
- Scientific calculators, including graphing calculators, may be used but may not be shared.

Instructions to Students:

- There should be 9 pages in this exam package. Check now.
- The last page is with formulas, and may be detached.
- Answer all of the questions in the space provided. If necessary, use the back of the previous page. Clearly indicate where you have continued the question.
- Attempt every question, but budget your time carefully. Part marks will be given for correct partial solutions.
- Relax! Give your best work, with good form.

The following rubric will be used to evaluate the communication mark:

	Outstanding 10 9.5 8.9 8.4	Good 7.8 7.5 7.2	Acceptable 6.8 6.5 6.2	Poor 5.8 5.5 5.2	Unacceptable 4.0 3.0	Incomplete 0
TECHNICAL CORRECTN ESS OF SOLUTIONS	All or almost all solutions are technically correct	Most solutions are technically correct	Some solutions are technically correct	Few solutions are technically correct	No solutions are correct or many left blank	All or most solutions are blank
PRESENTAT ION OF SOLUTIONS	Solutions to <u>all</u> or <u>almost all</u> problems can stand alone	Solutions to most problems can stand alone	Solutions to some problems can stand alone	Solutions to few problems stand alone	No evidence of presentation or many solutions left blank	All or most solutions are blank

Section	Marks Earned	Marks Available
Section 1 – Multiple Choice		16
Section 2 – True / False		6
Section 3 – Full Solutions		62
Communication + Info Sheet		10
TOTAL		94

MBF 3C

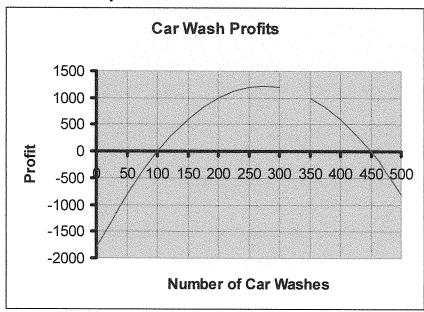
SECTION 1:

MULTIPLE CHOICE - 20 Marks

(Circle answers on your SCANTRON CARD)

Questions 1 to 5 refer to the example below

The fundraising director of a high school has compiled data about the school's car washes. The data was sent to be analyzed by the Mathematics Team and they produced a written report for the director. The report was emailed to the director but unfortunately, only the graph below was received by the director.



- 1. What is the maximum profit possible?
- (a) 275 cars
- (b) \$1500
- (c) \$100 and \$450
- (d) \$1225
- 2. How many cars need to be washed in order to make this profit?
- (a) 275 cars
- (b) 200 cars
- (c) 100 cars
- (d) 500
- 3. If the director would like to make \$1000 profit, how many cars need to be washed?
- (a) 200 cars
- (b) 350 cars
- (c) 200 or 350 cars
- (d) 275 cars
- 4. A company is said to "break even" when revenue equals expenses, or when profit equals zero. How many car washes are needed for the fundraiser to break even?
- (a) 275 cars
- (b) 100 cars
- (c) 100 or 450 cars
- (d) 0 cars
- 5. **[PROBLEM SOLVING]** Which of the following equations would best describe the above graph?

(a)
$$y = 0.5(x - 275)^2 + 1225$$

(b)
$$y = -275(x-100)^2 + 450$$

(c)
$$y = -0.04(x - 275)^2 + 1225$$

(d)
$$y = 2(x - 275)^2 + 1225$$

- 6. The factored form of $y = x^2 5x + 6$ is
- (a) y = (x 3)(x + 2)
- (b) y = (x-3)(x-2)
- (c) y = (x + 3)(x + 2)
- (d) y = (x-2)(x+3)
- 7. The equation which represents an exponential relationship is
- (a) $y = 2^x$
- (b) $y = x^2$
- (c) y = 2x
- (d) y = x + 2

- 8. An equivalent form of $\frac{1}{a^5}$ is
- (a) a^{5}
- (b) $-a^5$
- (c) $\frac{5}{a}$

(d) a^{-5}

Questions 9 to 13 refer to the example below

An RRSP is an investment offered by many financial institutions. In a particular RRSP, which is compounded monthly, the amount in dollars A in the RRSP after n months is given by the equation $A = 5000(1.005)^n$

9. What is the principal of the investment?

- (a) \$1.005
 - (b) \$5000
- (c) \$5025
- (d) \$0

10. What is the amount in the RRSP after 3 years?

- (a) \$5000
- (b) \$5075.38
- (c) \$5807.00
- (d) \$5983.40

11. What is the amount in the RRSP after 5.5 years?

- (a) \$6949.12
- (b) \$5139.06
- (c) \$61 668.67
- (d) \$5983.40

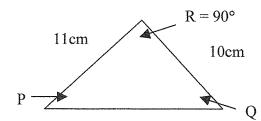
12. How much interest will the RRSP have earned in one year?

- (a) \$5308.39
- (b) \$5025.00
- (c) \$25.00
- (d) \$308.39

13. What is the annual interest rate (compounded monthly) of this RRSP?

- (a) 0.5%
- (b) 5%
- (c) 6%
- (d) 0.06%

Questions 14 to 16 refer to the triangle below



14. The measure of the missing side is

- (a) 221cm
- (b) 14.87cm
- (c) 21cm
- (d) 1cm

15. The measure of the angle at vertex P is

- (a) 42.3°
- (b) 43.2°
- (c) 47.7°
- (d) 132.3°

16. The measure of the angle at vertex Q is

- (a) 42.3°
- (b) 43.2°
- (c) 47.7°
- (d) 132.3°

SECTION 2: TRUE AND FALSE – 6 Marks

(On your SCANTRON CARD, select A for TRUE, B for FALSE)

TRUE FALSE

- A B
- 1. The first differences are the same in a linear relation.
- A B
- 2. The first differences are the same in a quadratic relation.
- A B
- 3. The second differences are the same in an exponential relation.
- A B
- 4. The graph of $y = 3^x$ has a y-intercept of (0, 1)
- A B
- 5. The parabola y = 3(x 4)(x + 2) has x intercepts at (-2, 0) and (4, 0)
- A B
- 6. Doubling the interest rate will half the time it takes for an investment to double its value.

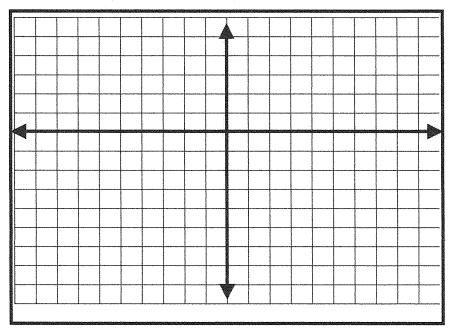
SECTION 3:

FULL SOLUTION - 62 Marks

(Answer on the exam paper)

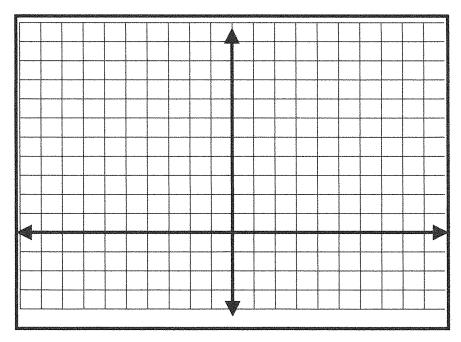
- 1. Let a parabola be defined by the equation $y = 2(x 3)^2 8$
- (a) graph the parabola on the grid provided

[3 marks]



- (b) write the equation of the parabola in Standard Form [3 marks]
- (c) factor the expression in (b) and write the equation of the parabola in Factored Form [2 marks]
- 2. Using the grid below, graph the equation $y = 3^x$

[2 marks]



3. Evaluate the following.

[3 marks]

(a) 2^{-4}

(b) 1.03⁴

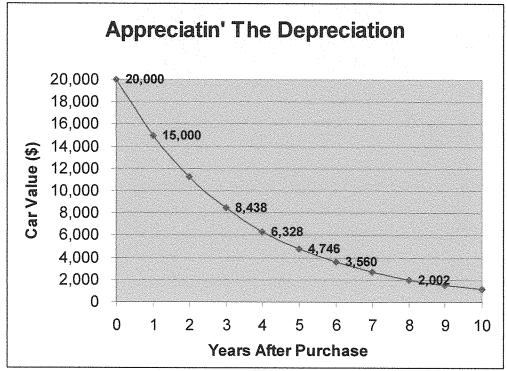
- (c) 14.432^0
- 4. Evaluate, but do not write as a decimal.

[3 marks]

(a) 5^{-2}

(b)
$$\left(\frac{2}{3}\right)^2 \times \left(\frac{1}{5}\right)^3$$

5. The graph below shows the projected value of a car over the next 10 years.



- (a) {COMMUNICATING} Does this graph show traits of a linear relationship, quadratic relationship, or exponential relationship? Explain your reasoning. [2 marks]
- (b) Using the graph, estimate the car's value

[4 marks]

- a. originally
- b. after 2 years
- c. after 7 years
- d. after 10 years
- (c) Based on the graph, what percent of its value does the car lose each year?

[1 mark]

6. Simplify the following. Write each answer as an expression with a single positive exponent. [3 marks]

(a)
$$4^4 \times 4^5$$

(b)
$$(3^2)^6$$

(c)
$$\frac{2^4 \times 2}{2^9}$$

- 7. (a) How much money is needed in a bank account (which pays 4.8%/a compounded monthly) in order to have \$5000 after 4 years? [2 marks]
 - (b) How much interest was earned in this situation?

[1 mark]

8. You are trying to explain to a friend the power of compound interest. So you use the following example: You will invest \$500 into two bank accounts which each pay 6% interest per year, but the first (Account A) pays simple interest and the second (Account B) pays the interest compounded yearly. How much is in each account after 3 years?

Use the table below to demonstrate the difference between simple and compound interest. [3 marks]

BANK ACCOUNT A			
Year	Amount in Account		
0	\$500		
1			
2			
3			

BANK ACCOUNT B				
Year	Amount in Account			
0	\$500			
1				
2				
3				

- 9. Many financial experts advise that the first step to becoming financially "secure" is to eliminate all credit card debt. They even suggest adding a credit card debt to another loan, such as a mortgage or a line of credit, in order to eliminate the credit card debt.
- (a) Why is it so important to eliminate credit card debt?

[1 mark]

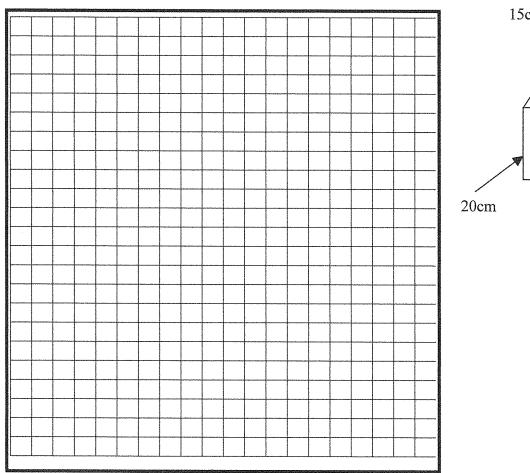
- (b) Some credit cards offer incentives to use them. Give an example of such an incentive and describe how it works. [2 marks]
- (c) Other than interest what other costs can be associated with a credit card? [1 mark]
- 10. You own a car that consumes diesel fuel at a rate of 6.7L/100km. On a 2500km trip to the East Coast you estimate that the average fuel price will be \$0.93/L. What will be your estimated fuel cost to drive the 2500km trip? [2 marks]

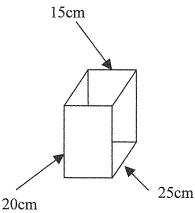
11. Many people struggle over the decision of acquiring a vehicle. Usually the three main choices involve buying a used vehicle, buying a new vehicle or leasing a new vehicle. In the table below provide two positive aspects (pros) and two negative aspects (cons) for each of the options.

[6 marks]

	Buying Used	Buying New	Leasing New
Pros			
Cons			

12. Using the grid below, and the scale of 1 grid square = 5cm, create a net for the rectangular prism shown. [3 marks]

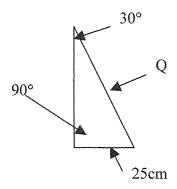




13. A surveyor is marking a rectangular plot which is scheduled for development. Standing on one corner the surveyor uses a laser pointer to determine that the diagonal corner of the rectangular plot lies 300m away at an angle of 40 degrees to the short side of the rectangle. Draw a diagram, and find the dimensions of the rectangular plot. [4 marks]

14. Solve for the measure of Q

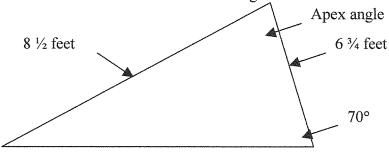
[3 marks]



- 15. **(SELECTING TOOLS...)** Captain Jack Sparrow is anchored at Isle de los Nombres 40 miles north of Tortuga when he spies billowing smoke in the distance, at a bearing of N60°W. He sets sail in that direction and, 20 miles later, finds the charred remains of a fishing vessel. As he mutters "Scurvy Pirates!" to himself, he wonders how long it will take him to get back to Tortuga.
- (a) Sketch the positions of Tortuga, Isle de los Nombres and the burnt fishing vessel in a correctly labeled diagram [2 marks]
- (b) Calculate how far the fishing vessel is from Tortuga

[2 marks]

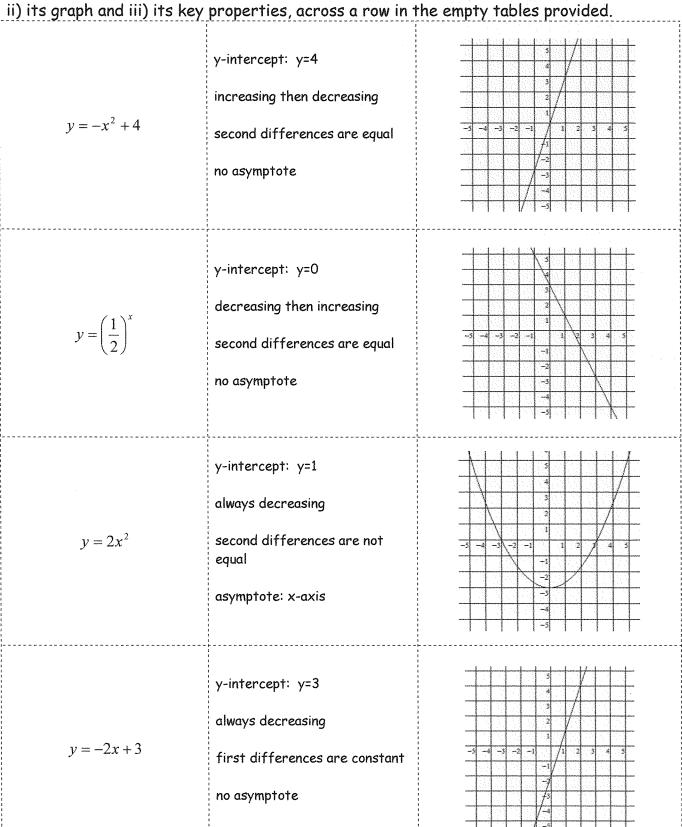
- (c) Captain Jack's ship can sail at 16mph. How many minutes will it take for him to reach Tortuga? [1 mark]
- 16. The carpenter on the Curb Appeal home improvement program wants to change the roof of a garden shed to give it more... well... curb appeal. He designs the roof shown in the diagram below but then remembers that an apex angle greater than 70° requires additional truss structures for stability. These additional truss structures are expensive and the homeowner wants to avoid any unnecessary cost. Will the carpenter need to redesign the roof? Justify your answer with mathematical reasoning. [3 marks]



SPACE FOR FORMULA SHEET

LINEAR, QUADRATIC AND EXPONENTIAL MATCHING CHALLENGE

Cut out the squares along the dotted lines. Assemble the puzzle to match i) the equation ii) its graph and iii) its key properties, across a row in the empty tables provided.



Unit 3 – Exponential Relations

$y = 6(2^x)$	y-intercept: y=6 always increasing second differences are not equal asymptote: x-axis	-5 -4 -3 -2 -1 1 2 3 4 5 -1 -2 -3 -4 -4 -4 -5 -5 -4 -5 -5 -5 -4 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5
$y = \frac{1}{3}x^2 - 3$	y-intercept: y=-2 always increasing first differences are constant no asymptote	-5 -4 -3 -2 -1 s 2 3 4 5 -1 -2 -3 -3 -4 -5 -5
y = 3x	y-intercept: y=6 always decreasing second differences are not equal asymptote: x-axis	5 4 4 7 3 7 2 7 1 3 2 3 4 5 7 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
y = 3x - 2	y-intercept: y=0 always increasing first differences are constant no asymptote	-6 -5 -8 -3 -2 -3 1 2 3 4 5 -1 -2 1 -2 1 -2 1 -2 1 -2 1 -2 1 -2 1

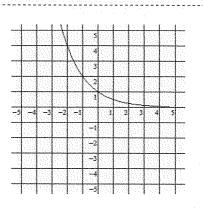
٠.		\mathbf{a}^{x}
ν	=	2"

y-intercept: y=-3

decreasing then increasing

second differences are equal

no asymptote



$$y = 6\left(\frac{1}{2}\right)^x$$

y-intercept: y=1
always increasing
second differences not equal
asymptote: x-axis

